An Operations Research-Based Approach to the Allocation of COVID-19 Vaccines

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The global scientific community has been successful in their efforts to develop, test, and commercialize vaccines for COVID-19. However, the limited supply of these vaccines remains to be a widespread problem as different nations have started their respective vaccine rollouts. Policymakers continue to deal with the difficult task of determining how to allocate them. This research work will present how the use of mathematical models can provide valuable decision support under such conditions.

Both a linear programming model and a nonlinear programming model have been developed to determine the optimal allocation of COVID-19 vaccines that minimize fatalities and COVID-19 transmission, respectively. These scenarios have to be dealt with when not enough vaccines are available, and the pandemic is still in progress. The model is capable of handling large scale allocation problems such as those intended for the general population of a country. It could also be scaled down for organizations such as private companies or universities. The model also considers multiple vaccines with different levels of efficacy. The distribution of vaccines reduces transmission and relative infectiousness of individuals across different age groups. A hypothetical case study is solved to illustrate the computational capability of the models. The results indicate that priority should be given to the elderly when fatalities are minimized. In contrast, the younger population should then be prioritized when the objective shifts to suppressing contagion.

1. Introduction

The COVID-19 pandemic has been an unprecedented crisis that continues to have significant global impact. The outbreak has put at risk the achievement of the Sustainable Development Goals (Barbier and Burgess, 2020). Many countries have taken different measures such as border closures and lockdowns to mitigate the spread of the virus. Ultimately, vaccination is proving to be one of the most effective solutions to manage the pandemic. With more than 50 % of the US population receiving at least one COVID-19 vaccine shot (Mendez and Rattner, 2021), case counts have continued to decline, so much so that the CDC had dropped mask rules in most parts of the country. Meanwhile, the UK have started to record no new deaths due to the virus since the start of the pandemic, with 59 % of the population receiving at least one shot (Marsh and Gayle, 2021).

The global scientific community has been successful in their efforts to develop, test, and commercialize vaccines for COVID-19. However, the limited supply of these vaccines remains to be an overarching problem as different nations have started their respective vaccine rollouts. Yu et al. (2020) discussed the various challenges that the pharmaceutical industry will face to respond to the crisis. While countries such as the US, UK, and other members of the European Union have eased restrictions due to relatively successful vaccination programs, the rest of the world is still grappling with renewed surges of infections coupled with vaccine supply shortages. Mathieu et al. (2021) reported large differences in the scale of vaccination rates, with most developing countries inoculating just 5–10 % of their population.

The emergence of COVID-19 variants have further complicated vaccination programs, with different nations already planning for booster doses for their citizens (Howard, 2021). Jian et al. (2021) state that such prolonged programs necessitate leaders to take a life cycle perspective to ensure sustainability in the management of

vaccines. They should concurrently recognize the effects of their vaccination programs to energy, environmental, economic and social equity (4Es). The authors add that taking a life cycle approach will not only minimize the 4E footprints but would also lead to a more efficient and equitable distribution of vaccines.

Quantitative models have been used to provide valuable decision support to achieve the aforementioned objectives. At the onset of the pandemic, when vaccines were still unavailable, Sy et al. (2020) presented an optimization model for the distribution of repurposed drugs that the medical community uses to treat COVID-19 infections. They showed that allocating the drugs based on identified risk factors led to a decrease in disease severity and improvement in the utilization of healthcare facilities. Enayati and Ozaltin (2020) developed an influenza vaccine model that minimizes the number of vaccine doses to be distributed subject to the containment of an outbreak in its early stages. The authors used the reproduction number as a metric for disease transmission.

This research alternatively presents an extension to Sy et al. (2020) and focuses on the optimal allocation of COVID-19 vaccines within a community or area. Flexibility is integrated in the design of the model such that it allows users to decide on the scope of the allocation problem. It is able to handle vaccine allocation for the general population of a country or be scaled down for the purpose of allocating within an organization such as private companies or universities. Two optimization models are developed, with each one minimizing either disease transmissibility or resulting fatalities. The models address prioritization in the distribution of vaccines subject to risk factors in the population, supply availability, and efficacy levels of the vaccines. The rest of the paper is structured as follows: Section 2 discusses the formulation of the linear programming (LP) model which minimizes the fatalities and the nonlinear programming (NLP) model that minimizes the reproduction number. Section 3 presents the computational experiments on both models and subsequent results using a hypothetical case study. Finally, the paper provides a synthesis of the research work and recommendations for further studies in Section 4.

2. Allocation models for COVID-19 vaccines

The models are intended to provide recommendations of vaccine allocation in the general population with respect to two alternative objectives. The first model considers the minimization of the number of fatalities due to COVID-19. The second objective seeks to minimize disease transmission by looking at the reproduction number of the spread of the virus. In both models, it is assumed that there are j types of vaccines having varying degrees of efficacy, h_j.

2.1 Minimizing the number of fatalities

In the first model, the population is divided into age groups i, where each group n_i has an associated fatality rate α_i. The fatality rate is then multiplied to the difference between the total susceptible population and the number of people considered to be immune due to vaccination m_{ij}. Eq(1) represents the objective function minimizing the number of fatalities Z, where k_{ii} denotes the contact rates within the age groups. Liu et al. (2020) used these contact rates to characterize disease transmission within a community. This research work adopts the usage of the contact rates, which are identified and classified into "next generation matrices". These contact rates differ with respect to environmental settings such as schools, households, workplaces, and public areas.

$$\text{Min } Z = \sum_i \left\{ \alpha_i \left(\sum_j (k_{ii} n_i - m_{ij}) \right) \right\} \tag{1}$$

The constraints of the model are shown in Eq(2) to Eq(5).

$$h_j * (n_i * f_{ij}) = m_{ij} \qquad \forall\, i, j \tag{2}$$

$$\sum_i f_{ij} = 1 \qquad \forall\, j \tag{3}$$

$$\sum_i (n_i * f_{ij}) \leq \beta_j \quad \forall j \tag{4}$$

$$h_j, m_{ij} \geq 0 \qquad \forall\, i, j \tag{5}$$

Eq(2) calculates for the number of people in age group i considered to have gained immunity due to inoculation of vaccine j. This is defined in the left-hand side, where f_{ij} denotes the proportion in age group i that is assigned to vaccine j, subject to the efficacy rate of that particular vaccine. Eq(3) ensures that the proportion sums up to 1. Eq(4) defines the supply limitations of each vaccine type denoted by β_j. Finally, Eq(5) represents the nonnegativity requirement for the decision variables.

2.2. Minimizing the reproduction number

The second model considers an alternative objective by minimizing disease transmission. The reproduction number R_0 has often been used as a metric for disease outbreaks. It denotes the expected number of secondary infections generated by an infectious individual (Enayati and Ozaltin, 2020). Hence, an emerging outbreak will prevail if the reproduction number is greater than 1. Otherwise, the epidemic is considered contained and the outbreak will subside (Duijzer et al., 2016). The reproduction number has also been used as a barometer to measure how well a country is managing the COVID-19 pandemic. For instance, British Prime Minister Boris Johnson mentioned that COVID-19 alert levels would be "primarily determined" by the number of cases, and by the reproduction number (Adam, 2020). Eq(6) shows the objective function of the second model which is to minimize this reproduction number.

Enayati and Ozaltin (2020) related the reproduction number to an eigenvalue that is dependent on the next generation matrix. The matrix likewise leads to corresponding eigenvectors v_i. A similar approach is adopted such that Eq(7) relates the next generation matrix to the overall disease transmission observed in the community. The left-hand side of the constraint shows that the allocation of the vaccines (subject to their efficacy levels) reduces the contact rates in each age group. This then leads to an overall reduction of the reproduction rate in the system. Readers are referred to the aforementioned work for a more detailed discussion on its formulation.

It should be noted that this formulation results in a nonlinear programming model due to the bilinear terms seen from both sides of the constraint. Eq(8) represents a technical constraint equating the eigenvalue as a sum of the eigenvectors calculated from the aforementioned matrix. In addition, Eq(9) to Eq(11) correspond to the constraints previously defined in the first model (Eq(3) to Eq(5)). These have been retained in this subsequent model formulation.

$$\text{Min } R_0 \tag{6}$$

$$\sum_j (1 - h_j) f_{ij} * \sum_i K_{ii} v_i \leq R_o * v_i \quad \forall\, i \tag{7}$$

$$\sum_i v_i = R_o \tag{8}$$

$$\sum_i f_{ij} = 1 \qquad \forall\, j \tag{9}$$

$$\sum_i (n_i * f_{ij}) \leq \beta_j \quad \forall j \tag{10}$$

$$h_j, m_{ij} \geq 0 \qquad \forall\, i, j \tag{11}$$

3. Computational Experiments

A hypothetical case study from Sy et al. (2020) has been used to demonstrate the capability of both models to allocate different types of COVID-19 vaccines to the general population. The computational experiments have been implemented using the commercial optimization software LINGO 18.0, which is capable of handling both linear and nonlinear programming models (Gau and Schrage, 2004). Table 1 shows the next generation matrix used for both models, in which the population has been divided into six age groups.

Table 1: Next generation matrix for contact rates in public areas

Group	Age	0-24	25-34	35-44	45-54	55-59	>=60	Population	Mortality rate (%)
1	0-24	0.6	0.1	0.1	0.1	0.1	0.1	77	0.0002
2	25-34	0.2	1.7	0.3	0.2	0.2	0.1	241	0.0010
3	35-44	0.4	0.3	0.5	0.4	0.3	0.2	375	0.0038
4	45-54	0.2	0.1	0.3	0.2	0.1	0.1	204	0.0106
5	55-59	0.1	0.1	0.1	0.1	0.1	0.1	85	0.0480
6	>=60	0.1	0.1	0.1	0.1	0.1	0.1	103	0.3281

The numbers inside the matrix can be interpreted as the expected number of secondary infections brought about by people belonging to respective age groups. For instance, a person belonging in the 25-34 group will, on the average, infect 1.7 people in that same age group. Age groups having higher mobility such as 25-34 and 35-44

would naturally have higher contact rates. The respective mortality rates for each age group, used in both case studies, have likewise been provided in the table

3.1 Allocation of vaccines to minimize the number of fatalities due to COVID-19

In the first model, it was assumed that there is only a single vaccine to be allocated, with an efficacy rating of 95 %. Scenario analysis which takes into account different levels of supply availability of the said vaccine was conducted. A percentage rating of 10 means that only 10 % of the population would have access to the vaccine. The results are shown in Figure 1 and Figure 2.

Most nations have prioritized vaccinating the elderly since they are at a higher risk of hospitalization or dying if diagnosed with COVID-19 (CDC, 2021). This observation is affirmed by the results under the objective of minimizing fatalities. Figure 1 shows that priority is given to Group 6 when supply level is low. It is only after inoculating everyone in that group will the allocation shift to another age group. In this case, age Group 4 is then prioritized over Group 5. This could be explained by looking at the mortality and contact rates of the respective age groups. While Group 5 may have a higher mortality rate, its associated contact rates are lower than Group 4. Whereas Group 4's higher contact rates imply a greater number of people getting infected and subsequently dying from COVID-19. Furthermore, no other age groups will be allocated the vaccine even when supply levels increase. This suggests that even if the younger age groups have higher contact rates, the low mortality rates attributed to them will lead to an insignificant change in the number of fatalities.
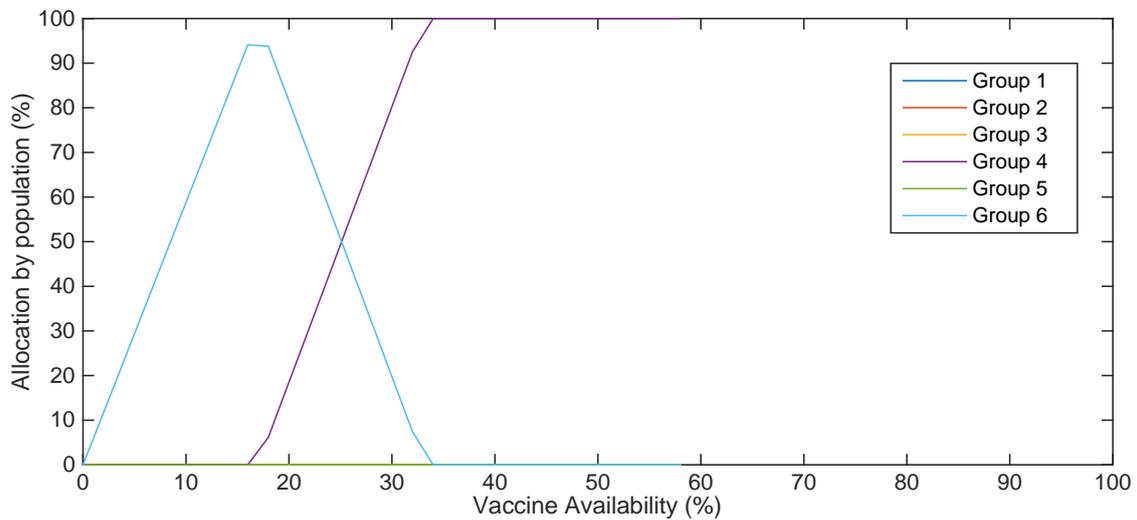

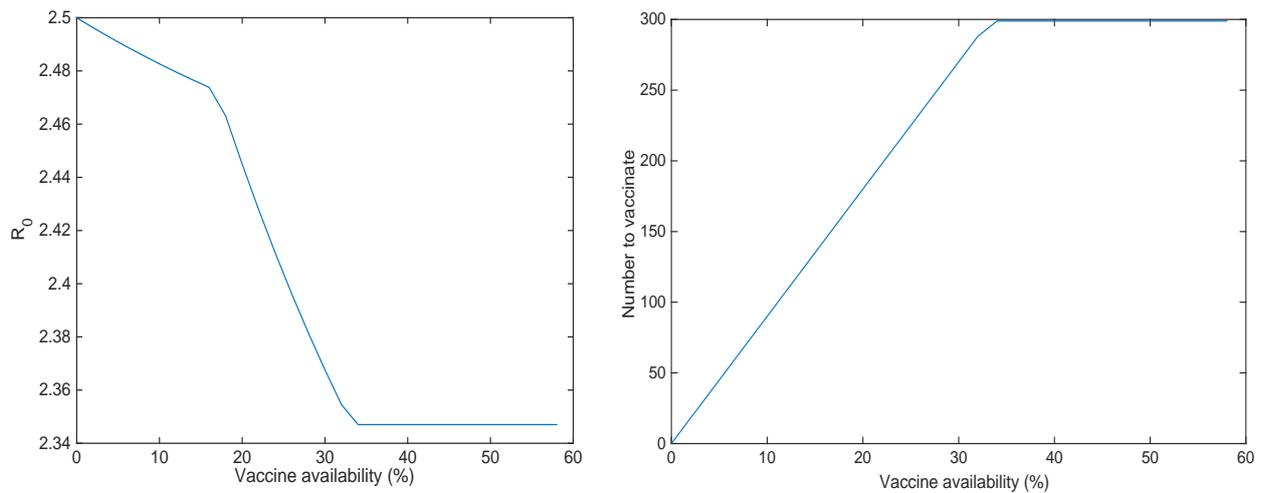Figure 1: Vaccine allocation under varying levels of supply availability

Figure 2: Reproduction number of recommended number of vaccines to deploy with respect to availability

The resulting reproduction number R_0 and the number of people that should be vaccinated are examined in Figure 2. It can be seen that minimizing the number of fatalities due to COVID-19 has little effect in controlling transmission. Starting at an R_0 of 2.5, the lowest R_0 was observed to be around 2.35 at the highest level of supply availability. This suggests that the disease outbreak has not been contained even if the number of deaths are being minimized. This observation is primarily because no allocation was provided to the groups having the highest mobility and contact rates. It can also be observed that the number of people that should be vaccinated plateaus to around 300, reinforcing the observation from Figure 1. One can then conclude that simply looking at the number of deaths is insufficient to control transmission and eventually contain and manage the virus effectively in a given community.

3.2 Allocation of vaccines to suppress contagion of COVID-19

An additional vaccine has been added to facilitate the computational experiments of the second model. The effect of supply availability to the allocation of the vaccines was again observed. It was assumed that Vaccine 1 has a 95 % efficacy while Vaccine 2 has a 90 % efficacy. Furthermore, only 30-60 people will have access to Vaccine 1 while 100-200 people will have access to Vaccine 2.

Table 2 shows the results of the model runs. At the lowest supply levels, the minimum R_0 was 1.24 with only Group 2 (25-34) being given the vaccines. This is in contrast to the first model wherein priority was given to Group 6 or the elderly. In this subsequent run, both Group 6 and 4 were not allocated any vaccine doses. A similar trend was observed when supply levels are increased. Group 1 (0-24) followed by Group 3 (35-44) will be the succeeding groups to be given the vaccines. The corresponding R_0 values were 1.06 and 0.97, respectively. The results imply that the contact rates are the primary drivers of how the allocation will proceed regardless of vaccine availability.

Table 2: Allocation when R_0 is minimized

Supply (# of people) Vac 1, Vac 2	0-24	25-34	35-44	45-54	55-59	60 and above	R_0
30, 100	-	30 , 100	-	-	-	-	1.24
45, 150	11, 12	34 , 138	-	-	-	-	1.06
60, 200	12, 17	37 , 159	12, 24	-	-	-	0.97

Availability of vaccines directly impacts the containment of the virus. More importantly, when one wants to minimize virus transmission, priority should be given to the groups having the highest mobility. This is indicated by the contact rates assumed for these groups in Table 1. Groups 1 and 2 had the highest contact rates which are also indicative of the general mobility seen amongst those who belong in these age groups. The allocation also shows preference being given to Vaccine 1 since it has a higher efficacy level than Vaccine 2. It was seen that the model tries to first utilize Vaccine 1 for Group 2 before allocating Vaccine 2 to the other groups.

4 Conclusions

The limited supply of COVID-19 vaccines remains to be an overarching problem of different nations in their respective vaccine rollouts. Policymakers continue to deal with the difficult task of determining how to allocate them to the general public. The research work demonstrated how the use of quantitative modelling can provide valuable decision support considering accepted metrics such as the reproduction number and fatalities in controlling and managing the COVID-19 pandemic.

Two models have been developed for each of the aforementioned objectives. The results from each model underscore the tradeoff presented by transmission and fatalities. Trying to minimize one objective over the other leads to an entirely different allocation plan. Priority will be given to the elderly when fatalities are minimized. Furthermore, diminishing returns in the number of fatalities were observed once vaccine availability increases beyond 20 %. In contrast, the younger population will then be prioritized when the objective shifts to suppressing contagion. This can be attributed to the higher degree of mobility this group represents, wherein mobility has often been equated to the rate of spreading the virus in different community settings.

It should be acknowledged that there are many sources of uncertainty and uncontrollable variables presented by this pandemic. Comprehensive scenario analysis should be further performed in the succeeding stages of this research work. Specifically, one can look at other groupings aside from age. The types of vaccines could likewise be increased beyond what was considered in this study. The possibility of mixing the types of vaccines between the first and second dose can be a further extension of the paper. These models should continuously be updated as new peer-reviewed data and information become available. Finally, model deployment represents an important dimension of this research work. Hence, a web-based application for these allocation models is

currently under development that will allow users and decision makers to facilitate allocation decisions for COVID-19 vaccines in an effective and efficient manner.

Nomenclature

h_j	Efficacy of vaccine j	n_i	Total number of people in age group i
f_{ij}	Proportion of age group i that receives vaccine j	k_{ii}	Contact rate from group i to group i
β_j	Availability of vaccine j	R_0	Reproduction number
m_{ij}	Number of people in age group i considered immune due to receiving vaccine j	v_i	Eigenvector of age group i
α_i	Mortality rate of age group i		

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