

# Carbon Footprint Analysis of a Laboratory Plate Heat Exchanger Control

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More than 100 countries worldwide have pledged carbon neutrality by 2050 to reduce the negative environmental impacts of carbon emissions. To reach this goal, every part of the industry must prepare to transform and take precautions. Heat exchangers are one of the most common parts of industrial production. The industrial operation of heat exchangers still offers lots of opportunities to minimize their carbon footprints. This goal can be achieved by their optimized operation. This work investigates the operation of a laboratory plate heat exchanger controlled by a proportional-integral-derivative (PID) controller and by an optimization-based model predictive controller (MPC). The analysis is performed using both, the simulations, and the laboratory experiments. The carbon footprint and energy consumption of the heat exchanger are analysed and compared. The paper investigates whether the optimization-based approach subject to the constraints on manipulated variables and controlled variables, considering economic criteria, simultaneously, reduces the carbon footprint of a laboratory heat exchanger control.

## 1. Introduction

To achieve the aim of the EU plan for a climate-neutral economy by 2050, it is necessary to adopt the concepts of low carbon energy systems in industrial production (Oluleye, 2020). The implementation of carbon reduction technologies is insufficient in comparison to the possible opportunities. Therefore, to increase the efficacy, they should be realised in a systematic way balancing the investment and operating costs (Lameh et al., 2020). Heat exchangers are widely used in various branches of industry. The control of heat exchangers represents a challenging task due to their non-linear and asymmetric behaviour, affected by various uncertain and time-varying parameters, such as fouling (Trafczynski et al., 2018). The challenges of heat exchanger control and the analysis of its energetic consumption are provided in Oravec et al. (2020). In Fratzak et al. (2018), the experimental control of liquid-liquid heat exchanger is conducted considering balance-based adaptive controller. The model predictive control (MPC) attracted a high interest of both, academia, and industry in the past three decades, e.g., see Morato et al., (2020). Successful industrial implementations of MPC, and its numerous variations, lead to improved control performance, reduced costs, and decreased energy consumption (Forbes et al., 2015). A novel data-driven robust control framework based on the model predictive control design for greenhouse temperature and CO<sub>2</sub> concentration was proposed in Chen and You (2020).

This paper analyses the operation of a laboratory plate heat exchanger controlled by a proportional-integral-derivative (PID) controller and by an optimization-based MPC. The analysis is performed using both, the simulations, and the laboratory experiments. The carbon footprint and energy consumption of the heat exchanger are analysed and compared. The paper investigates whether the optimization-based approach subject to the constraints on manipulated variables and controlled variables, considering economic criteria, simultaneously, reduces the carbon footprint of a laboratory heat exchanger control.

## 2. Controlled plate heat exchanger

### 2.1 Laboratory plate heat exchanger

The heat exchanger is a plant that ensures heat transfer from the heating or cooled liquid to the heated or cooling liquid. The considered laboratory plate heat exchanger is manufactured by Armfield, see Armfield (2007) and it is classified as a three-stage indirect liquid-liquid plate heat exchanger. The diagram of the considered plate heat exchanger is depicted in Figure 1 and both, hot and cold, liquids are water. Two retention tanks store the input cold liquid (Figure 1, device II), the input hot liquid is preheated using a heating coil inside a tank (Figure 1, device III). The dosing of hot and cold liquid is ensured using peristaltic pumps (Figure 1, device IV and device V). In the main closed-loop control setup, depicted in Figure 1 with blue colour, the considered controlled variable is the output temperature  $T_1$  of the cold liquid. The manipulated variable, i.e., the control input, is the flow rate  $q$  of the hot liquid. The additional closed-loop control is depicted in Figure 1 in green colour. This closed loop ensured a constant temperature  $T_2$  of the input fluid using a proportional controller. Further technical details are listed in Oravec et al. (2016).

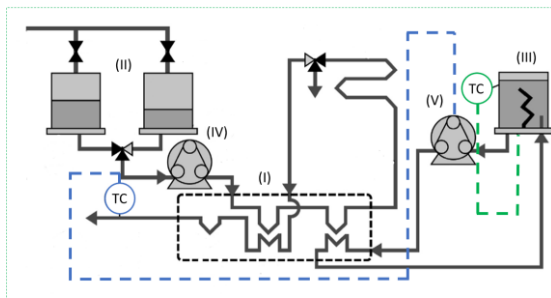


Figure 1: Diagram of the laboratory plate heat exchanger of Armfield PCT23 (I), two retention tanks storing input cold fluid (II), retention tank for preheating input hot fluid (III), peristaltic pump dosing cold fluid (IV), and peristaltic pump dosing hot fluid (V).

### 2.2 Mathematical model of the plate heat exchanger

For the controller design, it is necessary to obtain the proper mathematical model. In this case, experimentally measured step responses were considered to determine the parameters of the system. The considered plant is characteristic of non-linear and asymmetric behaviour. Therefore, multiple increasing and decreasing step changes of manipulated variable were evaluated. The final model was computed as the average of minimal and maximal values of the system gain  $Z$  and time constant  $T$ . The value of time delay was neglected. Table 1 summarizes the boundary values of the identified model, i.e., the minimum and maximum values of the system gain and time constant, and the average values of the nominal model of the plant. The nominal model of the heat exchanger was transformed into a state-space system in the discrete time-domain:

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k), \quad x(0) = x_0, \quad (1)$$

where  $k$  is the discrete-time increment,  $x(k)$  is the vector of measured system states,  $u(k)$  is the manipulated variable, i.e., volumetric flow rate, and  $y(k)$  is the system output, i.e., measurable output temperature of the heated liquid. The matrices  $A$ ,  $B$ ,  $C$  of appropriate dimensions represent the system, input, and output matrix, respectively.

Table 1: Parameters of the uncertain system model in the discrete-time domain.

System parameters	$Z$ [°Cs/ml]	$T$ [s]
Minimum	0.32	13.16
Maximum	0.51	29.40
Average	0.41	21.28

## 3. Controller Design

This section describes the considered controllers, i.e., PI controller and MPC controller. They were chosen to compare the control performance of the most common industrial controller – PI controller and a more complex optimisation-based controller - MPC.

### 3.1 PI Controller Design

First, a simple PI controller was employed to control the heat exchanger. When considering the PI control, the manipulated variable, i.e., control input  $u(t)$ , is computed in the continuous time domain according to the following control law:

$$u(t) = K_P e(t) + \frac{K_P}{T_I} \int_0^t e(t) dt. \quad (2)$$

In the defined PI control law,  $K_P$ ,  $T_I$  are the proportional and integral coefficients, and  $e(t)$  is the control error computed as follows:

$$e(t) = y(t) - r(t), \quad (3)$$

where  $r(t)$  is the reference variable. The parameters  $K_P$  and  $T_I$  define the control performance of the designed controller. The presence of an integral part  $T_I$  ensures offset-free reference tracking. In this case study, the parameters  $K_P$  and  $T_I$  were designed and tuned with a systematic experimental approach, see Astrom and Haglund (1994). First step of tuning is to slowly increase the value of parameter  $K_P$  until the controlled variable in the steady-state reaches approximately 80% of the reference variable, while the integral action is disabled. The next step is to enable integral action and to increase the value of ratio  $K_P/T_I$  representing the integral action, until the controlled variable removes the steady-state tracking error and, simultaneously, the control trajectory reaches the reference value in minimum settling shortest time. The disadvantage of the method is that it takes lots of experiments to determine the values of  $K_P$  and  $T_I$ .

### 3.2 MPC Design

To offer an alternative to a simple PI controller, optimisation based MPC was designed. To ensure the offset-free tracking of the reference variable, integral action was considered during the controller synthesis. The vector of system states was augmented in the following way:

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ x_1 \end{bmatrix} = \begin{bmatrix} x(k) \\ \sum_{j=0}^k e(k) \end{bmatrix}, \quad (4)$$

where  $\tilde{x}$  is the augmented vector of states,  $x_1$  is the integral action state. The nominal system in Eq(1) was rewritten subject to the augmented vector of states:

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k), \quad y(k) = \tilde{C}\tilde{x}(k), \quad \tilde{x}(0) = \tilde{x}_0, \quad (5)$$

where  $\tilde{A}$ ,  $\tilde{B}$ ,  $\tilde{C}$  are state-space matrices constructed with respect to the integral action. MPC optimizes a sequence of optimal control inputs at each sampling time. In the optimisation problem, MPC considers the behaviour of the model and respects constraints on outputs, inputs, and states. Even though MPC evaluates a whole sequence of optimal control inputs, it applies only the first computed control input, to ensure its predictive properties. The objective function of the optimisation problem is formulated in the following way:

$$\min_{u(k)} \sum_{k=0}^{N-1} (Q\|\tilde{x}(k)\|_2^2 + R\|u(k)\|_2^2), \quad (6)$$

where  $N$  is the prediction horizon that determines the length of the optimized sequence of control inputs. The matrices  $Q$  and  $R$  represent the tuning parameters of MPC, both are positive definite and diagonal. These matrices determine the control performance of the designed MPC. The manipulated variables, the system states and outputs could be constrained within symmetric inequalities in the form:

$$y_{\min} \leq y(k) \leq y_{\max}, \quad u_{\min} \leq u(k) \leq u_{\max}, \quad x_{\min} \leq x(k) \leq x_{\max}, \quad (7)$$

where  $y_{\min}$ ,  $y_{\max}$ ,  $u_{\min}$ ,  $u_{\max}$  and  $x_{\min}$ ,  $x_{\max}$  are the limit values of the corresponding variables. As MPC is a predictive controller, another constraint is represented by the augmented model of the plant defined in Eq(5).

## 4. The Analysis of the Carbon Footprint

The control of the heat exchanger is compared subject to the carbon footprint generated by each of the considered controllers. This section describes the computation of the carbon footprint, which is, in fact, only an approximation, and it is based on several assumptions. In the laboratory conditions, the initial step of the control of a heat exchanger is to heat the hot liquid from laboratory temperature approximately 20 °C to the reference

temperature of the hot liquid  $T_{2,ref} \approx 70$  °C. The heating is ensured using a P controller, where the energy of the heating coil  $P$  in kW is the manipulated variable and  $T_2$  is the controlled variable. The heating process of the hot liquid is depicted in Figure 2.

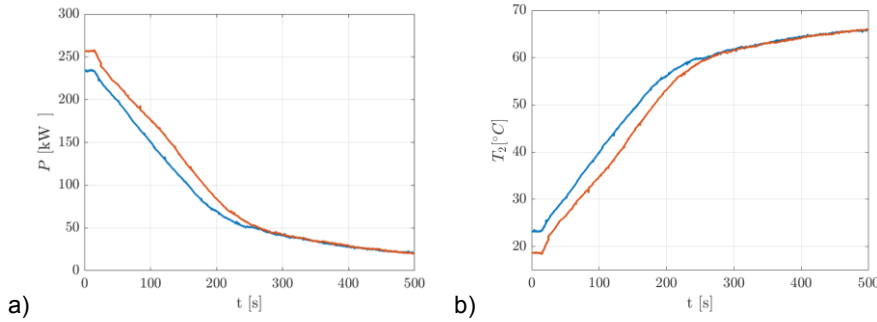


Figure 2: The energy consumption of the heating coil (a) and the temperature of the hot liquid (b) first experiment (red), second experiment (blue).

The main simplifying assumption in the computation of the carbon emissions is the fact that coefficient  $c$  is computed for a setup where a fresh liquid at laboratory temperature enters the heating tank to be heated to  $T_{2,ref} \approx 70$  °C. In fact, the hot liquid enters the heating tank again, after it exited the heat exchanger as can be seen in Figure 1. To compute the carbon footprint of the heat exchanger, first, we need to determine the coefficient  $c$  in kWh/ml, which reflects the amount of consumed energy required to heat 1 ml of liquid. The heated liquid is stored in a retention tank (Figure 1, device III) with volume  $V_{heater} = 2,650$  ml from laboratory temperature to the reference temperature of the hot liquid  $T_{2,ref}$ . The coefficient  $c$  can be determined as follows:

$$c = \frac{1}{V_{heater}} \int_0^{t_c} P(t) dt, \quad (8)$$

where  $t_c$  is the time span of the experiment. As can be seen in Figure 2, the time span is  $t_c = 500$  s. From this time the temperature of the heated liquid was settled and at this time the water was fully heated to the setpoint temperature. To increase the accuracy of the computation of carbon footprint, the heating process of the hot liquid was measured twice. In Figure 2, one experiment is depicted in red colour and the other one in blue. The coefficient  $c$  was computed for both experiments using Eq(8). The final value of the constant  $c$  is computed as an average value. The next step is to determine how much of the hot liquid is fed into the heat exchanger during control  $q_{total}$ . As the flow rate of the hot liquid is the manipulated variable, the value of  $q_{total}$  is determined by the controller and is computed as follows:

$$q_{total} = \int_0^{t_c} q(t) dt \approx \sum_{k=0}^N t_s q(k), \quad N = \frac{t_c}{t_s}, \quad (9)$$

where  $t_c$  is the time span of the experiment,  $t_s$  is the sampling time. As can be seen in Figure 3 and Figure 4, the time span is  $t_c = 200$  s when considering simulation results and  $t_c = 450$  s when considering experimental results. The consumption of the hot fluid is computed in the discrete-time domain when the MPC controller is employed, and in the continuous time domain when the PID controller is considered.

The coefficient  $c$  in Eq(8) determines how much energy is needed to heat 1 ml of the liquid,  $q_{total}$  reflects how much liquid is consumed during the control. Therefore, the multiplication of these parameters determines how much energy is consumed to heat the laboratory plate heat exchanger. The last step is to determine the emission factor. The emission factor  $f_{CO_2}$  is the amount of CO<sub>2</sub> in tons emitted into the atmosphere during the production of 1 MWh of electricity. This factor depends on the percentage shares of each energy source in electricity generation. The emission factor is different for every country and changes each month. The experimental results of this case study were measured in November 2020 at Slovak University of Technology in Bratislava (Karaffová, 2021). According to the data provided by the company supplying the electricity, the corresponding emission factor was (ZSE, 2020):

$$f_{CO_2} = 0.252 \text{ t MWh}^{-1}. \quad (10)$$

Based on all the defined parameters, the amount of carbon emissions associated with heat exchanger control is computed according to the following formula:

$$m_{CO_2} = c q_{total} f_{CO_2}. \quad (11)$$

## 5. Results and discussion

The numerical simulations and experimental results of the closed-loop control were evaluated using a CPU i5-8250U 1.80 GHz, 8 GB RAM. The *MATLAB/Simulink* R2019a programming environment was considered to generate the numerical simulations and experimental results. The formulation of the optimization problems was handled by the *YALMIP* toolbox (Löfberg, 2004). The optimization problems were solved by the MATLAB built-in solver *quadprog*. The constraints on the manipulated variable in Eq(7) were:  $0 \leq q(k) \leq 12 \text{ ml s}^{-1}$ . In this case, we did not consider constraints on the system states and outputs. In the simulation results, the pairs of the weighting matrices in Eq(6) were set to:  $Q = \text{diag}([10, 1])$ ,  $R = 0.1$ , the prediction horizon  $N = 10$ , and the sampling time was  $t_s = 0.1 \text{ s}$ . The parameters  $K_p$  and  $T_I$  were tuned to correspond:  $K_p = 10$  and  $T_I = 2$ . In the experimental results, the pairs of the weighting matrices were set to:  $Q = \text{diag}([0.5, 0.1])$ ,  $R = 10$ , the prediction horizon  $N = 10$ , and the sampling time during the experiments was  $t_s = 3 \text{ s}$ . The parameters  $K_p$  and  $T_I$  remained the same.

The reference tracking problem was analysed subject to step changes of the reference value:  $40 \text{ }^\circ\text{C} \rightarrow 45 \text{ }^\circ\text{C}$ . The simulation results are depicted in Figure 3 and experimental results are depicted in Figure 4. Table 2 summarizes the evaluated values of the carbon footprint for the designed controllers.

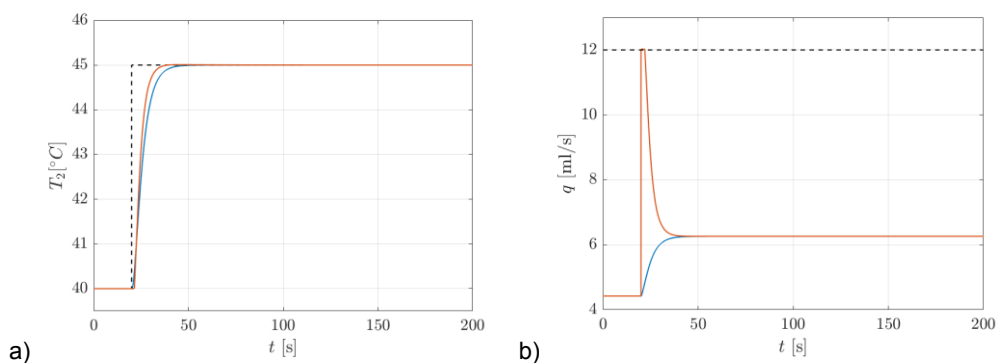


Figure 3: Results of numerical simulations: (a) controlled variable and (b) manipulated variable: PI controller (red), MPC (blue), reference/constraints (dashed black).

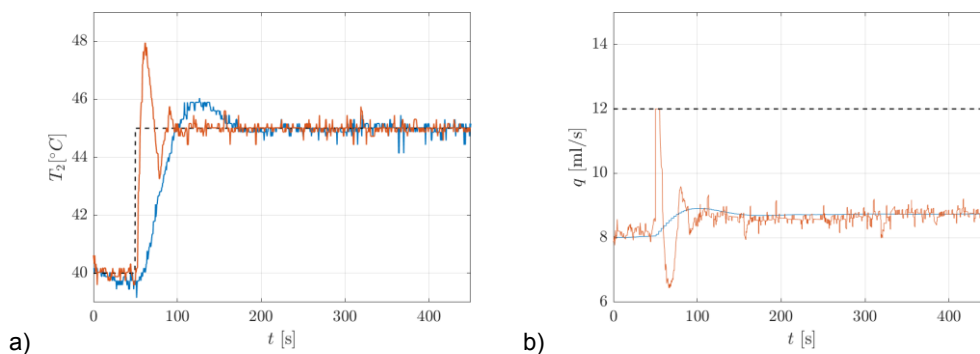


Figure 4: Results of laboratory implementation: (a) controlled variable and (b) manipulated variable: PI controller (red), MPC (blue), reference/constraints (dashed black).

Table 2: The evaluation of carbon footprint for the designed controllers.

Controller	$m_{\text{CO}_2}$ – simulations [g]	$m_{\text{CO}_2}$ – experiments [g]
PI	19.5	58
MPC	8.5	59

As can be seen in Figures 3-4, the designed controllers removed the steady-state control error in both simulations and in experimental results. Although the steady-state control error was removed, the overshoot of the controlled variable was observed in the experimental results. PI controller was not able to consider constraints during its design. On the other hand, in the case of the MPC, the constraints on manipulated variable

were considered during the design and satisfied during the control. The generated control performance (Figure 3 and 4) may be further tuned to remove the overshoot. The carbon footprint could also be decreased by more extensive tuning and by increasing the weighting matrix  $R$ , in the MPC formulation. A significant advantage of MPC is its ability to directly consider the minimization of carbon footprint in its cost function and its ability to respect constraints.

## 6. Conclusions

This paper analyses the operation of a laboratory plate heat exchanger controlled by the PI controller and by an optimisation based MPC controller. Moreover, we have investigated the carbon footprint of the heat exchanger control to compare the designed controllers. The analysis included both, the simulations, and the laboratory experiments. The simulation results confirmed the optimization-based approach subject to the constraints on manipulated variables, considering economic criteria, reduces the carbon footprint of a laboratory plate heat exchanger control. The experimental results, however, generated comparable results for both types of controllers. Nevertheless, a significant advantage of MPC is its ability to directly consider the minimization of carbon footprint in its cost function and its ability to respect constraints. Therefore, by more intensive experimental tuning the carbon footprint of a heat exchanger could be further decreased. Further research will focus on more intensive tuning of a designed MPC controller, implementation, and analysis of other control methods.

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## References

- Armfield, 2007, PCT 23: Process Plant Trainer, manual, Armfield Limited, Ringwood, UK.
- Astrom K.J., Hagglund T., 1994, PID Controllers, Theory, design and Tuning (2<sup>nd</sup> Edition), ISA - The Instrumentation, Systems and Automation Society, Research Triangle Park, North Carolina.
- Chen W.H., You F., 2020, Data-driven robust optimization for greenhouse temperature control using model predictive control. *Chemical Engineering Transactions*, 81, 721–726.
- Forbes M.G., Patwardhan R., Hamadah H., Gopaluni R.B., 2015, Model predictive control in Industry: challenges and opportunities, *IFAC-PapersOnLine*, 48(8), 531-538.
- Fratzak M., Czczot J., Nowak P., Metzger M., 2018, Practical validation of the effective control of liquid–liquid heat exchangers by distributed parameter balance-based adaptive controller, *Applied Thermal Engineering*, 129, 549-556.
- Karaffová K., 2021 Analysis of carbon footprint generated by advanced control of heat exchanger (in Slovak), Master's thesis, supervisor Oravec J., Slovak University of Technology in Bratislava.
- Lameh M., Al-Mohannadi D.M., Linke P., 2020, Developing Minimum Cost Targets for Carbon Reduction in Different Geographical Regions, *Chemical Engineering Transactions*, 81, 97–102.
- Löfberg, J., 2004, Yalmip: A Toolbox for Modelling and Optimization in MATLAB. *Proc. of the CACSD Conference*, Taipei, Taiwan, 284 – 289.
- Morato M.M., Normey-Rico J.E., Senane O., 2020, Model predictive control design for linear parameter varying systems: A survey, *Annual Reviews in Control* 49, 64-80.
- Oluleye G., 2020, A Novel Optimisation Framework to Support Increased Uptake of Low Carbon Industrial Energy Systems, *Chemical Engineering Transactions*, 81, 1063–1068.
- Oravec J., Horváthová M., Bakošová M., 2020, Energy efficient convex-lifting-based robust control of a heat exchanger, *Energy*, 201, 117566.
- Oravec J., Bakošová M., Mészáros A., Miková N., 2016, Experimental Investigation of Alternative Robust Model Predictive Control of a Heat Exchanger, *Applied Thermal Engineering*, 105, 774 – 782.
- Trafczynski, M., Markowski, M., Urbaniec, K., Grabarczyk, R., 2018, Energy saving potential and the efficacy of using different control strategies for the heat exchanger network operation, *Chemical Engineering Transactions*, 70, 823-828.
- ZSE Energia, a.s. <<https://www.zse.sk/english>> accessed 02.11.2020.