

# Mathematical Decision Framework for Integrated Solar Thermal System Networks

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The rising energy demand and the depletion of fossil fuels have resulted in technology development to harness solar energy. Solar thermal technology provides a compelling alternative for energy conservation to provide heat energy to residential, commercial, and industrial processes. Designing an optimum solar thermal system for industrial operations is complex due to intermittent solar irradiance and temperature variance of process demand. A mathematical decision framework is needed to aid users in the decision-making process for the optimisation approach to design an integrated solar thermal network to identify the best configuration and optimal design to fulfil multiple sources and demand scenarios. This study aims to develop a decision-making framework for solar heat networks and optimal thermal energy storage (TES). The framework was applied to an illustrative case study with two scenarios based on full and 75 % load of heat demand. Based on the result of the case study, the framework can assist decision-making in designing an integrated solar thermal network. The results show that the excess solar yield from Plant 3 can be shared to Plant 2 at a range of 54.93–84.45 kW. The optimal capacity of TES is 183.6 m<sup>3</sup>, which can fulfil the demand in both scenarios. The decision framework successfully analysed and designed an integrated solar thermal network.

## 1. Introduction

Solar heat offers immense potential for various industrial processes. Most industrial heat is required in the low-medium temperature range, with temperatures not surpassing 250 °C (Schweiger et al. 2001), supplied by solar thermal technology. The application of solar thermal comes with several challenges. The energy harvest ability depends on the type and size of the solar thermal collector, its efficiency, ambient temperature, and mass flow rate. The source also relies on the amount of solar radiation available at a specific location, which varies daily and seasonally. Besides, the process parameters, heat demand for utilities and process, and available heat thermal generated from solar thermal are vital to design an optimal solar thermal system. Furthermore, a centralised thermal storage tank should be designed to the appropriate size. These tanks should not be too enormous to be particularly expensive and cover wide areas. The storage tanks should also not be too small to hold too little heat, or the heat would dissipate quickly. This problem can be solved by studying and analysing the integrated solar thermal networks design using suitable tools.

Optimisation tools can be used in the design of optimal and energy-efficient systems. Hence, the tools have been widely extended and become a standard approach for advanced design and optimisation in different industries. Petruschke et al. (2014) proposed a hybrid optimisation method for renewable energy system synthesis that comprises a heuristic-based pre-selection step to simplify and reduce suitable technology candidates and subsequent superstructure optimisation. A method for preliminary evaluation was also developed by Nemet et al. (2015), where a sequential approach was used for targeting the optimal solar collector area and size of the thermal storage tank. Besides, flowsheet superstructure and integration options for concurrent optimisation of the maximum share of feasible direct and indirect solar use through heat storage in industrial activities were performed by Abikoye et al. (2019). The models were applied to a dairy plant case study for two separate integration possibilities of targeting one and multiple cold stream utility requirements.

Nonetheless, several details on solar thermal technology are missing from the most recent process integration studies. Considering the changeable condition, the solar thermal integration should be built using a suitable process integration approach by optimising the amount of heat thermal that can be supplied by solar energy to reduce the dependence on fossil fuels.

Pinch Analysis and mathematical models are examples of numerical- and graphical-based process integration. Generally, Pinch Analysis can only be used to design a solar thermal system, whereas mathematic programming effectively understands many variables, and it is a rapid approach for evaluating an integrated solar thermal energy network and multi-period process heat demand. On the other hand, the mathematical model can be used for assessing solar system efficiency based on daily solar radiation analysis to model the optimal design of solar collectors and storage systems operation. Furthermore, the model can manage all design parameters for problems involving decision-making to obtain the best solar thermal integration configurations and the simulation can be repeated with different energy-saving targets.

Researchers have adopted mathematical modelling to optimise or simulate solar thermal integration in various industrial sectors. Isafiade et al. (2015) employed the multi-period stage-wise superstructure with an objective function incorporating simultaneous minimisation of annual investment and operation costs. Compared to the case without solar thermal integration, the study found that a reasonable amount of cost savings could be achieved. Bolognese et al. (2020) performed modelling analysis using MATLAB to demonstrate the dynamics and robust integration of solar heat for an industrial process located in a topographically complex territory. The study also evaluated the position of the sun and components of incident angle, and also compared three solar technologies. Tóth and Farkas (2019) compared two mathematical models of solar thermal devices containing two different heat storages. These studies focused on individual solar thermal system integration but did not consider thermal heat sources may be integrated from multiple sources and supplied to various demands.

This study developed a new framework for an optimisation model to demonstrate the feasible installation and validate the efficacy of proposed models for a centralised and decentralised thermal energy system that integrated industries with a similar heat demand range. Based on multiple desired heat demands, the multi-objective model will determine the amount of thermal energy generated based on the area available to install solar collectors and identify centralised energy storage capacity for an illustrative case study.

## 2. Problem description

The decentralisation of heat supply from centralised thermal storage to different heat demands can be met in an integrated solar thermal system by a combination of supply from multiple sources. Figure 1 illustrates an example of a typical conventional individual and an integrated solar thermal system. In an integrated system, the heat generated from the  $n$ th solar collectors,  $S_n$ , is stored in the  $n$ th thermal storage,  $T_n$ , where  $n$  is the number of solar collectors and thermal storage respectively before transferred to  $n$ th plants,  $D_n$ . The heat sources for the process can be supplied by a combination of solar and fossil fuels. In this system, fossil fuels are employed as a backup depending on how much of the process heat demand is met by solar energy. The problem is typically complex and challenging to solve, and a simultaneous solution is required for finding an optimal integrated solar thermal system. Mathematical modelling will be used for more holistic system analysis.

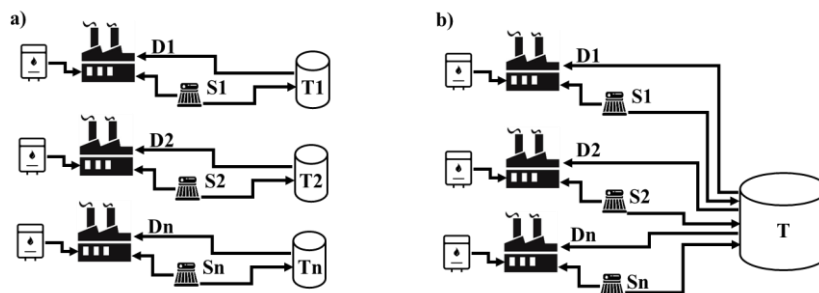


Figure 1: An example of (a) conventional and (b) an integrated solar thermal system with multiple supply and demand

## 3. Mathematical formulation

Several parameters, such as the global horizontal irradiation (GHI) and ambient temperatures ( $T_a$ ) for the plant location, solar collector efficiency ( $\eta_{coll}$ ), and its thermal coefficients ( $c_1$ ,  $c_2$ ) were determined to synthesise an optimal solar thermal network. The average GHI was used to represent the daytime of an operational year to determine the area required for the solar collector installation. The data were retrieved from the Photovoltaic

Geographical Information System (PVGIS) developed by the European Commission Joint Research Centre (2021). The mathematical decision models were formulated in MATLAB based on the data obtained to determine the solar collector output and the optimum thermal energy storage (TES) size.

Various assumptions and simplifications were applied to address the complex multiple supply and demand problem. Firstly, the availability of solar thermal energy is time-of-day and season-of-year dependent. For a tropical country, the actual amount of solar radiation that can be harnessed as heat energy may vary, especially during the monsoon season. Therefore, in this model, every solar collector is assumed to receive the same amount of solar radiation for the whole year. Secondly, the actual heat capacity flowrates of process streams may change from time to time due to plant start-ups and shut-downs, change in product quality, and demand. Therefore, in this study, the flow rates and the daily demand are assumed to be constant. Third, it is assumed that there are 5 % heat losses during storage. Finally, as the model does not consider economic analysis for the piping cost, the distance between all plants to the TES is assumed to be the same.

### 3.1 Decision-making framework

The main objective of the model is to evaluate the feasibility of an integrated solar thermal network, i.e., the number of solar collectors needed, the excess heat that can be transferred to the centralised thermal storage, and to determine the storage capacity necessary to obtain an optimal configuration. The equations will then be coded into MATLAB, and the results will be generated with solvers. The decision-making methods are shown in Figure 2.

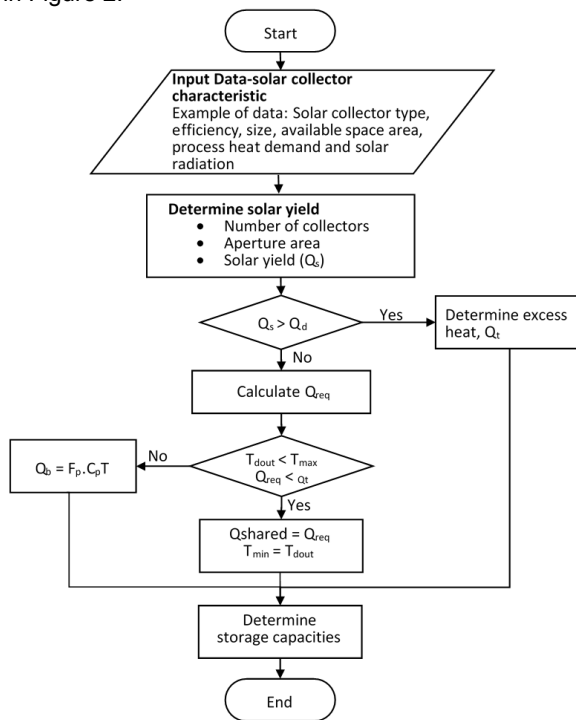


Figure 2: Algorithm to determine an integrated solar thermal system with multiple supply and demand

The number of collectors and the amount of potential heat generated by each source must be calculated to determine whether it can meet internal demand. The first step is the data collection and determination of input parameters. The solar yield ( $Q_s$ ) will be determined using Eq(1) based on the number of collectors ( $C$ ), collector aperture area ( $C_a$ ), solar radiation data, and solar loop temperatures. The sizing was performed based on available space area. The typical area utilisation factor for Malaysia is used for this study which is 0.8 for flat plate collectors.

$$Q_s = \frac{G_{max} \cdot \left(\frac{C}{C_a}\right) \cdot \eta_{coll}}{24} \quad (1)$$

Where  $G_{max}$  is assumed as the maximum irradiation (7.67 kWh/m<sup>2</sup>d) for the calculation of aperture area, and the efficiency of flat plate collectors,  $\eta_{coll}$  is defined as 60 % (Browmik and Amin, 2017). The number of collectors

was calculated based on the available area using Eq(2). This equation ensures that the collector area is not under-designed for days or seasons with low solar radiation and ambient temperatures.

$$C = \frac{(W - 2 * D_r)}{C_w + D_a} \cdot \frac{(L - 2 * D_r)}{D_c} \quad (2)$$

Where  $W$  is the width to place collectors per row,  $L$  is the length to place rows,  $D_r$  is the distance from collectors to the edge of roof or fence,  $C_w$  is the collector width,  $D_a$  is the distance between collectors (side by side), and  $D_c$  is the distance for no mutual shading. In the next step, the amount of energy generated by solar will be assessed to ensure it is sufficient for internal consumption,  $Q_d$ . If  $Q_s$  is greater than  $Q_d$ , it indicates that there is an excess of heat generated by solar, which can be transferred to TES. Based on the excess heat, the size of TES and  $Q_t$  can be determined by using Eq(3).

$$Q_t = F_w \cdot C_{pw} \cdot \left( T_{lout} - \frac{Q_d}{(F_w \cdot C_{pw})} \right) - \left( T_{lout} - \frac{Q_s}{(F_w \cdot C_{pw})} \right) \quad (3)$$

If the energy generated by solar is insufficient to meet the demand, the amount of required energy,  $Q_{req}$ , will be computed using Eq(4). The decision on whether,  $Q_{req}$  can be supplied by the energy from TES,  $Q_t$  or conventional heat supply,  $Q_b$  depends on whether  $Q_{req}$  is less than  $Q_t$  and the desired process temperature must be less than the maximum storage temperature,  $T_{max}$ . The final step is to determine the capacity of TES using Eq(4). The decision to build TES depends on the sharing possibility between plants, which rely on storage and demand temperature.

$$V_t = \frac{Q_t}{C_{pw} \cdot \rho_w \cdot (T_{max} - T_{min})} \quad (4)$$

#### 4. Case study

The case study is designed to validate the decision for an integrated solar thermal network involving three types of sources and demand, as illustrated in Figure 2. Each plant is equipped with its solar collectors. The constraints of this study are the area for solar installation, process conditions, and demand.

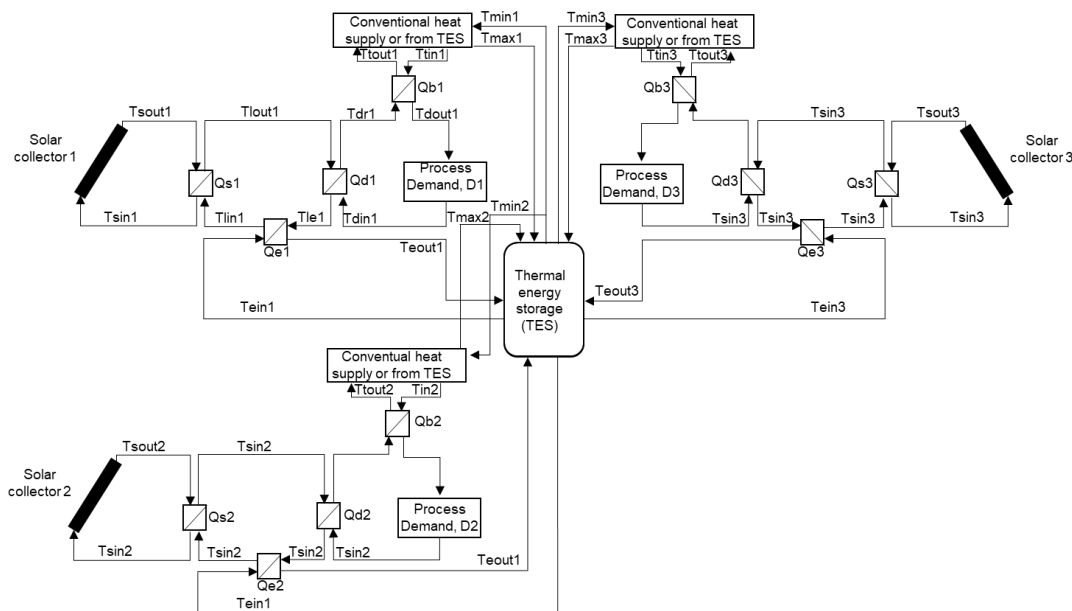


Figure 2: An integrated solar thermal network with a centralised TES

The analysis is constrained by the fact that the available area for solar collector installation is limited. The collector loop was used to transfer heat from the solar collector and integrate it into the process through utility exchangers. This implies that a 10 °C minimum approach temperature is allowed for feasible heat exchange. Each plant requires a specific temperature range. Solar energy will take precedence in supplying heat for interior consumption. If extra heat is generated, it is transferred to TES and used to meet other demands. The storage

area was designed to accommodate one-day storage. Storing this heat for a period of 24 h will incur some heat losses. Therefore, a 5 % loss of heat is assumed during storage. The time series of a typical meteorological year (TMY) by PVGIS is used for solar radiation in Johor Bahru, Johor, Malaysia, for 2015. The demand and availability of solar radiation for each day (in hours) are summarised in Table 1. In this paper, multiple demands have been studied based on full and 75 % load of heat demand scenarios. Due to the fact that the solar thermal energy profile is periodical, with significant variation throughout the year and the load profiles of various industries are relatively constant, an assumption has been made for this case study. 75 % load heat demand profile is assumed to represent heat load reduction during plant break down and maintenance.

Table 1: Main conditions and process demand for the case study

Plant	1	2	3
Inlet temperature (°C)	30	30	45
Outlet temperature (°C)	60	45	65
Area (m <sup>2</sup> )	900	400	2,000
Full load of heat demand profile (MJ)	445	425	695
75 % load of heat demand profile (MJ)	281	243	374

## 5. Results and conclusions

Figure 3 shows the trend of daily thermal energy demand that is potentially satisfied by the solar collectors. In addition, all solar collectors were connected to storage models, and the outlet temperatures of the storage were compared. In the present stage of the work, the mathematical decision-making was carried out by employing the calculated data to meet the decision parameter.

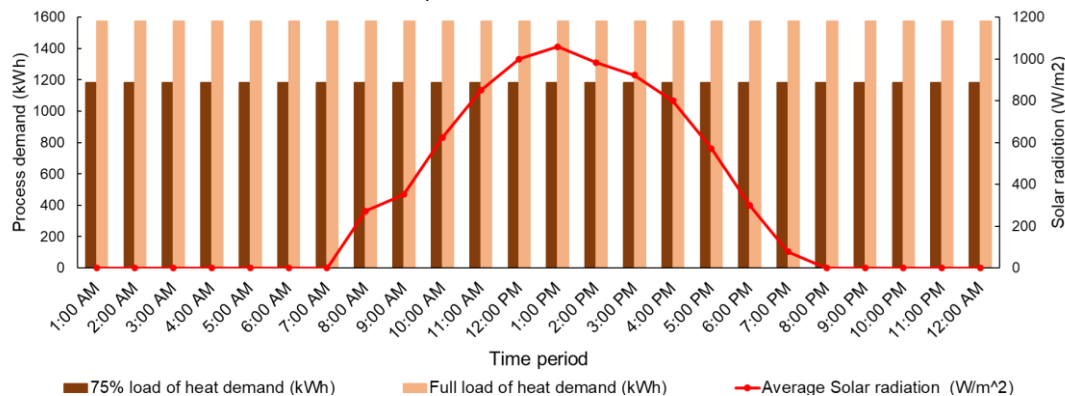


Figure 3: Solar irradiation and process demand

Table 2 summarises the value of optical efficiency ( $c_0$ ) and thermal losses ( $c_1$  and  $c_2$ ) coefficients used to compute the collectors efficiencies for flat plate collectors model Arcon HT-A 28/10 manufactured by ARCON Solar A/S. The input data, parameters, and variables are defined in Table 2. The heat generated and the number of collectors are listed in Table 3.

Table 2: Value of collector efficiency coefficients and parameters used in the simulations

Variable	Value	Unit
Heat capacities of water, $C_{pw}$	4.2	kJ/kg.K
Mass flowrate of water, $F_w$	4.5	kg/s
Density of water, $\rho$	958	kg/m <sup>3</sup>

Table 3: Number of solar collectors and heat production for all three plant

Plant	Number of collectors	Solar yield (kW)
1	44	105.63
2	14	33.61
3	114	273.68

The mathematical decision has been applied to evaluate the feasibility of integrating three solar thermal systems with various sources and demands. The results were assessed based on scenario 1 (full load of heat demand) and scenario 2 (75 % load of heat demand) from Plant 1–3, as shown in Table 4. For scenario 1, the results revealed that Plant 1 could be fully supplied by solar thermal, whereas Plant 3 has an excess of 169.69 kW, and Plant 2 required an additional 33.81 kW that can be supplied by TES.

Table 4: Heat required and heat excess for both scenarios

Supply	Scenario 1			Scenario 2		
	1	2	3	1	2	3
Plant						
Heat required/heat excess (kW)	26.31	84.45	-86.18	-27.51	33.81	-169.69

A negative value in Table 4 represents heat excess, whereas a positive value indicates heat required. For scenario 1, only Plant 3 has 86.18 kW of excess heat shared with TES while Plant 2 can receive 84.45 kW from TES. On the other hand, 26.31 kW of heat required by Plant 1 can only be fulfilled by conventional heat supply due to the higher temperature at the demand side than the TES temperature. One of the factors for consideration in the decision for sharing is the demand temperature. If the temperature of the demand is higher than TES, the heat exchange cannot happen. This is due to principle of second law of thermodynamics, the heat flow from hot stream to cold stream. The total amount of external heat required is 26.31 kW for Plant 1 only, while at 75 % load of heat demand, no external heat supply is needed as it can be fulfilled by solar thermal. The amount of heat supplied to TES will be lower when the heat is used for the process. In addition, the designed capacity of TES for scenario 1 is 183.6 m<sup>3</sup> and 227.5 m<sup>3</sup> for scenario 2. Based on both scenarios, it can be concluded that the solar thermal network integration required a 26.31 kW backup from the conventional heat supply. The optimal size for TES is 183.6 m<sup>3</sup>, which is large enough to provide the required heat for both scenarios.

## 6. Conclusions

In this paper, a mathematical decision framework for the integration of solar thermal networks has been developed. Based on full and 75 % load of heat demand scenarios, the proposed mathematical decision framework successfully evaluated an integrated solar thermal network with different sources and needs. The result of the case study indicates that the framework can assist decision-making in designing an integrated solar thermal network. The results show an opportunity to share the excess solar yield to other plants with a minimum size of TES. This study can be further improved by considering other factors, such as distance, economic analysis, and the multi-period analysis of solar radiation using actual data for validation.

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