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Robust Control of Continuous Stirred Tank Reactor with Jacket Cooling

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Continuous Stirred-Tank Reactors (CSTR) belong to basic technological equipment frequently used in the the production of various types of chemicals. These systems are quite complex with many nonlinearities. So, the conventional linear control with fixed parameters can be questionable or unacceptable. The solution should be found in so-called “non-traditional” control approaches like Adaptive, Robust, Fuzzy or Artificial Intelligent methods. One way is the utilization of selftuning adaptive schemes but computations are quite difficult, clumsy and time-consuming. This paper brings an alternative principle called robust approach. This approach considers a linear system with parametric uncertainty which covers a family of all feasible plants. Then a controller with fix parameters is designed so that for all possible plants the acceptable control behavior is obtained. The two degree of freedom (2DOF) structure for the control law was chosen. All calculation and simulations of mathematical models and control responses was performed in the Matlab and Simulink environment.

1. Introduction

The plants in technological processes and especially in chemical and biochemical industry usually have nonlinear behavior that causes difficulties in the control of such processes. The negative property of nonlinearity can be overcome with the linearization of nonlinear models. On the other hand, this simplification could result in an inaccurate description of the system. The utilization of adaptive (e.g. selftuning) schemes brings more difficult, clumsy and time-consuming computations (Åström and Wittenmark, 2008). The control design using a hybrid adaptive control principle was used in Vojtesek et al. (2017) where the originally nonlinear system was represented by the external linear model with recursively identified parameters and the pole-placement method adjustment principle was applied. A practically favored approach to overcome the loss of the model accuracy, compensated by its structure simplicity, consists in the utilization of a model with uncertainty. There are more ways of incorporating the uncertainty into the mathematical model available, see Barmish (1994), Bhattacharyya (2017). The popular group of uncertain systems is known as the systems with parametric uncertainty, which means the model structure is fixed but its parameters can vary, typically within some prescribed intervals. Then, the natural task is to find a controller, called a robust controller, that ensures the preserving some important closed-loop properties (e.g. stability) for the whole assumed family of controlled plants, see Grimble (2006).
The main aim of this paper is in the design a robustly stabilizing controller for the CSTR with the cooling in the jacket, modelled as a system with parametric uncertainty, by means of algebraic approach.
The paper is organized as follows. In Section 3 a mathematical model of CSTR is described. Then Section 3 outlines principles of uncertainty, robust control and control design in R_{PS}. Section 4 is devoted to simulation example and discussion of results. Finally, section 5 offers some concluding remarks.

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2. Continuous Stirred Tank Reactor

The controlled system is a CSTR which is schematically displayed in Figure 1. The reaction inside the reactor is called van der Vusse reaction and can be described by the following scheme:

$$\begin{aligned} &A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ &2A \xrightarrow{k_3} D \end{aligned} \tag{1}$$

The mathematical description of the process is quite complex and there must be introduced some simplifications. Suppose that the reactant is perfectly mixed, all densities, heat capacities and transfer coefficients are constant throughout the reaction. In fact, they are not constant but they usually vary only in a small range and this variation can be neglected.

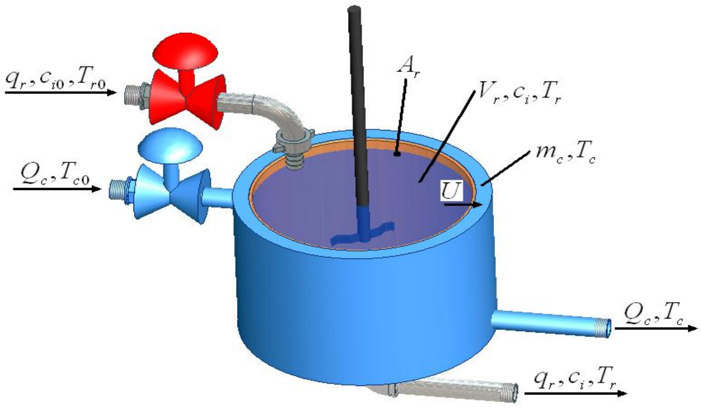

Figure 1: Continuous stirred tank reactor with cooling in the jacket.

Under these assumptions, the mathematical model can introduced by four material and heat balances inside the reactor (see e.g. Russell and Denn, 1972 or Vojtesek et al. 2017):

$$\begin{aligned} \frac{dc_A}{dt} &= \frac{q_r}{V_r}\left(c_{A0} - c_A\right) - k_1 c_A - k_3 c_A{}^2 \\ \frac{dc_B}{dt} &= -\frac{q_r}{V_r} c_B + k_1 c_A - k_2 c_B \\ \frac{dT_r}{dt} &= \frac{q_r}{V_r}\left(T_{r0} - T_r\right) - \frac{h_r}{\rho_r c_{pr}} + \frac{A_r U}{V_r \rho_r c_{pr}}\left(T_c - T_r\right) \\ \frac{dT_c}{dt} &= \frac{1}{m_c c_{pc}}\left(Q_c + A_r U\left(T_r - T_c\right)\right), \quad \textit{where } 0 \le c_A,\ 0 \le c_B \end{aligned} \tag{2}$$

This mathematical model of the reactor belongs to the class of *lumped-parameter nonlinear systems*, see e.g. Ingham et al. (2000) because it is described by a set of ODE. Nonlinearity can be found in reaction rates (k_j), which are described via the Arrhenius law:

$$k_j\left(T_r\right) = k_{0j} \cdot \exp\left(\frac{-E_j}{RT_r}\right), \text{ for } j = 1, 2, 3 \tag{3}$$

where k_0 represent pre-exponential factors and E are activation energies.
The reaction heat (h_r) in Eq. (2) is expressed as:

$$h_r = h_1 \cdot k_1 \cdot c_A + h_2 \cdot k_2 \cdot c_B + h_3 \cdot k_3 \cdot c_A^2 \tag{4}$$

where h_i means reaction enthalpies.

The initial conditions for the set of ODE (2) are $c_A\left(0\right)=c_A^s,\, c_B\left(0\right)=c_B^s,\, T_r\left(0\right)=T_r^s,\, T_c\left(0\right)=T_c^s$ (5)

Parameters of CSTR and more details can be found in Vojtesek et al. (2017).

3. Robust Control

3.1 Models with Parametric Uncertainty

Systems with parametric uncertainty represent effective and popular way of considering the uncertainty in the mathematical model of a real plant, see e.g. Barmish (1994) or Matušů and Prokop (2013, 2014). The utilization of such models supposes known structure (and order) of the transfer function but not precise knowledge of real parameters, which can be bounded by intervals with minimal and maximal possible values. They can be described by a transfer function:

$$G(s,q)=\frac{b(s,q)}{a(s,q)} \tag{6}$$

where $b(s,q)$ and $a(s,q)$ denote polynomials in *s* (Laplace transform)with coefficients depending on *q*, which is a vector of real uncertain parameters. Typically, this vector is confined by some uncertainty bounding set which is generally a ball in some appropriate norm. The combination of the uncertain system (e.g. transfer function (6)) with an uncertainty bounding set gives so-called family of systems, see e.g. Barmish (1994). A special and frequent case of system with parametric uncertainty is interval plants. Its parameters can vary independently on each other within given bounds, i.e.: $a_i\in\left[a_i^-;a_i^+\right],\, b_i\in\left[b_i^-;b_i^+\right]$,where $b_i^-, b_i^+, a_i^-, a_i^+$ represent lower and upper limits for parameters of numerator and denominator, respectively.

3.2 Control Structure and Design

The 2DOF closed-loop control system with separated feedback and feedforward parts of the controller is well known, see Kučera (1993), Prokop and Corriou (1997) and the control law is governed by:

$$P(s)U(s)=R(s)W(s)-Q(s)Y(s) \tag{7}$$

The transfer functions $G(s)$ =*B(s)/A(s),* $C_b(s)=Q(s)/P(s)$, and $C_f(s)=R(s)/Q(s)$ represent controlled plant, feedback part of the controller, and feedforward part of the controller, respectively and the signals *w*(*s*), *n*(*s*), and *v*(*s*) are reference, load disturbance, and disturbance signal. The traditional (one degree of freedom) feedback system is obtained by R=Q. However, there are many relevant evidence that the feedforward part brings positive improvements in control responses, see e.g. Gorez, (2003) or Matušů and Prokop (2013, 2014). The control synthesis itself is based on the algebraic ideas of Vidyasagar (1985), and Kučera (1993). Subsequently, the specific tuning rules has been developed and analyzed e.g. in and Prokop and Corriou (1997) or Matušů and Prokop (2013). Besides, the controller tuning rules for the case of law order controlled plant under assumption of either purely reference tracking problem or reference tracking and load disturbance rejection together have been already studied e.g. Kučera, (1993) or Matušů and Prokop (2013). The control design technique supposes the description of linear systems by means of the ring of proper and stable rational functions (R_{PS}). The conversion from the ring of polynomials to R_{PS} can be performed very simply (see e.g. Vidyasagar, 1985 or Prokop and Corriou, 1997) according to:

$$G(s)=\frac{b(s)}{a(s)}=\frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}}=\frac{B(s)}{A(s)},\quad m>0,\;\; n=\max\left\{\deg(a),\deg(b)\right\} \tag{8}$$

The parameter $m>0$ will be later used as a controller-tuning knob. The value of the tuning knob has relevant influence on the control behavior of control responses. The algebraic analysis, see e.g. Prokop and Corriou, (1997) or Matušů and Prokop (2013, 2014) leads to the first Diophantine equation:

$$A(s)P(s)+B(s)Q(s)=1 \tag{9}$$

with a general solution $P(s)=P_0(s)+B(s)T(s)$, $Q(s)=Q_0(s)-A(s)T(s)$, where $T(s)$ is an arbitrary member of (the ring) R_{PS} and the pair $P_0(s)$, $Q_0(s)$ represents any particular solution of (9). This principle is known as Youla – Kučera parameterization of all stabilizing controllers. Thus, all possible solutions of the Diophantine equation give all stabilizing feedback controllers. Since the feedback part of the controller is responsible not only for stabilization but also for disturbance rejection, the convenient controller from the set of all stabilizing ones can be chosen on the basis of divisibility conditions. The requirement of the reference tracking is obtained by the second Diophantine equation (F_w is the reference denominator), see e.g.Kučera (1993):

$$F_w(s)Z(s)+B(s)R(s)=1 \tag{10}$$

3.3 Robust Stability

Stability of the feedback loop is the crucial requirement in all control applications. Naturally, the feedback loop can be stable when the controlled and/or control plant is unstable. In the case of uncertainty of controlled plants, robust stability means that not only one fixed closed-loop system is stable but also whole family of closed-loop control systems is ensured to be stable. Since the stability of linear systems can be investigated by means of stability of its characteristic polynomials, the main object of interest from the robust stability viewpoint is the uncertain continuous-time closed-loop characteristic polynomial $p(s,q)=\Sigma\, \rho_i(q)\, s^i$. Details can be found in e.g. Ackermann (1993) or Bhattacharyya (2017).
However, there is a very universal graphical approach applicable for all, even in complicated cases. It is known as the value set concept in combination with the zero exclusion condition see e.g. Barmish (1994) or Matušů and Prokop (2011). In other words, $p(j\omega,Q)$ is the image of Q under $p(j\omega,\cdot)$. Practical construction of the value sets then means to substitute s for $j\omega$, fix ω and let the vector of uncertain parameters q range over the set Q. The zero exclusion condition for Hurwitz stability of family of continuous-time polynomials says see e.g. Barmish, (1994): Assume invariant degree of polynomials in the family, pathwise connected uncertainty bounding set Q, continuous coefficient functions $\rho_i(q)$ for $i=0,1,2,\ldots,n$ and at least one stable member $p(s,q^0)$. Then the family P is robustly stable if and only if the complex plane origin is excluded from the value set $p(j\omega,Q)$ at all frequencies $\omega\geq 0$, that is P is robustly stable if and only if

$$0\notin p(j\omega,Q)\quad \forall\omega\geq 0 \tag{11}$$

4. Simulations and discussion

4.1 Simulation example and results

The CSTR was identified in Vojtěšek et al., (2017) as a second order system with the transfer function:

$$G(s)=\frac{b(s)}{a(s)}=\frac{b_1 s+b_0}{a_2 s^2+a_1 s+a_0} \tag{12}$$

with nominal parameters: a_2 = 1, a_1 = 1.4550, a_0 = 0.3072, b_1 = -0.0037, b_0 = -0.0095. The intervals for uncertain perturbations were obtained by deeper analysis of the dynamic behaviour and they result in the following ones:

a_2 = [0.8; 1.2], a_1 = [1.164; 1.746], a_0 = [0.24576; 0.36864],
b_1 = [-0.00296; -0.00444], b_0 = [-0.0076; -0.0114] (13)

The first 2DOF controller has been designed for the nominal plant and the tuning parameter m = 0.8. The feedback and feedforward parts of the controller:

$$C_b(s)=\frac{q_2s^2+q_1s+q_0}{s^2+p_1s}=\frac{-102.4519s^2-154.6167s-43.1158}{s^2+1.3659s},\quad C_f(s)=\frac{r_2s^2+r_1s+r_0}{s^2+p_1s}=\frac{-67.3684s^2-107.7895s-43.1158}{s^2+1.3659s} \tag{14}$$

The second 2DOF controller was generated by tuning parameter m = 1.2 in the form:

$$C_b(s)=\frac{q_2s^2+q_1s+q_0}{s^2+p_1s}=\frac{-326.4929s^2-573.464s-218.2737}{s^2+2.137s},\quad C_f(s)=\frac{r_2s^2+r_1s+r_0}{s^2+p_1s}=\frac{-151.5789s^2-363.7895s-218.2737}{s^2+2.137s} \tag{15}$$

Figure 2 shows the output controlled variables for both tuning parameters. The red lines depict the nominal plant responses and black shadows are responses for the whole uncertain family (13). The load disturbance n = 50 was injected in the time t = 40 and it is evident that no permanent error is observed.

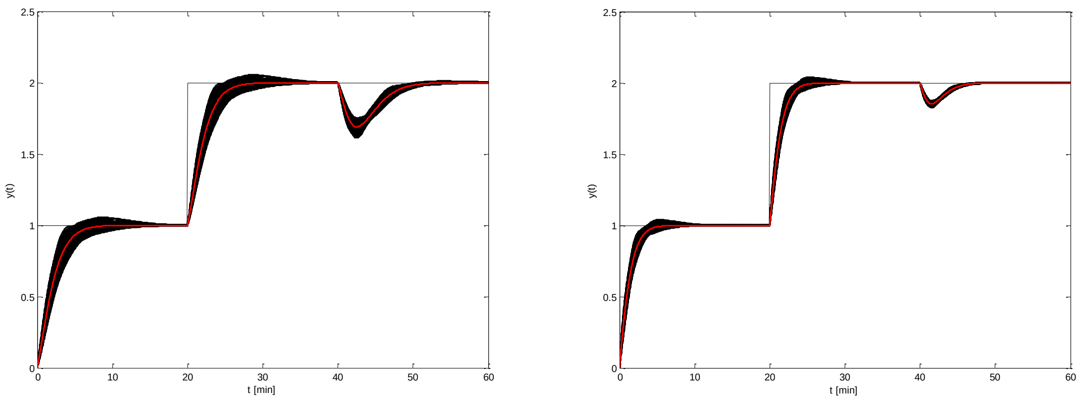

Figure 2: Set of output controlled variables for m=0.8 (left) and m=1.2 (right)

4.2 Analysis and Discussion

Simulation results proved that the fix robust controller can be designed for a wide family of interval systems. The results are shown in Figure 2 for two values of the tuning parameter m > 0. The choice of the tuning parameter m > 0 was found empirically and experimentally. Until now, there is no exact theory how to obtain the optimal value (see e.g. Prokop and Corriou, 1997). In order to verify the practical usability of both designed controllers, they were applied not only to the linearized model, but also to the original nonlinear model of CSTR. The control results for this nonlinear case are shown and mutually compared in Figure 4. The red line corresponds to the value m=0.8, while the blue line represents the value m=1.2. All simulations confirm that lower values of the parameter m give slower responses of the control behaviour. The price for the faster response is the more aggressive (higher) control inputs. Figure 3 shows the zoomed value sets for the family of closed-loop characteristic polynomials with the affine linear uncertainty structure. It demonstrates the robust stability of both designed control systems.

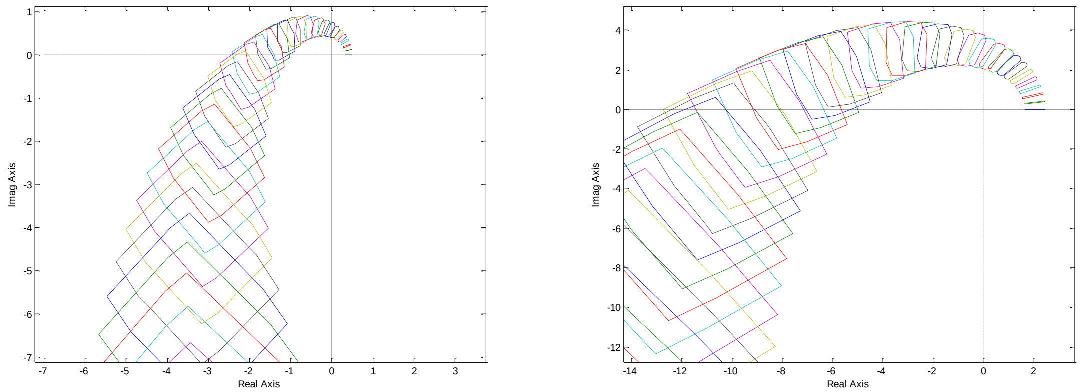

Figure 3: Zoomed value sets for m=0.8 (left) and m=1.2 (right)

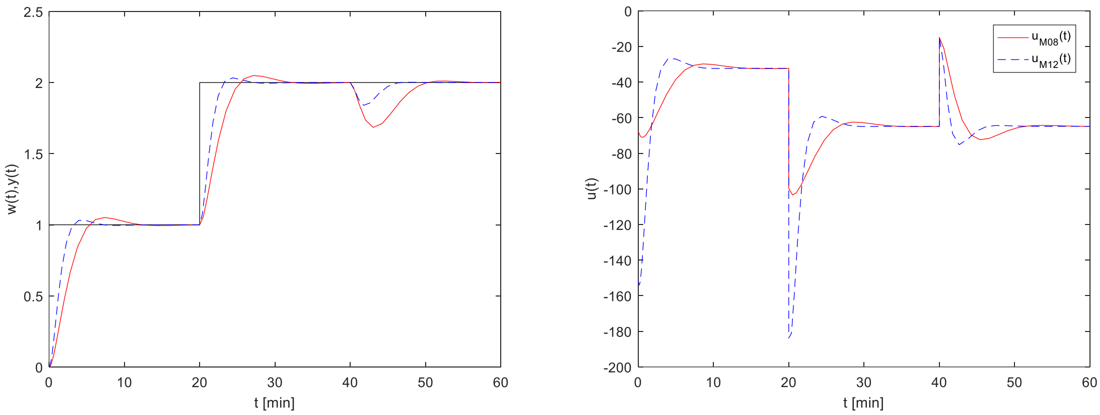

Figure 4: Control of the original nonlinear model for both values of m – comparison of the output controlled variables (upper) and the input control variables (bottom)

5. Conclusions

The paper has been focused on application of continuous-time 2DOF robust control algorithms designed in R_{PS} to systems with parametric uncertainty. The synthesis method itself is accompanied by the graphical approach to robust stability analysis based on the value set concept and the zero exclusion condition. As an application, two designed controllers were applied to control of nonlinear Continuous Stirred Tank Reactor.

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