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Heating Optimisation of a Multi-Zone Building's Thermal Comfort Under Stochastic Condition using Data-Driven Model Predictive Control

Guoqing Hu*, Fengqi You

Cornell University, Ithaca, NY 14853, USA
gh429@cornell.edu

Model Predictive Control (MPC) has gained popularity in recent years and is widely adopted in building control. This study proposes a novel data-driven robust MPC to make the optimal heating plan, specifically for the multi-zone single-floor building. In this study, the room temperature and relative humidity (RH) will be highly valued in the optimisation decision. To better incorporate RH into the state-space model (SSM), the linear relations between RH and other room temperature parameters in the thermal zones are formulated, ensuring the better linear fitting of SSM to the original nonlinear model. Afterward, k-means clustered, principal component analysis (PCA), and kernel density estimation (KDE) based data-driven uncertainty set is constructed and applied to MPC. The other three kinds of MPC's are compared to our proposed data-driven robust MPC (RMPC), including conventional RMPC, k-means clustered, data-driven RMPC (KM-DDRMPC), PCA and KDE based data-driven RMPC (PKDDRMPC). The results demonstrate that the optimality of our proposed k-means clustered, PCA and KDE based data-driven RMPC (KM-PKDDRMPC), which consumes 9.8 % to 17.9 % less energy in controlling both temperature and RH, compared to other data-driven robust MPC's, and essentially follow the constraints which certainty equivalent MPC and conventional RMPC cannot conform.

1. Introduction

The need for energy usage is increasing with the growing population in recent years (Shi et al., 2016), especially for the building control, which occupies 40 % of the total energy production (Shaikh et al., 2014). According to the EIA report in 2019, heating and humidity control dominate energy usage, contributing 30 % of total power consumption. Controlling temperature is essential to the building control since overheating is another problem that consumes significant energy and deteriorates the living condition.

Among all possible control methods, model predictive control (MPC) provides the new scope for controlling the building temperature (Chu and You, 2015), saving a tremendous amount of energy usage compared to the rule-based control strategies (Prívara et al., 2011). However, the conventional MPC does not possess the capability of hedging against the uncertainty, i.e. being applied under stochastic conditions (Oravec et al., 2016). In building control, weather information is treated as one of major source of disturbances since ambient weather has a remarkable impact to the building temperature and relative humidity, plus the weather information can never be perfectly predicted. In summary, there are some limitations of current research: first, although there is some research which studied for the humidity control, they did not focus on multi-zone building control. Finally, Yang et al. (2019) only focused on the application of conventional robust MPC (RMPC) to the building control, which is proved as "conservative" by Chen and You (2020), indicating conventional RMPC uses more power to follow the constraints under the stochastic condition. Finally, much of the research did not consider RH as one of the factors in building control. the importance of relative humidity (RH) cannot be ignored because RH is tightly related to thermal comfort (Jing et al., 2012).

Therefore, in contrast to previous studies, we focus on developing the better control strategy to multi-zone building's room temperature and RH under realistic condition, k-mean clustered, principal component analysis (PCA) and kernel density estimation (KDE) based data-driven RMPC (KM-PKDDRMPC). We apply this model

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to the multi-zone building's SSM, which incorporates both room temperature and RH. In this work, the SSM of the building is generated from based on both building element construction and the study of the dynamic airflow within the building. Afterward, the uncertainty set is constructed based on the historical forecast error to the weather information, i.e., the differences between forecast and real-measured values. This uncertainty set can be further clustered by the k-means algorithm, and PCA combined with KDE can return the polyhedral-shaped applied to the RMPC. The optimisation problem at each control horizon is solved with the help of affine disturbance policy (ADF). The contribution of this paper is summarised as follows:

- A novel data-driven robust model predictive control framework with disjunctive uncertainty to control the multi-zone building's room temperature and RH;
- A simulation of multi-zone building's temperature and RH control based on actual weather data demonstrates better control performance of KM-PKDDRMPC comparing to other MPC's

2. Model formulation

2.1 Complete state-space model

The BRCM MATLAB toolbox is adopted for finding the state space matrix (SSM). BRCM can generate the linear resistance-capacitance models from self-designed building geometry construction. The following dynamic multi-input multi-output system can be returned:

$$x_{t+1} = Ax_t + B_u u_t + B_v v_t + B_w w_t \tag{1}$$

Where A is the state matrix that correlates state variables x_t to SSM. The state variables returned from BRCM are room temperature, wall temperature, floor temperature and ceil temperature. B_u, B_v, B_w are control input matrix, disturbance matrix, and uncertainty matrix, respectively, corresponding to u_t, v_t, w_t, which are control input, disturbances, and uncertainty. The control inputs include heater, radiator, humidifiers and dehumidifiers; the disturbances are from ambient temperature and ambient relative humidity condition. Uncertainties are the forecasted temperature and relative humidity errors. Meanwhile, RH within each room is calculated based on the air dynamic within the building (Cengel, 1997). The mass of airflow is initially found as:

$$m_{air,t-1} = \frac{Q_{t-1}}{c_p \Delta T} \tag{2}$$

ΔT is calculated as follows:

$$\Delta T = \max\left(\left(T_{room,t} + \delta T - T_{air,t}\right), 0\right) \tag{3}$$

Unlike in previous research, $m_{air,t-1}$ is not regarded as a constant because the simulation process is conducted in the winter season. The constant intake airflow rate indicates that the room is constantly exchanging the air with a colder ambient environment. The heater, most of the time, is active to maintain room temperature within the thermal comfort standard. Alternatively, we assume the difference between the room temperature and heated air from the air heating unit (AHU) is constant. Subsequently, the heating airflow can be turned off when heating is not necessary. When the mass of airflow is calculated, the mass of water vapor brought by airflow can be found by the following equation:

$$m_{AC,in,t-1} = \rho_{AC,in,t-1} \cdot RH_{out,t-1} \cdot \frac{m_{air,t-1}}{\rho_{air}} \tag{4}$$

And so can be found the mass of water vapor taken away by airflow:

$$m_{AC,out,t-1} = \rho_{water,sat,t-1} \cdot RH_{t-1} \cdot \frac{m_{air,t-1}}{\rho_{air}} \tag{5}$$

Where saturated vapor density (SVD) values are found through equation f, which is a linear equation of SVD values over temperature (T) expressed as follows:

$$\rho_{water} = f(T) = 1.0272T - 1.8959 \tag{6}$$

Afterward, the mass of water vapor stored in each room can be found as:

$$m_{water,t} = \rho_{water,sat,t-1} \cdot RH_{t-1} \cdot V_{room} + m_{humidifier} + m_{AC,in,t-1} - m_{dehumidifier} - m_{AC,out,t-1} \tag{7}$$

Eventually, RH values within each room at t can be found, which is the ratio of absolute and saturated water vapor density:

$$\rho_{abs,t} = \frac{m_{water,t}}{V_{room}},\ \rho_{sat,t} = f\left(T_{room,t}\right),\ RH_t = \frac{\rho_{abs,t}}{\rho_{sat,t}} \tag{8}$$

At this point, the RH values within each room can be found based on the room temperature, control input and room volume. The next step is to add system identification toolbox found in MATLAB to obtain the SSM required for the MPC. After SSM is obtained from system identification, the comparison of SSM to original nonlinear model is presented in Figure 1. The testing data, instead of training data, is used to ensure the feasibility of SSM to be applied in simulation within the real condition. The average value of mean absolute percentage error (MAPE) for RH in all rooms is 4.65 % and the average MAPE for temperature in all rooms is 0.95 %, indicating this model is acceptable for the MPC problem.

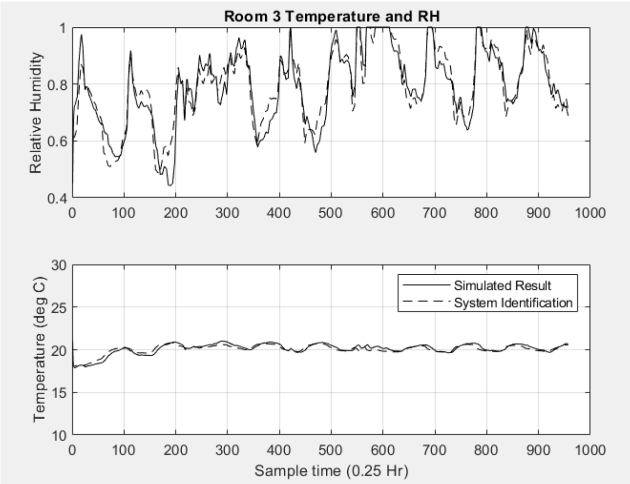

Figure 1. System identification on testing data. The MAPE for RH, in this case, is 5.11 %, and for temperature is 0.97 %

2.2 PCA and KDE based data-driven uncertainty sets clustered by K-means algorithm

Disjunctive uncertainty sets are constructed to better learn the trend of the uncertainty data (Ning and You, 2017). The K-means clustering method is adopted to cluster the uncertainty into multiple groups. The groups are identified by minimising the sum of intracluster variances, i.e., squared Euclidean Distance shown below:

$$D^* = \arg\min\left(\sum_{i=1}^{k}\sum_{w\in D_i}\left\|w-\mu_i\right\|^2\right) \tag{9}$$

Despite multiple groups of uncertainty data, the traditional norm-based uncertainty set cannot be applied directly to deal with the uncertainty data with varied structure and complexity. Therefore, PCA and KDE are adopted for coping with the data with polyhedral shapes (Zhao et al., 2019). PCA can then maximise the variance of the uncertainty under the same scale. The covariance matrix can be approximated as

$$S_i = \frac{1}{N-1} w_i^T w_i \tag{10}$$

As the covariance matrix S_i can be further decomposed as $S_i = Q_i \Lambda_i Q_i^T$, where Q_i's column contains all the eigenvectors, corresponding to the eigenvalues stored in the diagonal matrix Λ_i (Jolliffe and Cadima, 2016). The individual eigenvalue will represent the variance of this axis if data is projected on this eigenvector.
Finally, it can be further studied the distributional information of the uncertainty dataset within each component j within the cluster k via the KDE approach (Zhang et al., 2006):

$$f_{j,k} = \frac{1}{N}\sum_{n=1}^{N} K(\xi_{j,k}, p_{j,k}^{(n)}) \tag{11}$$

With probability density function, the cumulative density function will be written as follows (Ning et al., 2018):

$$F_{j,k}^{-1}(\alpha) = \min\{\xi_{j,k} \mid F_{j,k}(\xi_{j,k} \geq \alpha)\} \tag{12}$$

$$\mathrm{W}^k = \left\{ \mathbf{w}^k \in R^H \;\middle|\; \begin{array}{l} \mathbf{w}^k = \hat{\boldsymbol{\mu}}_k + Q_k \boldsymbol{\xi}_k,\ \boldsymbol{\xi}_k = \underline{\boldsymbol{\xi}}_k z^- + \overline{\boldsymbol{\xi}}_k z^+ \\ \mathbf{0} \leq z^+, z^- \leq \mathbf{1},\ \ z^+ + z^- \leq \mathbf{1},\ \ \mathbf{1}^T\left(z^+ + z^-\right) \leq \Gamma \\ \underline{\boldsymbol{\xi}} = \left[\hat{F}_{1,k}^{-1}(\alpha), \ldots, \hat{F}_{1,k}^{-1}(\alpha)\right]^T \\ \overline{\boldsymbol{\xi}} = \left[\hat{F}_{1,k}^{-1}(1-\alpha), \ldots, \hat{F}_{1,k}^{-1}(1-\alpha)\right]^T \end{array} \right\} \tag{13}$$

where α is the pre-specified small quantile parameter, ranging from 0 to 0.5, and ξ is the inferred latent variable. The uncertainty set W_k within cluster k can be formulated under the introduction of forward and backward deviation variable z^+ and z^- (Ning and You, 2018). There are multiple approaches to data-driven robust optimisation (Ning and You, 2019), such as support vector clustering-based approach (Shang and You, 2017), but we chose the proposed one due to the computational efficiency.

3. Control Strategy

After the acquisition of the SSM required for MPC and uncertainty sets, the next step is to develop the optimisation problem to get the control strategy to the multi-zone building. To ensure the tractability of the RMPC optimisation problem, ADF is adopted to get control input u_t based on past disturbances (Shang et al., 2019). The equation is expressed as following (Goulart et al., 2006):

$$u_i = h_i + \sum_{j=0}^{i-1} M_{i,j} w_j, \ \forall w \in \mathrm{W}^k \tag{14}$$

Where M is regulated as follows:

$$M = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ M_{1,0} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ M_{H,0} & M_{H,1} & \cdots & 0 \end{bmatrix} \tag{15}$$

Only the first u_0 will be applied for the control to the model and the rest will be discarded. The optimisation problem with ADF can be formulated as follows:

$$\begin{aligned} &\min \sum_{i \in B_u} c_i u_i + \lambda^T L \lambda \\ &\text{s.t. } F_u[Mw + h] \le f_u, \qquad \forall w \in \mathrm{W}^k \\ &\quad F_x[Ax_0 + B_u h + B_v v + (B_w + B_u M)w] \le f_x + \lambda, \quad \forall w \in \mathrm{W}^k \end{aligned} \tag{16}$$

Where F_x, F_u, f_x, f_u represent the state variable constraints matrix, control input constraints matrix, constraints for state variables, and constraints for the input. L is the weighted cost matrix that penalises the violation to the constraints. Λ is the slack variable that allows some extent of violation to the hard constraints.

4. Case Study

4.1 Problem statement

In this study, the single-floor multi-zone building located in Ithaca, New York, USA is selected for the simulation of close-loop data-driven RMPC to control the temperature and relative humidity in each individual room. The self-constructed floor plan can be referred to Figure 2

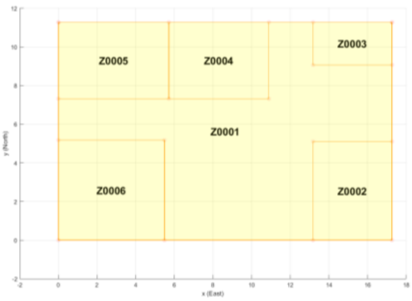

Figure 2. Floor plan of the single-floor multi-zone building

The constraints for the control conditions are: For the room temperature should be within 15 ℃ to 25 ℃, and relative humidity should sit in between 30 % to 60 %, according to ASHRAE Standard 62-2001.

4.2 Result and discussion

The model was simulated in Ithaca, New York, from 0:00 AM, November 1st, 2016 to 0:00 AM, on November 8th, 2016, ranging from precisely one week. The initial conditions for temperature values in all rooms are 21 ℃ and RH values are 40 %. Room 3's results are selected for demonstration, as shown in Fig 3. The rest of the information is summarised in Table 1 and Table 2. Based on the result, both certainty equivalence MPC (CEMPC) and RMPC violate the constraints more severely. Certainty equivalent MPC, which only considers the deterministic conditions, fails to compose the strategy against the prediction error from ambient temperature and RH. Meanwhile, the RMPC fails to obey the relative humidity constraints, indicating an irregular shape of the uncertainty data of relative humidity. On the other hand, the rest three control strategies can be more conservative in maintaining both temperature and relative humidity within the constraints. K-means clustered data-driven RMPC (KM-DDRMPC) will be the most conservative one since there is nearly no violation at all, but, it will have the highest power consumption across all control methods. On the other hand, though there are slightly more violation cases and more computation time, KM-PKDDRMPC will draw significantly less power in controlling the temperature and relative humidity compared to KM-PKDDRMPC and PCA coupled with KDE based data-driven RMPC (PKDDRMPC).

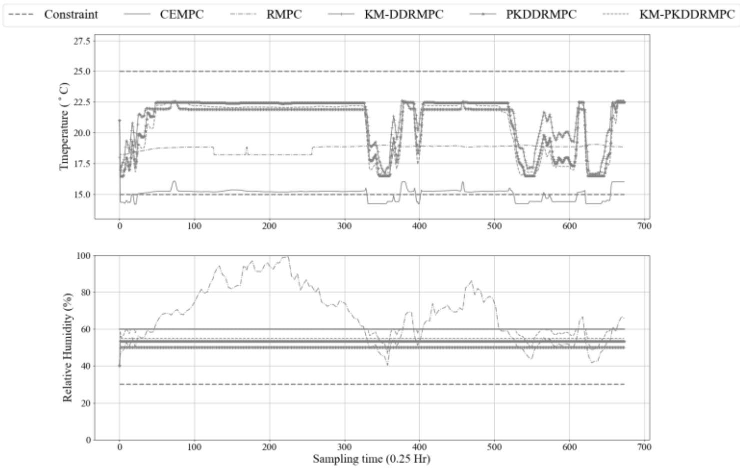

Figure 3. Multi-zone building control profile in Ithaca, New York, in the first week of November, 2016

Table 1. Constraint violation percentage of each controller

	Temperature	Relative humidity
CEMPC (%)	50.85	63.18
RMPC (%)	0.00	59.61
KM-DDRMPC (%)	0.12	0
PKDDRMPC (%)	1.55	0
KM-PKDDRMPC (%)	0.77	0

Table 2. Power consumption of each controller

	Temperature control	Relative humidity control
CEMPC (W)	116.87	132.09
RMPC (W)	114.14	0.00
KM-DDRMPC (W)	162.49	235.08
PKDDRMPC (W)	160.98	209.56
KM-PKDDRMPC (W)	159.10	201.84

5. Conclusions

In this work, we develop a KM-PKDDRMPC framework for the multi-zone building SSM, which includes indoor temperature and relative humidity control. In order to maintain temperature and relative humidity within the comfortable range, KM-PKDDRMPC is capable of handling the uncertainty sets from temperature and relative humidity forecast. The steady-state system with relative humidity is constructed with the help of system identification. Then the optimisation problem can be further developed with the SSM and disjunctive uncertainty sets. The proposed KM-PKDDRMPC was compared with the CEMPC and other MPC strategies, including RMPC, KM-DDRMPC, PKDDRMPC. The result demonstrated that the proposed KM-PKDDRMPC has outperformed the rest from the overall perspective, using 17.9 % less power consumption than KMDDRMPC and 9.8 % fewer compared to PKDDRMPC. Though CEMPC and RMPC have used less power, the high violation rate will exclude them from the final consideration to the practical application.

Nomenclature

SVD – Saturated vapor density
$m_{air,t-1}$ – mass of airflow at t-1, kg
Q_{t-1} – heat input at t-1, J
c_p – specific heat of air, kJ/(kg-K)
ΔT – temperature change, ℃
$T_{room,t}$ – room temperature at t, ℃
$T_{air,t}$ – ambient temperature at t, ℃
δT – temperature difference of room and AHU, ℃
ρ_{air} – air density, kg/m^3
RH_{t-1} – relative humidity in room at t-1,-
$RH_{out,t-1}$ – ambient relative humidity at t-1,-
$\rho_{water,sat,t-1}$ – SVD of room temperature at t-1, g/m^3
$\rho_{AC,sat,t-1}$ – SVD of ambient temperature at t-1, g/m^3
V_{room} – room volume, m^3
$m_{water,t}$ – the mass of water vapor at t, kg
$m_{humidifer}$ – the mass of water vapor provided by humidifier, kg
$m_{dehumidifier}$ – the mass of water vapor taken by dehumidifier, kg
$m_{AC,in,t-1}$ – the mass of water vapor provided by air circulation at t-1, kg
$m_{AC,out,t-1}$ – the mass of water vapor taken by air circulation at t-1, kg
$\rho_{abs,t}$ – absolute water vapor density at t, kg/m^3
$\rho_{sat,t}$ – SVD at t, kg/m^3

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