Seismic analysis using maximum likelihood of gutenberg-richter

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ARTICLE INFO	ABSTRACT
	An earthquake is one of catastrophe which often claim numerous lives and cause great damage to infrastructure. Multiple studies from various field have been conducted in order to make a precise prediction of earthquake occurrence, such as recognizing the natural phenomena symptoms leading to the shaking and ground rupture. However, up till now there is no definite method that can predict the time and place in which earthquake will occur. By assuming that the number of earthquake follow Gutenberg-Richter law, we work b-value derived using Maximum Likelihood Method to calculate the probability of earthquake happen in the next few years. The southern sea of D.I. Yogyakarta was divided into four areas to simplify the analysis. As the result, in the next five years the first and second area have high enough probability (>0.3) to undergo more than 6.0-magnitude earthquake.

1. Introduction

An earthquake is natural disaster which occur as result of seismic wave released by earth crust [1]. It happens mostly triggered by the rupture of geological faults and volcano activities which cause mild to severe tremor. Other causes such as landslide, mine blast, and nuclear test may result mild tremor.

To measure the velocity of seismic wave, an instrument called seismograph is used [2]. This instrument is installed under the earth's surface and is connected to seismogram (a device to record ground motion). Seismogram works by recording the ground motion which is visualized through vertical waves [3]. Unlike seismograph, seismogram is placed in an observation station. To define the size of the magnitude of earthquake, a scale named Richter scale is developed. This scale is determined from the base-10 logarithm of the amplitude of seismic waves [4].

The latitude of D.I. Yogyakarta, Indonesia is -7.797068, and the longitude is 110.370529. D.I. Yogyakarta is located in Java which lies on convergent plateau zone where the Indo-Australian Plate is sub ducted under the Eurasian Plate. The offshore plate movement will suppress the onshore plates which is located near D.I. Yogyakarta. Therefore, D.I. Yogyakarta has highly possibility to encounter earthquakes.

There are many considerable researches conducted to predict the place and time the earthquake likely happens. However, until now there is no definite method that can precisely predict the occurrence of earthquake. The least the researcher can do is to conjecture the potential tremor to occur. Thus, we can make any effort to reduce the impact of the catastrophe.

Aki and Utsu proposed estimator obtained from maximum likelihood method [5]. This estimator is called b-value which used to determine seismicity rate. Since it is an estimator, there are the uncertainty factor (bias) [6]–[8]. The probability of earthquake may be obtained since the seismicity rate had been calculated [6], [9].

The Gutenberg-Richter (GR) distribution is widely used to show statistical relation,

$$log_{10} N(M) = a - bM \tag{1}$$

Where *N(M)* is the number of earthquake with magnitude equal to or greater than *M, a* and *b* are constants which show level and the character of seismicity in the region of concern. Using Maximum Likelihood Method, the estimator called *b-value* is derived. Since *b-value* is obtained, we also can get the value of *a.*

Seismic activity of D.I. Yogyakarta is able to be quantified by parameters that are seismicity index, probability of tremor occurrence, and return period of the highest tremor within the area. The probability describes the risk of earthquake including only seismic information, without others factor such as geology structure, population density, quality of infrastructure, etc.

The hazard risk of tremor is useful for construction planning. Therefore, we can build the earthquake resistant buildings. Earthquakes are classified into some categories from minor to great. This research calculated the probability of tremor with 4.0 magnitude or more which potentially destructive.

The outline of this paper was arranged as follow, first introduction described the background of research. Second, problem formulation states what this research would do. Method, the third outline, explains the source of data and the tools which is used to analyze the data. The seismic analysis of D.I. Yogyakarta was provided in fourth section. The conclusion and open problem, successively was written in fifth and sixth section.

2. Problem Formulation

To describe the seismicity condition in D.I. Yogyakarta from 2011 – 2015, we need a series of measurable parameters. This study is to quantify these parameters that are seismicity index, earthquake probability, and return period. The *b-value* and *a* which are obtained from Maximum Likelihood Method, used to draw the seismicity index. This index represents the number of earthquake occurrence with magnitude equal to or more than M_0.

Since seismicity index is given, the problem is how to calculate the probability for an area to encounter a number of earthquakes with magnitude equal to or more than M_0 and the return period. Return period indicates the repetitive interval (in years) of earthquake to occur. In this research we analyzed the tremor which have magnitude equal to 4.0 (on the Richter scale) or more.

3. Method

3.1. Data

Earthquake data from 2011 – 2015 is obtained from Subsection Data and Information of Meteorology Climatology and Geophysics Agency (BMKG) of D.I. Yogyakarta. The epicenter of earthquake located in southern offshore of D.I. Yogyakarta divided into six areas.

1. Area 1   : -8.26° – -9,31° latitude and 110,01° – 110,47° longitude
2. Area 2   : -8,26° – -9,31° latitude and 110,47° – 110,93° longitude
3. Area 3   : -9,37° – -10,37° latitude and 110,01° – 110,47° longitude
4. Area 4   : -9,37° – -10,37° latitude and 110,47° – 110,93° longitude
5. Area 5   : -10,37° – -11,42° latitude and 110,47° – 110,93° longitude
6. Area 6   : -10,37° – -11,42° latitude and 110,01° – 110,47° longitude
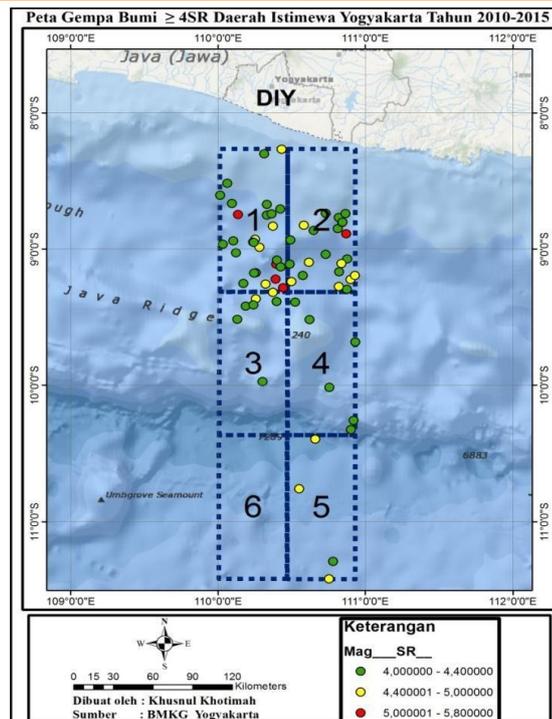
Fig. 1. The division of earthquake zone

We did not include area 5 and 6 into analysis because it has few tremors. There was even no tremor with magnitude more than 5.0 Richter. Moreover, area 5 and 6 is far from D.I. Yogyakarta onshore.

3.2. Gutenberg-Richter Law

In seismology, the Gutenberg-Richer (G-R) law holds [10],

$$log_{10}N\,(M) = a - b\,(M - M_0; M \geq M_0) \tag{2}$$

where $N\,(\,M\,)$ is the number of earthquake in particular magnitude M, M_0 is the minimum magnitude above which all earthquake within a certain region are recorded. Parameter b, called *b-value*, and parameter a are constant related to seismicity rate.

1) Maximum Likelihood Estimator (MLE)
Equation (1) implies that magnitude distributed exponentially [10]:

$$p(M) = \beta e^{-\beta(M-M_0)}; M \geq M_0 \tag{3}$$

where, $\beta = b\ln(10)$, $p(M)$ is the probability density function of M. MLE of *b-value* was derived by:

   a) The likelihood function defines as follows:

$$L(M) = \prod_{i-1}^{n} \beta e^{-\beta(Mi-M0)}$$

$$= \beta^n e^{-\beta \sum_{i=1}^{n}(Mi-M0)} \tag{4}$$

   b) The natural logarithm of likelihood function is given below:

$$\ln L\,(M) = n\,\lambda\nu\,\beta - \beta \sum_{i=1}^{n}(M_i - M_0) \tag{5}$$

c) Define the parameter β which optimized the log logarithm likelihood function by taking the derivative and setting it equal to 0.

$$\frac{\partial \ln L\,(M)}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n}(M_i - M_0) = 0 \tag{6}$$

$$\beta = \frac{1}{\bar{M} - M_0} \tag{7}$$

By substituting $\beta = b\ln(10)$ to equation (7), we obtain the MLE of β :

$$\hat{b} = \frac{1}{\bar{M} - M_0} \tag{8}$$

Thus we have MLE of parameter β which called b-value. Instead of equation (8), this research used the modified b-value below.

$$\hat{b} = \frac{\log e}{\bar{M} - (M_0 - \Delta M/2\,)} \tag{9}$$

In practice, magnitude is rounded into ΔM, usually ΔM = 0.1 . The uncertainty of b-value defines as follow [11]:

$$\hat{\sigma}_b^2 = \frac{\hat{b}}{\sqrt{N}} \tag{10}$$

where N is the number of earthquakes occurrence. Shi and Bolt provide formula to estimate the error of b-value that is [11]:

$$\hat{\sigma}_b = 2.30\hat{b}^2 \sqrt{\frac{\sum_{i=1}^{N}(M_i - \bar{M})^2}{N\,(N-1)}} \tag{11}$$

Compared equation (10), equation (11) have reliable estimation for b-value error. Parameter a and b in G-R distribution are constant. The value of a is formulated as follow [7]:

$$\hat{a} = \log_{10}\left(\frac{N(M)}{T}\right) + bM_0 \; ; M \geq M_0 \tag{12}$$

where T is time interval in which the observation recorded. Holding the estimation of parameter a and b, the seismicity index (or rate) of an area with magnitude M ≥ M0 can be written bellow:

$$N_1(M) = 10^{\hat{a} - \hat{b}M} \tag{13}$$

The formula to determine return period is given bellow:

$$\theta = \frac{1}{N_1(M)} \tag{14}$$

2) Probability
The probability of earthquake occurrence with magnitude equal to or more than M within T period, usually called cumulative distribution function, given as follow:

$$P(M,T) = 1 - e^{-N_1(M)T} \tag{15}$$

4. Seismicity Analysis of D.I. Yogyakarta

We define M0 = 4, since the data recorded was all the earthquakes with magnitude more than or equal to 4.0 occurred in different depth. The summary of the data is explained in Table 1.

Table 1.   Summary of earthquake data of D.I. Yogyakarta offshore 2011-2015

No	Area	Mmin	Mmax	$\bar{M}$	N(Mmin)
1	Area 1	4.0	5.8	4.435	31
2	Area 2	4.0	5.5	4.484	19
3	Area 3	4.0	4.5	4.300	6
4	Area 4	4.1	4.5	4.217	6

In area 1, earthquake with 4.0-magnitude until 5.8-magnitude happened in 10 km until 50 km depth, while the 5.8-magnitude earthquake happened in 50 km depth. The distribution of earthquake based on its depth is showed in Fig. 1. Meanwhile, others areas' description was provided in appendix.
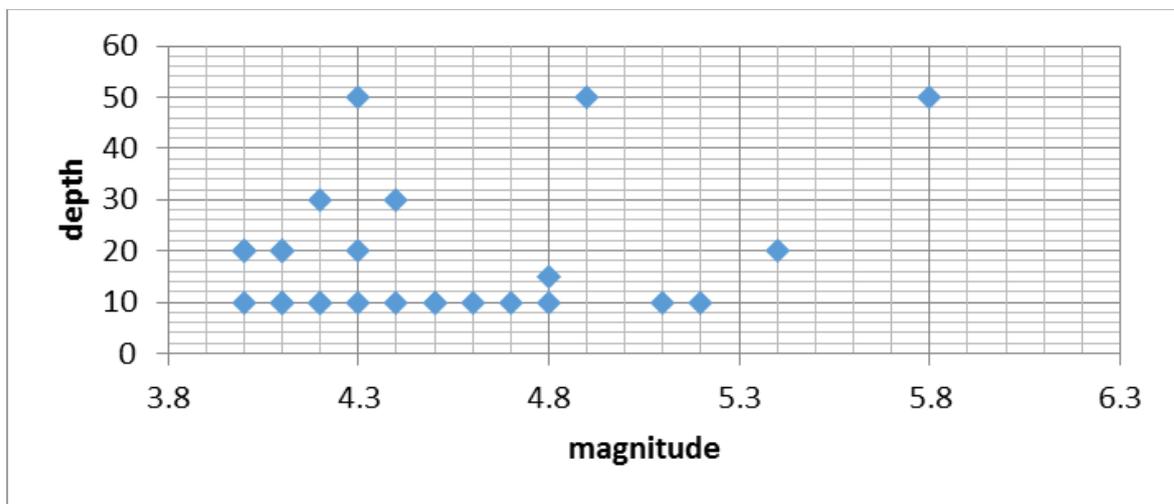
Fig. 2. The distribution of earthquakes based on its depth in area 1

Earthquake magnitudes are classified into some categories that are light (4.0-4.9), moderate (5.0-5.9), strong (6.0-6.9), major (7.0-7.9), and great (>8.0). Since the data recorded have more than or equal to 4.0-magnitude, we calculate the seismicity index for 4.1-magnitude.

Table 2.   Seismicity index for $M \geq 4.1$

No.	Area	M_0	b-value	a-value	$N_1 M$
1	Area 1	4	0.895	4.374	5.045
2	Area 2	4	0.813	3.833	3.151
3	Area 3	4	1.241	5.043	0.902
4	Area 4	4	1.627	6.585	0.825

According to Table 2, area 1 has seismicity index at 5.045 which means every year area 1 encounters about 5 earthquakes with magnitude more than or equal to 4.1. In area 3 and 4, tremor happen about once for every year. Meanwhile, in area 2, tremors happen four times within a year. Estimation of b-value error in every area is shown at Table 3.

Table 3.   Estimation of *b-value* error

No.	Area	M_0	*b-value*	^
1	Area 1	4	0.895	0.151
2	Area 2	4	0.813	0.147
3	Area 3	4	1.241	0.259
4	Area 4	4	1.627	0.290

The probability having tremor with magnitude more than or equal to 4.1 in the next of five years is showed in Table 4.

Table 4.   Probability of earthquake with magnitude more than or equal to 4.1

No.	Area	$N_1(M)$	$P(M \geq 4.1)$
1	Area 1	5.045	1.000
2	Area 2	3.151	1.000
3	Area 3	0.902	0.989
4	Area 4	0.825	0.984

In five years ahead, every area is almost certain for having earthquake with given magnitude. D.I. Yogyakarta encountered earthquake with 5.9-magnitude for about 57 seconds. This natural disaster caused a lots of deaths, about 5700, and injuries. The epicenter was located near area 1 and 2 in 33 km depth. Thus, we took the strong earthquake with minimum magnitude at 6.0 and calculate both seismicity index and the probability for experiencing this earthquake in five years ahead.

Table 5.   Seismicity index and probability for $M \geq 6.0$

No.	Area	$N_1 M$	$PM \; 6.0$
1	Area 1	0.100	0.395
2	Area 2	0.090	0.362
3	Area 3	0.004	0.020
4	Area 4	0.001	0.003

The earthquake with magnitude more than 6.0-magnitude may cause a lot of damage. Area 1 has the highest probability among the four area for going through this earthquake.

5. Conclusion

Employing Maximum Likelihood Method, we obtain MLE called b-value which is used to determine seismicity index. The b-value of earthquake with 4.1 magnitude or more of the four area ranged from 0.895 – 1.627. While seismicity index obtained is about 0.825 – 5.045, which means there are about 1 – 5 times earthquakes happen within a year. Since seismicity index provided, the probability was calculated. The probability for having more than or equal 4.1-magnitude tremor in five years ahead is almost certain for all areas.

The minimum magnitude for hazardous earthquake was given at 6.0-magnitude. The highest probability for encountering this earthquake is 0.395 which occurred in area 1. Meanwhile, the lowest probability is 0.003 which occurred in area 3. Whereas, area 1 is closer D.I. Yogyakarta onshore than area 3.

6. Open Problem

The formula for determining seismicity rate should consider the depth where the earthquake happen. The shallower the epicenter of earthquake, the more hazardous to the population.

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