

THE ROLE OF MULTI-FIDELITY MODELLING IN ADAPTATION AND RECOVERY OF ENGINEERING SYSTEMS

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ABSTRACT. Significant research has been conducted in identifying optimal recovery and adaptation decisions in disruptive scenarios using engineering models. In this context, an aspect that has been target of limited research is that of response times. Modelling is expected to grow progressively more complex as it becomes more accurate. Such complexity increases modelling efforts, and the promise of optimal adaptation and recovery may become hindered. The present work discusses the role of modelling fidelities in adaptation and recovery of systems, and in particular that of using a lower fidelity model that enables zero-time analyses of a system. A framework is proposed for using different fidelities in adaptation and recovery, considering system’s decision time requirements. The relevance of this analysis is researched in two traffic networks and results show that multi-fidelity models should be expected to play a key role in increasing the efficiency of optimal adaptation and recovery decisions.

KEYWORDS: Metamodels, multi-fidelity model, system adaptation, system recovery, systems.

1. INTRODUCTION

Adaptation and response of civil engineering systems has become one of the most prominent topics of research in recent years. Several methodologies and approaches have been proposed to tackle the need for adaptation and responsiveness, however, most of them fail to discuss one of the most relevant aspects of the system’s responsiveness, the decision time.

Engineering systems are complex in nature. They depend on multiple variables that can interact together and that originates a need for these to be treated as holistic. It is with no surprise that system analysis is recurrently highly complex, relying on intensive data analytics or costly modelling techniques. Due to this optimal decision-making for civil engineering systems can become a resource intensive task.

Most approaches for adaptation and response to natural or man-made disasters lay their foundation on the premise that under a disruption an optimal adaptation decision making scheme can be found that will maintain the functionality of a system at certain minimum operational level. This can be achieved through intervention, by adapting the system operational variables, or other alternative system approaches [1]. In all cases, some form of technique needs to inform decision making, and current trends of research indicate that there is a growing interest in the application of high-fidelity (HF) modelling as a tool to inform decisions. This is identified in the progressive interest for HF tools such as, Finite-Element Methods, and

approaches, such as digital twinning or prediction of complex adaptivity in systems.

Most of these techniques were introduced in order to enable prediction and decision-making that is as close as possible to real-time response. However, in practice, running a decision-making scheme in such HF models can be effort consuming. In the instance of needing to use real-time information and prediction of the system operation to adapt and prevent any perturbation a form of parametric analysis needs to be performed. As this procedure will still depend on the HF model built to have a accuracy on predicting the system’s response, then performing any analysis that will establish a new point of operation that maintains (or optimizes) its functionality is expected to have large cost.

Within this context, the present paper discusses the role that multi-fidelity (MF) techniques may have in efficient adaptation of engineering systems. MF, that is, a blend of HF and Low-Fidelity (LF) modelling, may allow to balance the effort that is required to set optimal adaptation and recovery decisions. In particular, decision-times are expected to have a significant influence on the efficiency of the decision-making schemes. In the case of adaptation and recovery of an engineering system to a disruptive event, stalling a decision for a time longer than strictly necessary may set the boundary between significant or minor impacts in the system. In order to discuss the role of MF models in the context described, Section 2 introduces the idea of MF modelling, Section 3 discusses a framework

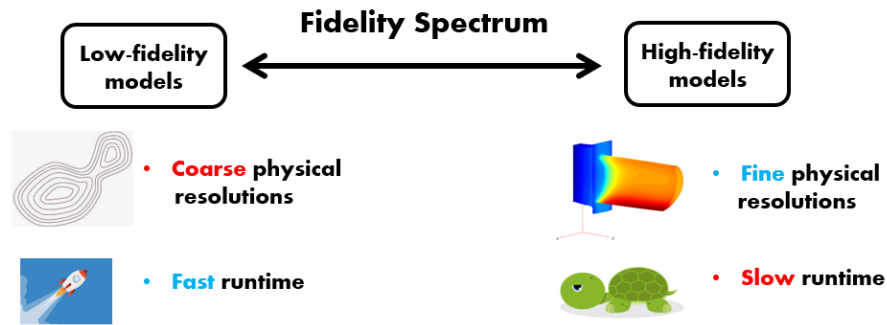


FIGURE 1. Multi-fidelity, from low-fidelity to high-fidelity models [2].

of application to adaptation and recovery of systems, Section 4 presents two examples of application to a traffic network, and finally, Section 5 draws the main conclusions of the work developed.

2. MULTI-FIDELITY (MF) MODELLING

MF modelling has been widely researched in order to tackle the increasing analysis costs that are imposed by progressively more complex and accurate modelling techniques. The fundamental idea of MF is to combine HF models (commonly characterized as of high-resolution, high analysis cost), with LF models (commonly characterized as of lower resolution, low computational cost), see Figure 1. HF models are expected to have better accuracy in practice, however, this does not mean that LF models cannot achieve adequate accuracy. In MF, both HF and LF are applied to balance the amount of effort required to perform an analysis based on the principle that an adequate amount of HF data may suffice an accurate analysis if complemented by LF data. Such usage of different levels of fidelity has been previously researched and identified to have an important role in fields that are characterised by large analysis efforts such as, uncertainty quantification and optimization [3, 4]. It should be expected to become relevant in all engineering analyses that depend on effort consuming models, and where the time necessary to inform decision-making schemes has a key role.

2.1. FORMS OF MULTI-FIDELITY MODELLING

MF models can appear in different forms, and the level of fidelity depends on a series of factors. It can be said that HF models represent the behaviour of a system with an adequate level of accuracy, and that LF models are frequently a less refined representation of these. LF models can be created by assuming simplified forms for HF models. This can be achieved through dimensionality reduction, linearisation, simpler physics models, coarser domains, partially converged results, among other [5]. [5] highlight also different forms in which a system can be distinguished in terms of levels of fidelity. These may depend on the physical assumptions (*e.g. may use different theories*), numerical assumptions (*e.g. more refined grids*

or different computational models that use same underlying theory), and also on the simple nature of the information (*e.g., experimental versus simulations*).

One alternative that has captivated interest in the field of MF modelling is metamodelling. It consists in creating surrogates of a function or model by characterizing the relationship between inputs and outputs and relies on metamodels as LF pairs to HF data [6]. An overview of popular metamodelling techniques is provided in [7], and [8] provides insight on the usage of MF paired with metamodelling. Different metamodels can be applied in this context. The interested reader is directed to the extensive literature on this topic [7] for a review of some examples. In the present work a kriging model is applied to discuss the relevance of MF alternatives in adaptation and recovery of systems.

3. INTEGRATION OF MF MODELLING IN ADAPTATION AND RESPONSE OF ENGINEERING SYSTEMS

Effective system response or adaptation is expected to rely on quick responses to damaging or disruptive events. When an informative state of the system is acquired, if decisions that are based on this state are not implemented fast enough, then this informative state may be lost as the system evolves. Moreover, if an optimal decision of recovery or adaptation that was based on this informative state is later obtained; then there is a large likelihood that it won't be optimal as the system evolved. To add that when an informative state of a system that is being affected by a disruption exists, from the perspective of the decision-maker, it is very difficult to not act and wait for an optimal evaluation of the decision-making scheme, even if this would be the most beneficial choice to achieve efficient decisions. As a result if an optimization for adaptation or recovery is pursued that aims at optimal decision-making, and if the results are delayed by running optimal searches for the decision variables in HF techniques, then loss of efficiency should be expected.

In order to illustrate the importance of these aspects, research on MF applications is implemented in a traffic network. Two networks are used to research on the effectiveness of MF in contexts of, respectively, recovery and adaptation. The Nguyen-Dupuis and

Sioux-Falls traffic networks are used. A more detailed description of the network parameters used in these can be found in [9]. The MF model applied combines as HF the traffic network user equilibrium, a non-linear model that determines the optimum state of equilibrium for the network users; and as LF model a kriging model, a surrogate of the HF model.

4. DISCUSSION ON RECOVERY AND ADAPTATION OF A TRAFFIC NETWORK TO A DISRUPTIVE EVENT

The MF relationship between the HF and LF in the contexts of adaptation and recover is highlighted in Figure 2. Two essential notions of time under decision-making schemes can be highlighted in this context:

- *slow-time*: where time is not key for the effectiveness of overall decision-making response.
- *fast-time*: where time has critical role in the overall effectiveness of the decision-making scheme.

A metamodeling discussion that uses a similar idea was previously implemented in the research of digital twinning in [10].

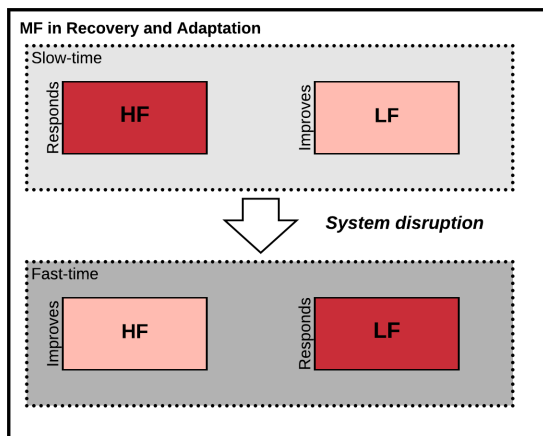


FIGURE 2. Framework for MF application in adaptation and response.

For most of the operation of a system, decision-making can be considered to be in slow-time. Any modelling of decision-making in such circumstance can rely in HF models and limited change to steady conditions should be expected. In this phase, HF can predict outcomes for the system at limited cost. Effectiveness will be largely unaffected by decision-times as the pairing of the system modelling with its operation is continuous. However, under a disruptive scenario the same assumptions about the relationship of decision-making schemes and effectiveness are not expected to hold. In unsteady conditions, the system will be changing, hence, an optimal decision on slow-time is unlikely to be optimal (to adapt or recover the system a decision-making scheme may be needed to

adapt the decision variables). In such scenario optimal decisions should have zero-time and be paired with the system unsteady conditions.

It is within this idea that MF modelling is framed in system recovery and adaptation. MF allows to control time in optimal decision-making, depending on the system demand. In slow-time the multi-fidelity technique can be improved as a predictor of the system response for fast-time. In this phase, which is representative of normal operational conditions, the HF model responds and the LF model improves using HF data (including potential disruptive scenarios).

To construct the MF model for this effect, it is assumed that a HF model $f(x)$ exists such that it is an exact approximation of the system behaviour $S(x)$ for any operational state x , then in slow-time a MF model can be built using metamodeling that includes a LF surrogate $G(x)$ of $f(x)$. This LF can use system inputs under foreseen and unforeseen scenarios of disruption or operation, x . If a global approximation to the space of x can be achieved, then the LF metamodel can act as an accurate predictor of $f(x)$ and $S(x)$ for the whole domain of operation.

Application of the kriging enables LF improvement in slow-time using its properties to enrich the LF model so that it responds accurately in fast-time. In this phase, it is possible to train the LF model in x using HF data. This relies on operational data and exploits the kriging unique Gaussian uncertainty. With kriging as a LF, it is possible to search the operational space x for points of large uncertainty. The LF has then the potential to systematically become a more accurate prediction of the HF model. As a result, in fast-time, the LF swaps to the responding position, and the HF is expected to improve collecting data from unforeseen occurrences. In this phase decision-times are critical, and the LF model, *i.e.* the Kriging, is expected to provide a sufficiently appropriate prediction in order to allow for a *quasi*-optimum (with relation to the LF-HF error) decision-making. Moreover, for any x that does not fulfil the accuracy condition in fast-time a HF sample can be taken to enrich the LF model. In fast-time, samples at HF should be minimized and only extracted when necessary.

Two examples of application, in recovery and adaptation, are studied as reference examples of the relevance of MF in the context described. These are presented in Sections 4.1 and 4.2.

4.1. RECOVERY OF TRAFFIC NETWORK IN A SCENARIO OF DAMAGE

In the first example of application, that concerns recovery of a traffic network, the Nguyen-Dupuis network is used. [11] researched this same traffic network in the context of reliability. It is assumed that a damage affects 7 of its links at 50% of their capacity; links 1,7,8,9,10, 12, 23 as per Figure 3. Then four recovery teams are to be deployed in order to mitigate the effects of this damage and to recover the network

functionality. This recovery will be as efficient as the decision-making that plans the recovery strategy. In practical terms, an efficient recovery should restore the network operational performance to the a stage of *pre-damage* as soon as possible. Such procedure depends on an efficient allocation of resources, which minimizes the loss of functionality in the network and improves its resilience [12, 13]. A damage in the network will increase the cost for users to travel from one node in the network to another, in particular if this trip passes through an affected link. From a decision-maker perspective the interest is to distribute the resources that will efficiently decrease the travel cost. If a damage occurs in the system at $t = 0$ then in this moment the decision-maker should set the optimum strategy in place to restore the network so that in $t=1$ its functionality starts to be recovered.

In the present case the LF model is defined using kriging where for x states of operation, the MF model uses the LF surrogate enriched on the HF model (the network user-equilibrium model) in slow-time. x encloses potential link capacities for the considered links. In the seven-dimensional operational state (*i.e.*, of damaged links), only 28 HF evaluations in slow-time are required to set an accurate LF model with a maximum relative error of prediction of a Latin Hypercube Sample (see example [13, 14]) of 100 points of 5%.

In order to discuss the relevance of the approach proposed, Figure 4 presents the results of recovery using the MF approach. The MF is used in fast-time to predict allocation decisions. The results of this approach are then compared with the result from decision-making that relies exclusively in the HF technique. In both cases a Genetic Algorithm (GA) is used to allocate teams for recovery.

Results show that recovery decisions for the MF approximate with small loss of accuracy the results provided by the HF. MF prediction refers to the value of prediction of the MF model, while MF-DM refers to the effective system recovery after the decision-making takes place, *i.e.*, real evaluation of MF prediction solution in the system. From $t = 1$ to $t = 3$ the MF prediction provides almost the same recovery efficiency as the HF (loss is below 0.1% in total travel cost), while at $t = 4$ there is a slightly larger loss of recovery cost when compared with the GA decision making supported by the HF model. The MF approach, however, consumes a small fraction of the decision time. In Figure 4-(c) it is possible to infer that the GA optimal decision for recovery (team allocation) is achieved in MF for each day in less than 2 seconds, regardless of the GA parameters. Such small decision-time has a large potential to increase the agility of this decision process. Finding the optimal decision with GA relying on the HF technique would impose a delay of the decision-making scheme stalling the recovery process. Decision-making that relies on the HF technique would require at least 25 minutes to achieve

similar results in the most critical days, even considering that this is a relatively low dimensional problem in a relatively simple network. It is possible to infer that the analysis time increases significantly when the optimisation search in the GA uses more points, which is an important consideration for analyses that include more decision variables and that may demand larger population sizes. When the population size of 100 individuals is applied jointly with the convergence of the fitness function for 50 generations, the effectiveness of the HF DM making decreases substantially (decision times increase by more than 3); while in practical terms the MF model is largely unaffected by this. It is still capable of enabling zero-time responses in practice. This may be an important consideration in more involved applications.

4.2. ADAPTATION OF TRAFFIC NETWORK UNDER INCOMPLETE INFORMATION ABOUT THE DAMAGE SCENARIO

The role of MF modelling in a context of adaptation is now discussed in the Sioux-Falls network, see Figure 5. This network includes 76 links and 24 nodes. In the present example, 14 OD pairs (with a demand of 200 users/hour for each OD) are considered in the modelling, each comprising 4 routes. The capacity of each link is set initially to be 50% larger than the OD pair demand.

The idea of the current implementation of MF modelling is to find a representative example of the role of different fidelities in adaptation of a system. In this case decision-making for adaptation should be studied considering a scenario of incomplete information. A damage is known to have occurred in the network, however its extent is uncertain. Existing information indicates that there is a likelihood of 70% that a scenario of minor damage occurred; where the number of affected links ($n(l_d)$) has 25% probability of being $n(l_d) \leq 4$, 35% probability of being $n(l_d) = 5$, 20% probability of being $n(l_d) = 6$, 15% probability of being $n(l_d) = 7$, and 5% probability of being $n(l_d) \geq 8$. In all possibilities the extent of the damage is expected to be 70% of the link capacity. Then, there is a 30% probability of a scenario of high damage having occurred; where there is 30% probability of $(n(l_d)) \leq 6$, 20% probability of $n(l_d) = 7$, a 20% probability of $n(l_d) = 8$, a 15% probability of $n(l_d) = 9$, and 15% probability of $n(l_d) \geq 10$. In all the cases of this scenario, a damage of capacity of 75% is expected. Figure 5 presents potentially affected links (with highlighted link numbers).

A decision should be made about how to adapt the network to this disruption. Decisions that use the HF model are highly accurate, but involve large computational cost. In this example of disruption, with 18 dimensions, that would cover the decision to adapt potentially disrupted link capacities. So, as soon as the decision-maker commits to search for an optimal decision, he will need to perform an optimization

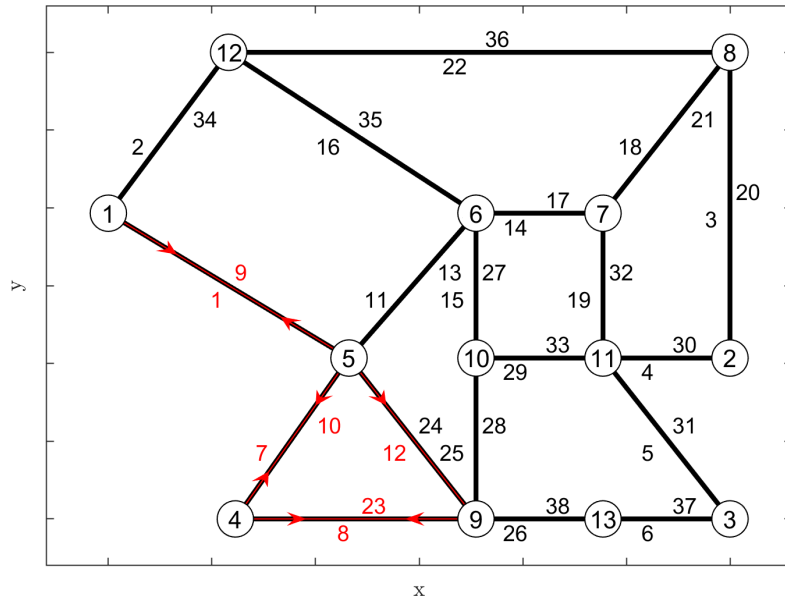


FIGURE 3. Perturbation considered in the Nguyen-Dupuis network. Red arrows represent the links that are damaged.

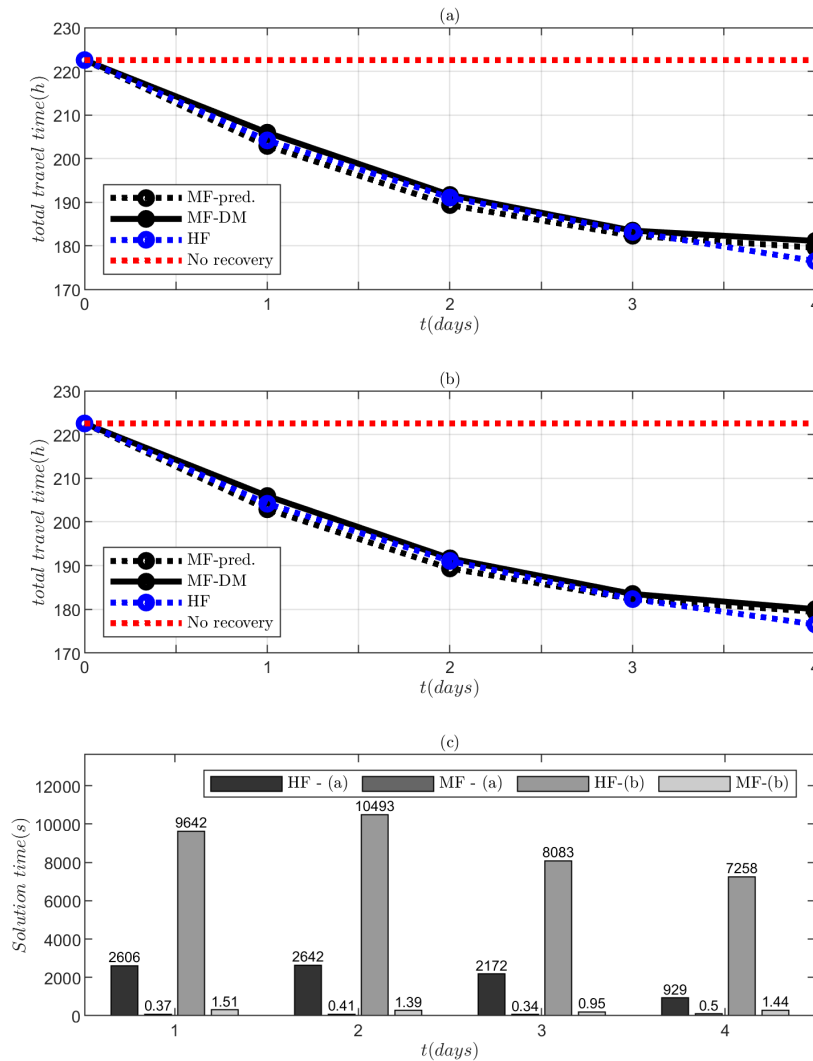


FIGURE 4. Results of the application of the MF approach to the problem of recovery. (a) Results based on a GA team distribution with a population size of 50 and that converge when no improvement in the fitness function occurs for 20 iterations. (b) Results based on a GA team distribution that uses a population size of 50 and converges when no improvement in the fitness function occurs for 20 iterations. (c) Computational times for (a)-(b).

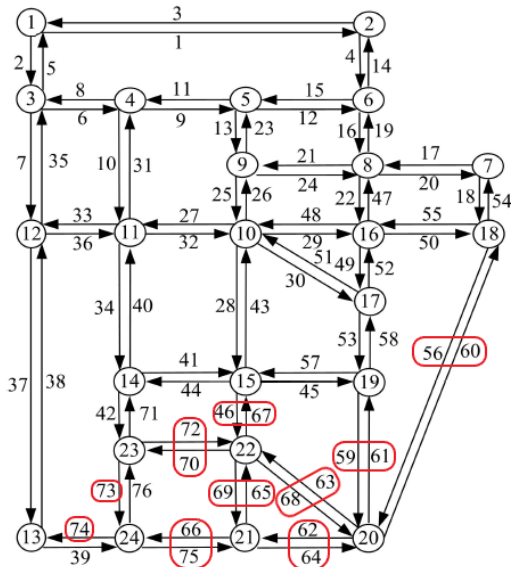


FIGURE 5. Sioux-Falls Network and uncertain scenario of damage simulated. The (red) rectangles identify the links that are potentially damaged in the uncertain disruption.

of the decision criteria on the HF model. There is limited adaptivity to this procedure, as changing the input conditions to it will involve a restart of the optimization calculations.

If a MF model is introduced, then decision-making into adaptation can become adaptive; and because running an optimization scheme on a LF model that is representative of the HF model is virtually cost-free, then, as new information becomes available it is possible to change decisions.

In the case of the Sioux-Falls with 18 potentially damaged links, the LF model is built in slow-time and runs *in tandem* with the HF model considering an x that includes the capacities of each link from a reduction of 80% in relation to the initial capacity to an increase of 50% of the initial capacity. It uses 3205 evaluations of the HF model and is able to validate in average a random global sample within 10% of absolute error in prediction. If new points are required these can be added in slow-time or fast-time.

Figure 6 presents the results for adaptation to the uncertain scenario presented in this example. When the system is damaged it is assumed that a budget of adaptation is available that can add extra capacity to the network links. This budget has a limit 300 users/hour, and any of the potentially damaged links cannot have its capacity enhanced in more than 150 users/per hour. The adaptation decision-making problem involves then finding the optimal usage of this budget to improve the capacity of the potentially damaged links in Figure 6, such that the loss in travel cost in the system is minimized. A Genetic Algorithm is used to allocate the additional capacity in the links considered in order to mitigate the effects of the perturbation. The population size is set to 100

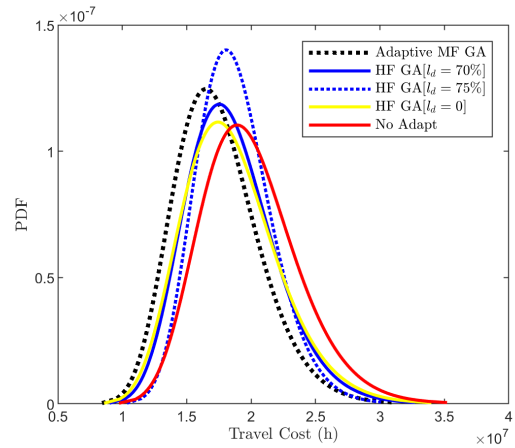


FIGURE 6. Results for the adaptation probability distribution function using HF and MF modelling.

individuals, complemented with convergence of the best fitness value for 50 iterations.

Loss of effectiveness for the adaptation decision is compared in terms of the reduction of travel cost that is possible to achieve with the it (*i.e.* distributing the budget available among links). The black dotted line presents the probability distribution of the response considering predictions using the MF model. Because the MF model is virtually costless to run, the optimal operational point can be determined as new information about the extent of the damage enters the decision-making scheme, that is, an optimal adaptation strategy can be defined for all possible scenarios at virtually no cost (zero-response time). This curve is to the left of the remaining adaptation decisions that rely on HF modelling combined with the same decision technique. Because these rely on the HF model it is not possible to change the input conditions of the decision-making scheme variables with the same level of flexibility. A single optimization to a specific scenario of damage will demand several hours to run. Three scenarios are therefore tested; to optimize the adaptation of the network considering that all links are damaged at 70% of their capacity, optimize the adaptation of the network considering that all links are damaged at 75% of their capacity, and optimize the adaptation of the network considering that no links are damaged. In relation to the case of no adaptation, all contribute to an important improvement of the response; nonetheless, still outperformed by the adaptation with MF. The capability to change *ad-hoc* the decision with new information means total adaptive capacity.

5. CONCLUSIONS

The present work discussed the role that multi-fidelity modelling may have in the future of system analysis, with explicit relation to decision-making in recovery and adaptation. While significant research has been directed to the need for accurate system models that can

accurately inform decision-making, limited research has been directed to the practicality of these as a tool to inform decision schemes. While modelling of systems becomes more complex and accurate, so does the cost of it (effort required). In particular if paired with an optimization scheme, achieving an optimal decision can be quite resource and effort-demanding. In practice, and considering that systems modelling is inherently complex, this may not provide an efficient alternative for decision-making in scenarios of disruption. Decisions in recovery and adaptation require zero-response times to be effective. As systems are dynamic, non-zero response times imply that at the moment a decision is achieved, there is a likelihood that it won't be optimal or near-optimal; a consequence of the system's evolution in time.

A metamodel was used to construct a multi-fidelity model, where it fulfilled the role of low-fidelity model, that enables zero-response times in recover and adaptation of a system. The ideas of slow-time and fast-time were introduced and the effectiveness of multi-fidelity models emphasized in two examples of traffic networks.

An adaptive system should include zero-time decision-making schemes, so that the optimal decision is enabled in each moment of operation, and any recovery or adaptation of the system is continuous with the system operation. Different layers of fidelity have a key role in enabling this system behaviour.

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