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PROBABILISTIC ASSESSMENT AND SENSITIVITY ANALYSIS OF EXISTING CONCRETE BRIDGE VIA SURROGATE MODELING

Lukáš Novák*, Drahomír Novák

Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic

* corresponding author: novak.l@fce.vutbr.cz

Abstract. The paper is focused on probabilistic assessment and sensitivity analysis of existing prestressed concrete bridge using surrogate model in form of Polynomial Chaos Expansion (PCE). The bridge was selected in the framework of the European Project INTERREG AUSTRIA-CZECH REPUBLIC "TCZ190 SAFEBRIDGE" focused on advanced numerical analysis of existing bridges represented by non-linear finite element model. In this study, surrogate model in form of PCE was created, which represents very efficient type of surrogate model. One of significant advantages of PCE is powerful post-processing including sensitivity and moment analysis of the response, which is important part of probabilistic analysis. The obtained numerical results of advanced stochastic analysis consisting of uncertainty quantification and sensitivity analysis of the existing bridge structure are presented in the paper.

Keywords: Concrete structures, polynomial chaos expansion, sensitivity analysis, statistical analysis.

1. Introduction

The development of computational methods for civil engineering has become more important than ever, since it is often necessary to employ advanced numerical methods for the design of new structures in order to fulfil the significantly increasing economical and safety requirements in the last decades. Moreover, there are a lot of structures, especially bridges, built in the last century, which must often be enhanced for higher loads assuming actual conditions of the structures. As a result of these industrial needs, researchers and civil engineers are more interested in advanced numerical methods to solve the mathematical models of structures – typically non-linear finite element method (NLFEM). Although NLFEM is a very accurate numerical method for solving differential equations, there is still a lack of knowledge of material characteristics (e.g. fracture energy), actual geometrical properties (e.g. position of reinforcement) and even mathematical models of some physical phenomena (e.g. fracture mechanics of quasi-brittle materials) collectively called uncertainties. As can be seen from the given examples, uncertainties play an important role, especially in the case of concrete structures. This lack of knowledge may generally lead to inaccurate results and even fatal failures despite the advanced numerical analysis performed by NLFEM.

In modern structural analysis, uncertainties are represented by random variables described by specific probability distribution, the structural system can then be seen as a mathematical function of a set of random parameters. Deterministic numerical analysis of structures must thus be enriched by stochastic analysis. The elementary task of stochastic analysis is to propagate uncertainties through a mathematical model in order to obtain statistical and/or sensitivity information of quantity of interest (QoI). Results of statistical analysis are important especially for semi-probabilistic design and assessment of structures, since it is necessary to estimate design quantile of structural resistance, which fulfils given safety requirements. Unfortunately statistical analysis of complex mathematical models (e.g. bridges) is highly computationally demanding or even not feasible in industrial applications, since the statistical analysis typically consists of large number of repetitive deterministic calculations.

On the one hand, it is possible to reduce the number of simulations as much as possible by simplified design methods such as Taylor Series Expansion or methods for estimation of coefficient of variation (ECoV). On the other hand, highly computational requirements per simulation can be significantly reduced by the surrogate model, which can be used as a computationally cheap approximation of the original mathematical model. Although there are various types of surogate models such as Support Vector Machine, Krigging or Artificial Neural Networks, it is beneficial to use approximations which can be easily used for analytical post-processing such as Polynomial Chaos Expansion (PCE) originally proposed by Norbert Wiener [1] offering efficient post-processing.

In this paper PCE is employed for sensitivity and statistical analysis of an existing prestressed concrete bridge. The bridge consists of 16 precast post-tensioned bridge girders and it is loaded by exceptional load according to national annex in order to estimate design value of resistance. The bridge is represented by NLFEM based on theory of non-linear fracture mechanics of quasi-brittle material. The obtained results and applied methodlogy might be interesting for civil engineers as well as for scientist dealing with semi-probabilistic analysis of existing structures.

2. Polynomial Chaos Expansion

Evaluation of mathematical model of QoI is often highly computationally demanding and thus it is necessary to create an efficient approximation. One of the most popular approach is PCE [1], which represents the output variable Y as a polynomial expansion g^{PCE} of an another random variable ξ called the germ with given distribution

$$Y = g(X) \approx g^{PCE}(\xi), \tag{1}$$

A set of polynomials, orthogonal with respect to the probability distribution of the germ, are used as a basis functions. The orthogonality condition for all $j \neq k$ is given by the inner product defined for any two functions ψ_j and ψ_k with respect to the probability density function of ξ

$$\langle \psi_j, \psi_k \rangle = \int \psi_j(\xi)\psi_k(\xi)p_\xi(\xi)\,\mathrm{d}\xi = 0. \tag{2}$$

Polynomials ψ orthogonal with respect to a selected probability distributions p_ξ can be chosen according to Wiener-Askey scheme [2] or created directly by Gram-Schmidt orthogonalization. In this paper we use normalized polynomials with inner product equal to the Kronecker delta δ_{jk}.

In the case of $\boldsymbol{X}$ and $\boldsymbol{\xi}$ being vectors containing M random variables, the polynomial $\Psi(\boldsymbol{\xi})$ is multivariate and it is built up as a tensor product of univariate orthogonal polynomials. The quantity of interest (QoI), i.e. the response of the mathematical model $Y = g(\boldsymbol{X})$, can then be represented, according to Ghanem and Spanos [3], as

$$Y = g(\boldsymbol{X}) = \sum_{\boldsymbol{\alpha}\in\mathbb{N}^M} \beta_{\boldsymbol{\alpha}}\Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}), \tag{3}$$

where $\boldsymbol{\alpha} \in \mathbb{N}^M$ is a set of integers called the *multi-index*, $\beta_{\boldsymbol{\alpha}}$ are deterministic coefficients and $\Psi_{\boldsymbol{\alpha}}$ are multivariate orthogonal polynomials.

Naturally, the approximating function given by Eq. (3) must be truncated to a finite number of terms P using e.g. total-order truncation by retaining only terms whose total degree $|\boldsymbol{\alpha}|$ is less than or equal to a given p:

$$\mathcal{A}^{M,p} = \left\{ \boldsymbol{\alpha} \in \mathbb{N}^M : |\boldsymbol{\alpha}| = \sum_{i=1}^{M} \alpha_i \leq p \right\}. \tag{4}$$

In case of high p and M, it possible to use additional "hyperbolic" reduction of the truncated set [4].

2.1. Non-intrusive approach

Truncated PCE can be seen as a linear regression model with deterministic coefficients $\boldsymbol{\beta}$, which can be thus obtained by ordinary least square (OLS) regression. Estimated $\boldsymbol{\beta}$ thus minimize the sum of the squares of the differences between the results of original mathematical model $\mathcal{Y}$ corresponding to he input random vector $\boldsymbol{X}$ together called the experimental design (ED) and the results of surrogate model. Specifically, the vector of deterministic coefficients $\boldsymbol{\beta}$ is calculated using data matrix $\boldsymbol{\Psi}$ as

$$\boldsymbol{\beta} = (\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\ \boldsymbol{\Psi}^T\mathcal{Y}. \tag{5}$$

The number of deterministic coefficients is directly connected to P, generally dependent on the number of input random variables M and the maximum total degree of polynomials p as can be seen in Eq. 4. Unfortunately, this leads to computationally highly demanding problems in case of large stochastic non-linear models. In order to reduce P, it is possible to select the best model represented by sparse set of basis functions. The best model selection is a broad scientific topic and several methods were proposed, here we use Least Angle Regression (LAR) [5] to find an optimal set of PCE terms as proposed by Blatman and Sudret [4].

2.2. Approximation Error Estimation

Naturally, it is necessary to measure the approximation error of PCEṠuch a measure can be further used for construction/selection of the best surrogate model. However, it might be highly computationally demanding to create a validation set containing several calculations of the original mathematical model. Therefore, it is beneficial to utilize accuracy measures, which do not need any additional simulations. Commonly used technique is the coefficient of determination R^2, which is well known from machine learning. However, this measure often leads to over-fitting and thus scientists are focused on more advanced techniques. The robust, computationally efficient and generally reliabile estimator is the leave-one-out cross validation error Q^2. The estimated error is based on residuals between predictions of the surrogate model and the results of original mathematical model measured on ED, while excluding one realization in construction of surrogate model. The errors are calculated for all realizations in ED and further the average error is estimated. It is clear that iterative process of PCEconstructions for all realizations in ED can be computationally demanding. Fortunately in case of PCE, it is possible to get Q^2 analytically from a single PCEbased on all realizations in ED as follows [6]:

$$Q^2 = 1 - \frac{\frac{1}{n_{\mathrm{sim}}}\sum_{i=1}^{n_{\mathrm{sim}}}\left[\frac{g\left(\boldsymbol{x}^{(i)}\right) - g^{\mathsf{PCE}}\left(\boldsymbol{x}^{(i)}\right)}{1-h_i}\right]^2}{\sigma^2_{Y,ED}}, \tag{6}$$

where $\sigma^2_{Y,ED}$ is a variance of experimental design obtained from results of the original mathematical model and h_i represents the ith diagonal term of the matrix $\mathbf{H} = \boldsymbol{\Psi}\left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^T$.

2.3. Statistical Moments

The PCE is famous for statistical analysis thank to its powerful and efficient post-processing allowing for analytical derivation of statistical moments of the QoI. The mean value is obtained from general formula of the first statistical moments as

$$\mu_Y = \langle Y^1 \rangle = \sum_{\boldsymbol{\alpha}\in\mathbb{N}^M} \beta_{\boldsymbol{\alpha}} \int \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})\, p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi}. \quad (7)$$

Considering the orthonormality of the polynomials

$$\int \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}) p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} = 0\, \forall \boldsymbol{\alpha} \neq 0, \quad \Psi_0 \equiv 1,$$

the original integration is reduced to simple post-processing of the PCE deterministic coefficients. Namely, the mean value is equal to the first deterministic coefficient of the expansion

$$\mu_Y = \langle Y^1 \rangle = \beta_{\mathbf{0}}. \quad (8)$$

The second raw statistical moment, $\langle Y^2 \rangle$, is written as

$$\langle Y^2 \rangle = \sum_{\boldsymbol{\alpha}\in\mathcal{A}} \beta_{\boldsymbol{\alpha}}^2 \int \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})^2 p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\ \mathrm{d}\boldsymbol{\xi} = \sum_{\boldsymbol{\alpha}\in\mathcal{A}} \beta_{\boldsymbol{\alpha}}^2 \langle \Psi_{\boldsymbol{\alpha}}, \Psi_{\boldsymbol{\alpha}} \rangle. \quad (9)$$

Similarly as in case of the mean value, it is possible to obtain the variance as the sum of all squared deterministic coefficients except the intercept (which represents the mean value), i.e.

$$\sigma_Y^2 = \sum_{\substack{\boldsymbol{\alpha}\in\mathcal{A}\\ \boldsymbol{\alpha}\neq\mathbf{0}}} \beta_{\boldsymbol{\alpha}}^2. \quad (10)$$

3. Sensitivity analysis

Once the PCE approximation is constructed, it is also possible to obtain sensitivity measure of input random variables. Although there are many sensitivity techniques beneficially coupled with surrogate models [7], the following two common and efficient techniques are employed in this paper: non-parametric rank-order correlation obtained by MC simulation and Sobol indices derived directly from PCE coefficients. Results of both techniques represent different information and thus they should be combined in order to correctly investigate the influence of input random variables.

3.1. Non-parametric Rank-order Correlation

The traditional sensitivity analysis method in statistics is represented by the correlation between an input variable and the quantity of interest of mathematical model. Although standard measurement via the Pearson correlation coefficient is simple and efficient enough for linear monotonic dependency, it is necessary to utilize a generalized measure for nonlinear monotonic relationships called the non-parametric Spearman rank-order correlation technique [8]. Obtained correlation coefficient ρ is in an interval $\langle -1, 1\rangle$. The higher absolute value of ρ corresponds to the stronger relationship between the two variables. If it is positive, then as an input variable increases, the QoI tends to increase. If it is negative, then as an input variable increases, the QoI tends to decrease.

3.2. Sobol Indices

One of the most important tasks in uncertainty quantification is the analysis of variance "the analysis of the influence of input variables on the variance of a mathematical model. Such information may be utilized to practically reduce the uncertainty of important input variables (material characteristics) used in mathematical model by experiments and measurements, which leads to a significant reduction in the uncertainty of the quantity of interest. Herein, the well-known ANOVA method represented by Sobol indices is employed. The Sobol indices method is widely used and well developed method for sensitivity analysis. However, it is still highly computationally demanding to evaluate Sobol indices via the classical double loop Monte Carlo method. Fortunately, there is a connection between PCE and the Hoeffding-Sobol decomposition [9] allowing for analytical derivation of Sobol indices.

PCE can be rewritten in the form of the Hoeffding-Sobol decomposition by a simple reordering of the terms:

$$g^{PCE}(\mathbf{x}) = \beta_0 + \sum_{\alpha\in A_{\mathbf{u}}} \beta_\alpha \Psi_\alpha(\xi), \quad (11)$$

where the set of basis multivariate polynomials dependent on selected input random variables $\mathbf{X_u}$ is

$$A_{\mathbf{u}} = \left\{\alpha \in A^{M,p} : \alpha_k \neq 0 \leftrightarrow k \in \mathbf{u}\right\}. \quad (12)$$

Therefore, the first order Sobol indices can be analytically obtained directly from PCE as follows [9]:

$$S_i = \frac{\sum_{\alpha\in A_i} \beta_\alpha^2}{\sigma_Y^2}, \quad (13)$$

where basis functions are selected as:

$$A_i = \left\{\alpha \in A^{M,p} : \alpha_i > 0, \alpha_{j\neq i} = 0\right\}. \quad (14)$$

Important information about the influence of input variables and all interactions can by expressed by total Sobol indices representing the first order influence and influence of all interactions, which can be obtained as

$$S_i^T = \frac{\sum_{\alpha\in A_i^T} \beta_\alpha^2}{\sigma_Y^2}, \quad (15)$$

where basis functions are selected as:

$$A_i^T = \left\{\alpha \in A^{M,p} : \alpha_i > 0\right\}. \quad (16)$$

Note that, the above expressions represent just a selection of the specific PCE coefficients associated to selected input random variable.

4. Post-tensioned Concrete Bridge

The PCE is employed for probabilistic assessment of an existing concrete bridge. The PCE is utilized as an alternative to semi-probabilistic assessment partly addressed in the simplified case-study of the selected bridge during the European Project INTERREG AUSTRIA-CZECH REPUBLIC "ATCZ190 SAFEBRIDGE". [20]. The bridge consists of three spans constructed from 16 bridge girders KA-61 in transverse direction. The crucial part of the bridge for assessment is the mid-span: 19.98 m long with total width 16.60 m. The geometry of a typical bridge girder KA-61 is created according to an original documentation describing also positions of reinforcement and tendons. The drawing together with the simplified cross-section is depicted in Fig. 1.

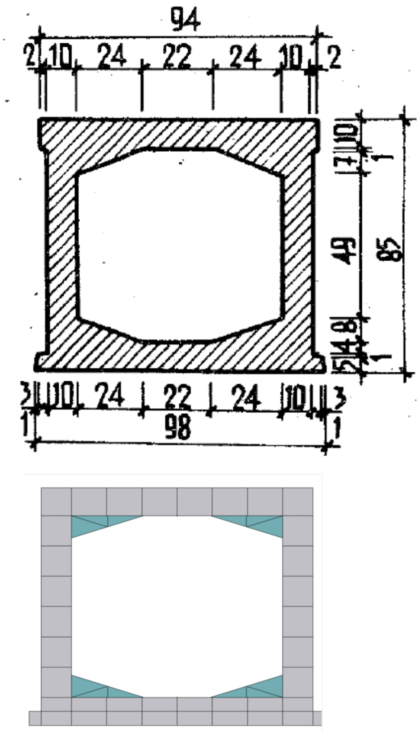

Figure 1. Cross-section of a singe bridge girder KA-61.

From structural point of view, it was necessary to create numerical model of the whole bridge span. The reason is that although the structure is symmetric, the individual bridge girders are not transversely prestressed, which leads to the different deflection of each girder in dependence on their distance to the loading position. In order to create numerical model reflecting their real connection conditions, the girders are connected by reinforcement according to original documentation together with a concrete mixture between single girders.

4.1. Finite Element Model

The cross-sections of girders KA-61 were simplified to regular shapes in order to reduce number of finite elements and to obtain regular mesh, see Fig. 1. Boundary conditions are assumed to be as a simply supported beam with elastic blocks. The geometry of elastic blocks and positions of loading plates are modeled according to bridge documentation and a national annex of Eurocode for load-bearing capacity of road bridges by exclusive loading (by six-axial truck).

The NLFEM is created in software ATENA Science including theory of non-linear fracture mechanics [10]. In order to reflect complex behavior of the bridge, the numerical model contains three construction phases as illustrated in Fig. 2:

(1.) prestressing of bridge girders and simultaneous application of the self-weight;

(2.) activating of the pavement and concrete among girders connecting bridge girders;

(3.) application of a load by a six-axial truck.

The major part of NLFEM is represented by 13,000 elements of hexahedra type and triangular ‘PRISM’ elements in the blue-colored parts of the cross-section (see Fig. 1). Hexahedra elements lead to better numerical stability of simulation and leads to easier construction of mesh compatible between two volumes connected by fixed contact, i.e. nodes of elements in both connected sub-volumes have same coordinates. The advantage of brick elements is that the structured mesh constructed from brick elements leads to a significantly lower number of finite elements in comparison to tetrahedra elements. Fracture-mechanical behavior of concrete is described by a non-linear mathematical model [10]. Reinforcement together with tendons are modeled as discrete 1D elements with positions and shape according to the original documentation.

The numerical model is analysed in order to investigate the following three limit states of the bridge:

(1.) the ultimate limit state (ULS) (peak of a load-deflection diagram);

(2.) the first occurrence of cracks in bridge girders (cracking);

(3.) the serviceability limit state of decompression defined according to Eurocode (SLS).

Note that obtained results are further reduced by dynamic amplification factor $\delta = 1.4$ in order to reflect that results are from static analysis.

4.2. Stochastic Model

The stochastic model contains 4 random material parameters of a concrete C50/60: Young’s modulus E ; compressive strength of concrete f_c; tensile strength of concrete f_{ct} and fracture energy G_f. Characteristic values of E, f_{ct}, G_f were determined from f_c according to formulas implemented in the fib Model Code 2010 [11] (G_f, E) and prEN 1992-1-1: 2021 (f_{ct}). The last random variable P represents prestressing losses with CoV according to JCSS: Probabilistic Model Code [12]. The stochastic model is summarized in

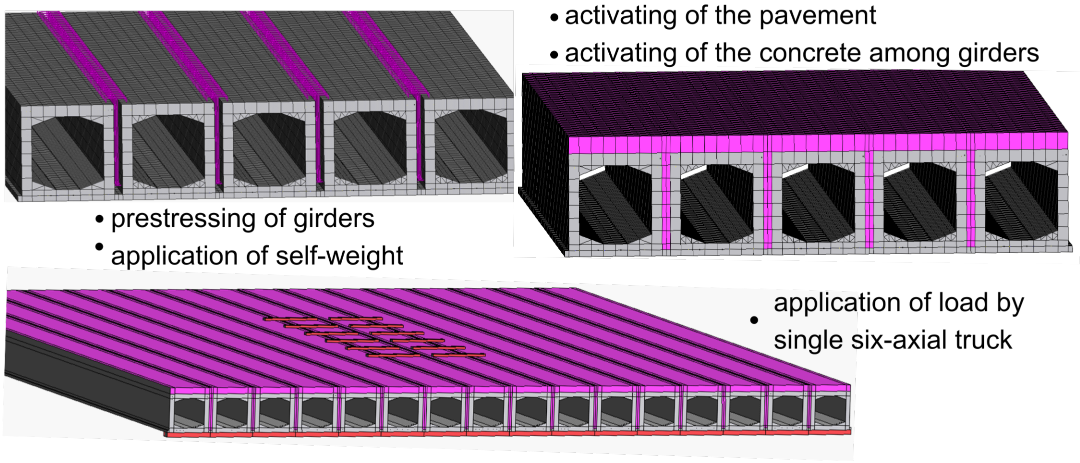

Figure 2. Three construction phases of the bridge represented by NLFEM.

Tab. 1. Mean values and coefficients of variation were obtained according to prEN 1992-1-1: 2021 (Annex A) for adjustment of partial factors for materials.

Var.	Mean	CoV [%]	Distrib.	Units
f_c	56	16	Lognormal	[MPa]
f_{ct}	3.64	22	Lognormal	[MPa]
E	36	16	Lognormal	[GPa]
G_f	195	22	Lognormal	[Jm2]
P	20	30	Normal	[%]

Table 1. Stochastic model of the numerical example.

In this first numerical study, there is an assumption of uncorrelated random variables.Though the correlation might play crucial role in case of concrete structures.

4.3. Numerical Results

The ED contains 30 numerical simulations generated by Latin Hypercube Sampling (LHS) [13] together with corresponding results of NLFEA. Note that each simulation takes approximately 24 hours. The PCE is created with maximal polynomial order $p = 5$ with the LAR algorithm for a selection of the best set of basis functions. The whole algorithm of adaptive construction of PCE connects state of art techniques into stand-alone software tool [14]. Obtained mean, CoV and design values together with PCE accuracy measured by Q^2 are summarized for all limit states in Tab 2. The design values of resistance R_d for each limit state in tons are determined as a quantile of Lognormal distribution with identified statistical moments and target reliability indices $\beta_{ULS} = 3.8$, $\beta_{crack} = 3.8$ and $\beta_{SLS} = 1.5$ according to EN 1990 [15]. Additionally, design values are reduced by global safety factor reflecting model uncertainties $\gamma_{R_d} = 1.06$ introduced originally in fib Model Code 2010 [11].

Once the PCE is created, it is also possible to easily obtain sensitivity indices as described in Section 3.2. Obtained sensitivity measures are summarized in Tab. 3.

	Mean	CoV [%]	Q^2	R_d
ULS	480	7.2	0.91	365
Cracking	400	8.4	0.96	290
SLS	150	14.1	0.98	120

Table 2. Obtained statistical moments and PCE accuracy for each limit state.

5. Discussion and Further Research

As can be seen from obtained accuracy, all three limit states are well approximated by PCE and thus one could use obtained statistical moments for derivation of design values of resistance. It can be seen, that a critical limit state (the lowest R_d) is represented by limit state of decompression, which is typical for prestressed structures. It is interesting that SLS has the highest CoV, though it is almost linear limit state. This could be explained by results of sensitivity analysis revealing that P (which has a high CoV) has absolutely dominant influence. The ULS and cracking limit state have lower CoV since there is also a signficant influence of concrete material characteristics and their interactions.

It is clear that sensitivity analysis plays important role in the probabilistic analysis of structures, however, it is important to understand obtained information from different types of sensitivity analysis [7]. Moreover, correlation among material characteristics may have a significant influence on obtained results [16] and thus further work will be focused on analysis assuming realistic correlation matrix of concrete material parameters similarly as in previous work of authors of this paper [17, 18]. Although $Q^2 > 0.9$ for all limit states, it should be improved in further work in order to obtain more reliable results, especially for sensitivity analysis. The recently proposed adaptive

	ULS			Cracking			SLS		
	S_i	$S_{T,i}$	ρ	S_i	$S_{T,i}$	ρ	S_i	$S_{T,i}$	ρ
f_{ct}	0.00	0.01	0.23	0.00	0.15	0.31	0.00	0.05	0.23
f_c	0.00	0.10	0.15	0.00	0.01	0.05	0.00	0.01	0.10
G_f	0.00	0.10	0.16	0.00	0.01	0.06	0.00	0.01	0.05
E	0.00	0.01	0.07	0.06	0.07	0.45	0.00	0.05	0.27
P	0.90	0.95	-0.90	0.77	0.90	-0.85	0.95	0.95	-0.95

TABLE 3. Results of sensitivity analysis via PCE. ρ represents non-parametric rank order correlation by Spearman coefficients, S_i first-order Sobol indices and $S_{T,i}$ total-order Sobol indices.

sequential sampling will be employed [19] for efficient extension of existing ED.

6. Conclusions

The paper presents practical application of PCE for probabilistic assessment of an existing prestressed concrete bridge. The bridge is represented by highly computationally demanding NLFEM reflecting theory of non-linear fracture mechanics of concrete. The stochastic model contains 5 random variables representing concrete characteristics and prestressing losses. The PCE is created by adaptive best model selection algorithm LAR (non-intrusive approach) with ED containing 30 samples generated by LHS. PCE was created for three limit states, which were further analysed in order to obtain statistical moments and sensitivity indices. The obtained statistical moments are used for the estimation of design values of resistance.

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