

## Implementation of State-feedback Controller on Quadruple Tank Modified I System

Juwari Purwo Sutikno<sup>\*a</sup>, Renanto Handogo<sup>a</sup>, Imam Arifin<sup>b</sup>, Santi Puteri Rahayu<sup>c</sup>, Vicky Dwi Putra<sup>a</sup>, Naufal Fauzan<sup>a</sup>, Anwarudin Hisyam<sup>d</sup>

<sup>a</sup>Department of Chemical Engineering, Institut Teknologi Sepuluh Nopember, 60111 Surabaya, Indonesia

<sup>b</sup>Department of Electrical Engineering, Institut Teknologi Sepuluh Nopember, 60111 Surabaya, Indonesia

<sup>c</sup>Department of Statistical, Institut Teknologi Sepuluh Nopember, 60111 Surabaya, Indonesia

<sup>d</sup>Faculty of Chemical and Natural Recourses, Univeristi Malaysia Pahang, Malaysia  
[juwari@dhem-eng.its.ac.id](mailto:juwari@dhem-eng.its.ac.id)

In general, industrial processes are multi-variable process, and are affected by the disturbances which may be detrimental to the process. The application of conventional control systems is often unable to cope with linear problems and obstacles associated to industrial processes. It produces poor performance while the market is demanding a better production quality. It is necessary to provide a more reliable control system to satisfy the demand. Model-based control system is considered to be able to offer a better performance for industrial problem solving. This study aims to make a comparison between close-loop responses for PID, State-Feedback and State-Feedback Lead-Lag Controller. The transfer function of the model for obtaining the tuning of Ziegler Nichols and Detuning McAvoy is acquired by doing step-test on open-loop system. It is concluded from the simulation that, at minimum phase, based on IAE, PID controller with Ziegler Nichols, tuning shows a better performance compared to both state-feedback controllers, particularly in level control. PID with Detuning McAvoy shows best performance for temperature control.

### 1. Introduction

Chemical plant consists of series of processes which converts raw materials to more valuable product. During the implementation, several process equipment often requires automatic control system. The system will control the operating conditions including temperature, level, pressure, flow rate, and composition that interact with each other. Controlling one parameter can be done indirectly by manipulating other parameters. It is necessary to maintain the stability of the process in order to obtain optimal results. In general, the process is characterised as a multivariable process, and is affected by the disturbance that can be detrimental to the process. Another common characteristic is the interaction between process variables and the time delay in the process.

A system can be classified into two types based on the input and output variables. The first type is a SISO (Single Input Single Output) and the second type is a MIMO (Multi Input Multi Output). Quadruple tank design is a well-known MIMO system which is suitable for non-linear control scheme. Some systems cannot be represented by a linear model so that the non-linear model is needed. Non-linear model in quadruple tank is caused by the square root relationship between flow and level in the tank. MIMO systems have interactions that caused some input variables that affect the use of more than one control variable. In the quadruple tank system, there are strong interactions between tank 1 and 3 and tank 2 and 4. This is because the input of the pump 1 fills the tank 1 and 3, then the output from the tank 3 fills the tank 1. The same interaction occurs between the tank 2 and 4 with water input from pump 2.

Quadruple-Tank becomes a benchmark system for analysing non-linear effects in a multivariable process (Johansson et al., 1999). This helps to realise multi loop system in industry. Quadruple tank process is used to demonstrate the effects of the interaction and performance limitations and describes the dynamic condition in multivariable control system. Vijula et al. (2013) derived mathematical modelling of quadruple tank with no

modification. Muhlis et al. (2016) implemented several controller design into quadruple tank modification 1 and 2.

## 2. Literature review

### 2.1 Modified I Quadruple Tank

Quadruple-tank modifications I system is made by adding a heater in tank 1 and a heater in tank 2. If the temperature inflow 1 and inflow 2 are different, it will get the asymmetric system.

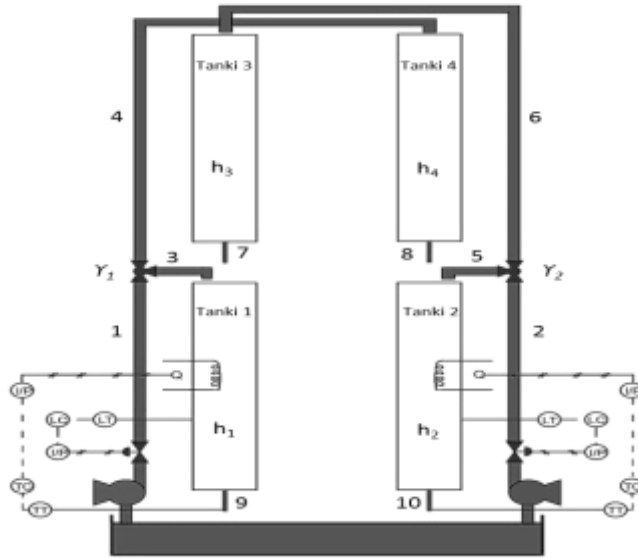


Figure 1: System quadruple-tank modification 1

Figure 1 shows the modified I quadruple tank system scheme. At stream 1 and stream 2, the flow is divided into two with a T-valve (flow splitter, stream 1 ( $u_1$ ) into a stream 3 ( $u_3$ ) and stream 4 ( $u_4$ ) while stream 2 ( $u_2$ ) into a stream 5 ( $u_5$ ) and flow 6 ( $u_6$ ). Controlled variable is the water level in the tank 1 ( $h_1$ ) and tank 2 ( $h_2$ ) as well as the temperature of the stream 9 ( $T_9$ ) and stream 10 ( $T_{10}$ ) while the manipulated variables are  $u_1$ ,  $u_2$ ,  $Q_1$  (heat into the tank 1), and  $Q_2$  (heat into the tank 2). This system is varied with different phases, which are non-minimum phase ( $Y_1 = 0,4$  and  $Y_2 = 0,4$ ) and minimum phase ( $Y_1 = 0,6$  and  $Y_2 = 0,6$ ) (Vijula et al., 2013). Gamma ( $Y$ ) indicates the stream ratio into the bottom tank with stream ratio into upper tank.

The mathematical models from Modified I Quadruple Tank System consist of mass balance and energy balance. The mass balance is given in Eqs(1) – (4):

$$A_1 \frac{dh_1}{dt} = -a_1 \sqrt{2gh_1} + a_3 \sqrt{2gh_3} + Y_1 k_1 u_1 \quad (1)$$

$$A_2 \frac{dh_2}{dt} = -a_2 \sqrt{2gh_2} + a_4 \sqrt{2gh_4} + Y_2 k_2 u_2 \quad (2)$$

$$A_3 \frac{dh_3}{dt} = -a_3 \sqrt{2gh_3} + (1 - Y_2) k_2 u_2 \quad (3)$$

$$A_4 \frac{dh_4}{dt} = -a_4 \sqrt{2gh_4} + (1 - Y_1) k_1 u_1 \quad (4)$$

Where  $A_i$  is tank area,  $h_i$  is water level,  $a_i$  is area of exit hole from tank,  $g$  is gravity acceleration and  $q_{in}$  is flow in to tank. Each pump provides flow proportional to the control signal.

$$q_{pump_i} = k_i u_i \quad (5)$$

where  $k_i$  is pump constant. The flow of the pump is divided according to two parameters  $Y_1$  and  $Y_2$ . Flow to tank 1 is  $Y_1 k_1 u_1$  and flow to tank 4 is  $(1 - Y_1) k_1 u_1$ . Symmetrically, flow to tank 2 is  $Y_2 k_2 u_2$  and flow to tank 3 is  $(1 - Y_2) k_2 u_2$ . Meanwhile the energy balance is obtained specifically for the dynamics of the temperature in the tank 1 and tank 2. The equations derived from energy balance of tank 1 and tank 2 is as the following Eqs(6) – (13) to denote the change on tank 1 and tank 2.

$$A_1\rho C \frac{dh_1.T_9}{dt} = W_3\rho C(T_3 - T_{ref}) + W_7\rho C(T_7 - T_{ref}) + Q_1 - W_9\rho C(T_9 - T_{ref}) \tag{6}$$

$$A_2\rho C \frac{dh_2.T_{10}}{dt} = W_5\rho C(T_5 - T_{ref}) + W_8\rho C(T_8 - T_{ref}) + Q_2 - W_{10}\rho C(T_{10} - T_{ref}) \tag{7}$$

where,

$$W_3 = \Upsilon_1 k_1 u_1 \tag{8}$$

$$W_7 = a_3 \sqrt{2gh_3} \tag{9}$$

$$W_9 = a_1 \sqrt{2gh_1} \tag{10}$$

$$W_5 = \Upsilon_2 k_2 u_2 \tag{11}$$

$$W_8 = a_4 \sqrt{2gh_4} \tag{12}$$

$$W_{10} = a_2 \sqrt{2gh_2} \tag{13}$$

**2.2 State-feedback control**

Figure 2 shows the block diagram of state-feedback controller (pole placement). The aim of this system is to determine control vector U so that the system response is appropriate to the design criteria. The above system can be turned into state space and output mode as given in Eqs(15) – (17):

$$x = Ax + Bu \tag{15}$$

$$y = Cx + Du \tag{16}$$

$$U = -Kx \tag{17}$$

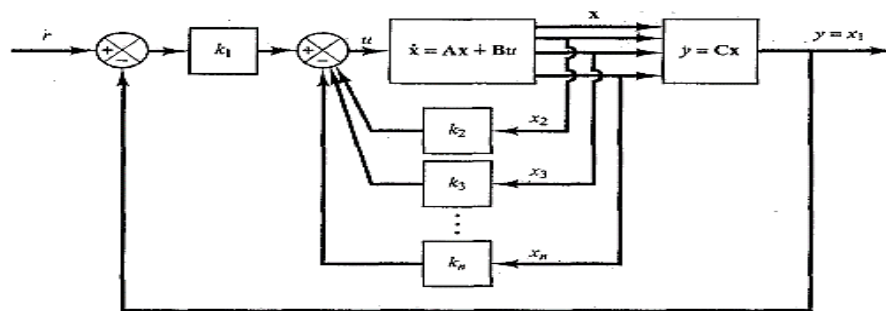


Figure 2: State-feedback controller gain block diagram

State Feedback Gain controller and state feedback lead-lag controller used state space model. From calculation, we obtained matrices A (Eq(18)) and B (Eq(19)):

$$A = \begin{pmatrix} -0.021843 & 0 & 0.054608 & 0 & 0 & 0 \\ 0 & -0.021843 & 0 & 0.054608 & 0 & 0 \\ 0 & 0 & -0.054608 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.054608 & 0 & 0 \\ 0.014918 & 0 & -1.365218 & 0 & -0.043687 & 0 \\ 0 & 0.014918 & 0 & -1.365218 & 0 & -0.043687 \end{pmatrix} \tag{18}$$

$$B = \begin{pmatrix} 19.09091 & 0 & 0 & 0 \\ 0 & 19.09091 & 0 & 0 \\ 0 & 12.72727 & 0 & 0 \\ 12.71717 & 0 & 0 & 0 \\ -477.2727 & 0 & 0.000018 & 0 \\ 0 & -477.2727 & 0 & 0.000018 \end{pmatrix} \tag{19}$$

**3. Result and discussion**

Transfer function identification is giving step in the open-loop system after reaching a steady state. Step test performed at t = 500 s given disturbance by changing the input signal. Input signals are converted to u<sub>1</sub>, u<sub>2</sub>.

$Q_1$ , and  $Q_2$ . Modified I Quadruple-Tank System is controlled by PID controller (Tuning Ziegler-Nichols and detuning McAvoy), State Feedback Gain controller, and State Feedback Lead-lag controller. The parameter of each controller can be seen in Table 1 below. The below parameters are used for closed-loop simulation. Set point is changed at each control variables. The responses of controlled variable i.e.  $h_1$ ,  $h_2$ ,  $T_9$ , and  $T_{10}$  are monitored when set point changed at 700 s.

Table 1: Controller Parameter

Pairing Controller	PID Tuning Ziegler-Nichols			PID detuning Mc Avoy			State Feedback Gain	State Feedback Lead-lag		
	Kc	$\tau_I$	$\tau_D$	Kc	$\tau_I$	$\tau_D$	K	K	$\tau_1$	$\tau_2$
$h_1 - u_1$	129.2	0.04	0.01	91.19	0.04	0.01	0.0003	0.0003	25	1
$h_2 - u_2$	129.2	0.04	0.01	91.19	0.04	0.01	0.0003	0.0003	25	1
$T_{10} - Q_2$	15.25	1.9	0.47	15.25	1.9	0.47	5	5	15	1
$T_9 - Q_1$	16.07	1.9	0.48	16.07	1.9	0.48	5	5	15	1

3.1 Set point change in  $h_1$  and  $h_2$

The change of level tank 1 ( $h_1$ ) is set from 0.40 m to 0.45 m. Simulation results at the change of set point  $h_1$  can be seen in Figure 3. Based on simulation results, the response process with PID controller, State Feedback Gain controller and State Feedback Lead-lag controller may reach set point without any offset for both level and temperature. From these results it can be said that the PID controller, State Feedback Gain controller and State Feedback Lead-lag controller are sufficient to overcome both level and temperature control of modified I Quadruple-Tank. The change of  $h_1$  set point does not affect temperature because the temperature of incoming water is equal to the set point temperature. There are several factors that show the superiority of the controller over other controllers, including the settling time, behaviour charts before reaching set point that is oscillating or not, and the integral of the absolute value of error (IAE) which shows the integral of the difference between the set point with control variable response. The value of IAE in closed loop minimum phase response of modified I Quadruple-Tank is shown in Table 2 and 3.

Based on the IAE value for  $h_1$  and  $h_2$  set point change at minimum phase PID controller Tuning Ziegler-Nichols is better than State Feedback Gain controller and State Feedback Lead-lag controller in controlling level.

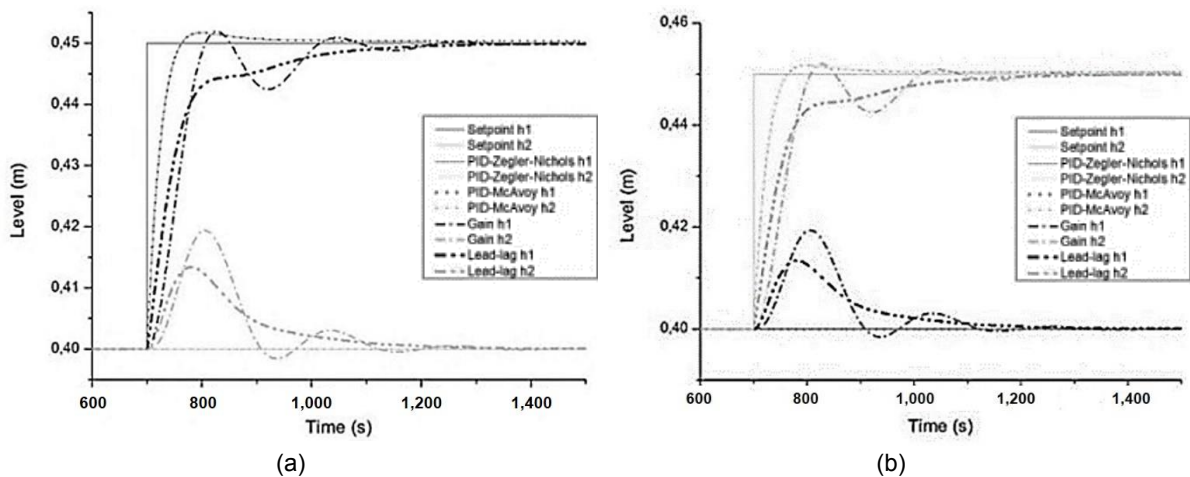


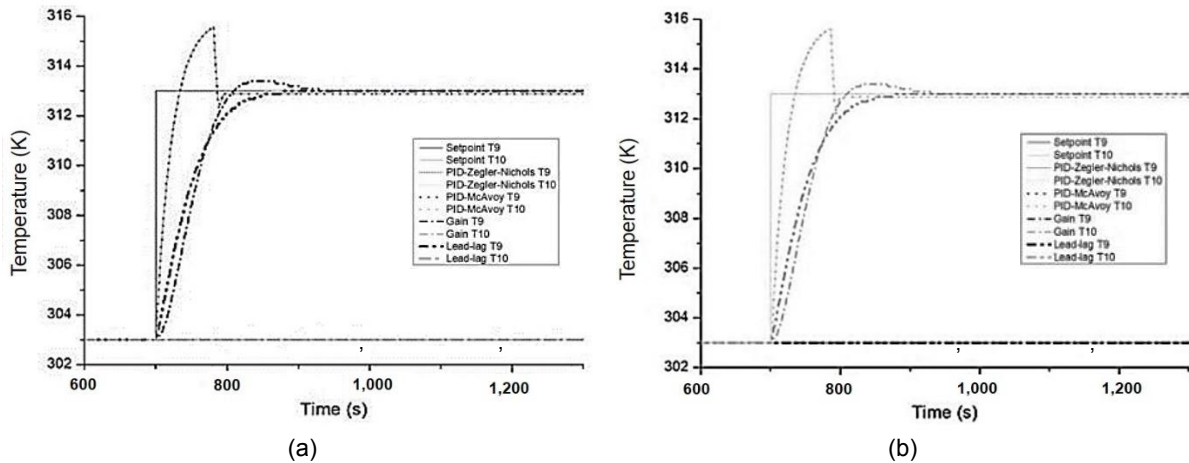
Figure 3: Simulation results at the change set point (a)  $h_1$  and (b)  $h_2$  for level at minimum phase

Table 2: IAE values for the change in set point  $h_1$

Controller	Integral absolute error			
	$h_1$	$h_2$	$T_9$	$T_{10}$
PID Tuning Ziegler-Nichols	1.63105	0.00030	0	0
PID detuning Mc Avoy	1.63111	0.00045	0	0
State Feedback Gain	3.6609	2.4753	54.8704	37.1093
State Feedback Lead-lag	3.4324	2.2882	29.7614	15.5771

Table 3: IAE values for the change in set point  $h_2$ 

Controller	Integral absolute error			
	$h_1$	$h_2$	$T_9$	$T_{10}$
PID Tuning Ziegler-Nichols	0.00030	1.63105	0	0
PID detuning Mc Avoy	0.00045	1.63111	0	0
State Feedback Gain	2.4753	3.6609	37.1093	54.8704
State Feedback Lead-lag	2.2882	3.4324	15.5771	29.7614

Figure 4: Simulation results at the change set point  $T_9$  (a) and  $T_{10}$  (b) for level at minimum phase

### 3.2 Set point change in $T_9$ and $T_{10}$

In the process given the change of set point  $T_9$  from 303 K to 313 K, simulation results at the change of set point  $T_9$  can be seen in Figure 4.

Based on simulation results, the response process with PID controller, State Feedback Gain controller and State Feedback Lead-lag controller may reach set point without any offset both level and temperature. A temperature set point ( $T_9$  and  $T_{10}$ ) change did not cause interference with the control level. There is no interaction between  $Q_1$  and  $Q_2$  with  $h_1$  and  $h_2$ . When the temperature set point changes, there is no change in level. Process response remains stable at set point. From these results it can be said that the PID controller, State Feedback Gain controller and State Feedback Lead-lag controller are sufficient to overcome both level and temperature control of modified I Quadruple-Tank. The value of IAE in closed loop minimum phase response modified I Quadruple-Tank to compare the performance of controller is presented in the Table 4 and 5.

Table 4: IAE values for the change in set point  $T_9$ 

Controller	Integral absolute error			
	$h_1$	$h_2$	$T_9$	$T_{10}$
PID Tuning Ziegler-Nichols	0.00014	0.00014	386.199	0
PID detuning Mc Avoy	0.00020	0.00020	386.197	0
State Feedback Gain	0	0	525.675	0
State Feedback Lead-lag	0	0	464.148	0

Table 5: IAE values for the change in set point  $T_{10}$ 

Controller	Integral absolute error			
	$h_1$	$h_2$	$T_9$	$T_{10}$
PID Tuning Ziegler-Nichols	0.00014	0.00014	0	406.734
PID detuning Mc Avoy	0.00020	0.00020	0	406.426
State Feedback Gain	0	0	0	525.675
State Feedback Lead-lag	0	0	0	464.148

Based on the IAE value for T9 and T10 set point change at minimum phase PID controller detuning McAvoy is better than State Feedback Gain controller and State Feedback Lead-lag controller in controlling temperature.

#### 4. Conclusions

In the system of quadruple tank modified I, there is no interaction between  $Q_1$  and  $Q_2$  with  $h_1$  and  $h_2$ . In term of controller, state space feedback controller has weakness i.e. the matrices parameters should be accurate to achieve a good controller. In minimum phase system, based on the IAE value, it was concluded that the result of PID controller with tuning Ziegler-Nichols is superior to State Feedback controller in controlling the level and PID controller with detuning Mc Avoy is superior to State Feedback controller in controlling the temperature.

#### Acknowledgments

The authors thank to Institut Teknologi Sepuluh Nopember Surabaya through the research grant No.01668/IT2.11/PN.08/2016 and Universiti Malaysia Pahang for the collaboration of the research.

#### Reference

- Muhlis G.A., Lestari N.F., Sutikno J.P., Handogo R., Hisyam A., 2016, Process Dynamic and Control Modified II Quadruple Tank on LabView Application, *ARNPN Journal of Engineering and Applied Sciences* 11 (8), 5251-5260.
- Johansson K.H., Alexander H., Olle W., Andre H., 1999, Teaching Multivariabel Control Using the Quadruple Tank Process, 38<sup>th</sup> IEEE Conference on Decision and Control, 7-10 December 1999, Phoenix, Arizona, DOI: 10.1109/CDC.1999.832889
- Vijula D.A., Anu K., Honey Mol P., Poorna Priya S., 2013, Mathematical Modelling of Quadruple Tank System, *International Journal of Emerging Technology and Advanced Engineering* 3 (12), 261-265.