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Proof of Lagrange Mean Value Theorem and its Application in Text Design

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At present, there are a lot of papers on Lagrange mean value theorem proving method, the paper On the application of the theorem is not in a few, but text designs from the perspective of curriculum explore proving a theorem and its application to the design of a text are rare. This paper first analyzes the objectives, tasks, methods, and then focuses on the teaching strategies and processes, Finally, it comes to the reflection, thus forming a complete, detailed and full text design on the proof of Lagrange mean value theorem and its application. In this paper, through study of the teaching content of the Lagrange mean value theorem, optimizing the teaching design, adopting it in the classroom teaching, the paper gives the classroom teaching practice in the specific practices, thereby improves the effect of classroom teaching, and it has a strong guiding significance in theory and practice.

1. Introduction

The differential mean value theorem is the theoretical basis of the application of the derivative, which is the bridge between the function and its derivative. It is an important tool to study the function overall by using derivative of the locality, and forms a relation between the increment of a function on an interval and the derivative of the function at a specific point of the interval. As a result, differential mean value theorem has important theoretical significance and application value.

2. Main text

2.1 Task analysis

"Lagrange mean value theorem and its application" belongs to the first section of the third chapter numbering in the first volume of "Higher mathematics" (6th Edition) (Department of mathematics, Tongji University, higher education press), the third chapter is the application of the differential mean value theorem and the derivative. The content of this chapter is the continuation of the last chapter. It mainly analyzes and studies the character of function and its graph and various forms by using derivative and differential (Farid, 2015). The theoretical basis of all of this is the differential mean value theorem, which plays an important role in the differential calculus, is also the content of the first section (Zhang, 2015). The differential mean value theorem includes Rolle theorem, Lagrange mean value theorem, Cauchy mean value theorem and Taylor mean value theorem. In the process of analysis and demonstration, the mean value theorem is widely used. The teaching task of this course is to study Lagrange mean value theorem and the application of theorem in equality and inequality (Mortici, 2011).

2.2. Objectives and basic requirements

Knowledge and Skill Goals: (1) through the study of this course, students should understand and master the conditions and conclusions of Lagrange mean value theorem; (2) to make the students understand Lagrange mean value theorem and the geometric meaning of Lagrange mean value theorem, and the connection and difference between Rolle theorem and Lagrange mean value theorem; (3) through the study of this course, students should learn how to use the mean value theorem to prove the equality and inequality.

Ability Training Goal: (1) to improve students' analysis ability and logical thinking ability of proving mathematical proposition with the cycle from conclusion to the conditions and the conclusion (Yi, 2015). The

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following section is to prove the conclusion of Lagrange mean value theorem: There is at least one point *f(b)-ξ(a<ξ<b)* Inside (*a*,*b*). In the process of making the equation

$$f(b)-f(a)=f^{'}(\xi)(b-a) \tag{1}$$

Established, Starting from the conclusion *f(b)-f(a)=f'(ξ)(b-a)*, through the analysis of this equation, the auxiliary function is constructed, and the condition that the function *f*(*x*) satisfies in the theorem is used to prove the Lagrange mean value theorem by Rolle theorem.

(2) To develop students' ability to use the construction of auxiliary function to prove the mathematical proposition while proving Lagrange mean value theorem (Heydari, 2013), (Tan, 2014) a relation between the Lagrange mean value theorem and Rolle's Theorem is found from the geometric meaning. In order to prove the Lagrange mean value theorem with the Rolle theorem, Starting from the equation *f(b)-f(a)=f'(ξ)(b-a)* itself , through deformation of the equation, and by making use of the property of the function and the derivative function, the auxiliary function

$$F(x)=f(x)-\frac{f(b)-f(a)}{b-a}x \tag{2}$$

As the auxiliary function *F*(*x*) satisfies the condition of Rolle's Theorem, we prove the Lagrange mean value theorem by using the Rolle theorem (Korobkov, 2001), (Elbrous, 2008) in this process, students are taught to prove the mathematical proposition with auxiliary function.

(3) To cultivate students' creative thinking ability. The college mathematics teaching should not only let the students know the existing research methods and conclusions, but also need to cultivate students' ability to discover, think about and solve new problems. In the course of teaching, I should take the following innovative try: In the process of proving the Lagrange mean value theorem, because the function *f*(*x*) does not satisfy the condition of Rolle theorem, I will start with the conclusion of the Lagrange mean value theorem, and guide the students to analyze the equation

f(b)-f(a)=f'(ξ)(b-a) with the help of the nature of the function and the derivative function, the auxiliary function *F(x)= f(x)-((f(b)-f(a))/(b-a))* is structured, and Lagrange mean value theorem is proved by Rolle theorem. The teaching material starts from the geometric meaning of the Lagrange mean value theorem, combined with the graphics

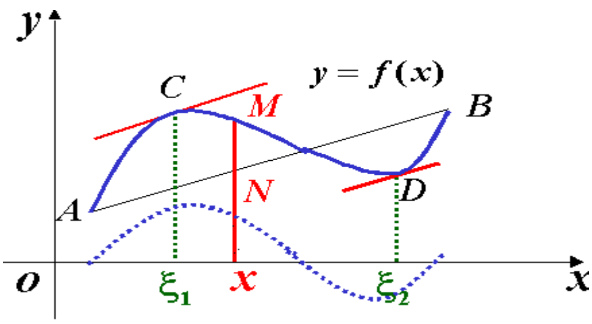

Figure 1: Geometric sketch of Lagrange mean value theorem

The auxiliary function

$$F(x)=f(x)-[f(a)+\frac{f(b)-f(a)}{b-a}(x-a)] \tag{3}$$

is constructed by analyzing the characteristic of the length function of the line segment NM. Similarly, to the function *F*(*x*), the Lagrange mean value theorem is proved by the Rolle theorem (Tong, 2000). In the process, I guide the students to compare the two kinds of auxiliary functions and detect the characteristics and the differences between them (Zabrdjko, 1996), and analyze the constructing context and perspective of the two auxiliary functions. By contrast, make the students experience different (Lupu, 2009), multi-angle method of constructing auxiliary function. Meanwhile, by inspiring students to think whether different auxiliary functions can be constructed to prove theorems, I guide the students to find the problem, propose the question, analyze question, and solve the problem, cultivating students' awareness of innovation and innovative thinking ability.

(4) To cultivate students' ability of comprehensive analysis of the problem and the comprehensive use of Through the proof of the theorem and the proof of two examples on the equality and inequality by the theorem, knowledge, and with the help of the assignments, the students are supposed to grasp the following two kinds of abilities of analyzing problems and the comprehensive use of knowledge.

2.3 Teaching Focus and Difficulty

Teaching Focus: Lagrange mean value theorem

Teaching Difficulty: the Construction of auxiliary function, the application of Lagrange mean value theorem

2.4 Teaching Strategies

Review the Old and Learn the New, Put forward the Question, Introduce New Courses: Looking back on the learned knowledge:

Review the Old: Rolle theorem If the function $f(x)$ is satisfied: (1) be continuous on the closed interval [a,b], (2) be derivable in the open interval (a,b), (3) be equal to the value of the function at the point of the interval, that is $f(a)=f(b)$.Then there is at least one point $\xi(a<\xi<b)$, in the interior (a,b) so that the derivative of the function $f(x)$ at that point is equal to zero, that is $f'(\xi)=0$.

Ask the question: if the third condition of Rolle's theorem is moved, what conclusion can we get (Comparative inspiration, to be discussed)

Introduce new courses: Lagrange mean value theorem and its application

Heuristic Induction, Theorem Proving: In each step of the class, including the discussion of the geometric significance of theorem, the construction of the auxiliary function from the analysis of the conclusion of theorem, the proof of the Lagrange mean value theorem by the Rolle theorem, the analysis and proof of the two examples of equality and inequality (Liu, 2015), I duly inspired and guided the students to think actively, participate in the classroom, and seek the functions that meet the conditions and the corresponding interval, thereby successfully applied theorem to prove the problems related.

Method Training, Migration and Expansion: Teacher-oriented class emphasizes the one-way knowledge transference of the teacher and the memorizing work on the students, is the teachers' "autocracy", "chalk and talk". While the students-oriented teaching reform attaches importance to comprehensive assistance from the teacher to the students and should provide students opportunities for autonomous learning on the students' part (Niu, 2015), and give students time and tasks for mental work in order to facilitate students to master the learning method and use of migration in the new situation. After the proof of the problems of the equality in example 1 with Lagrange mean value theorem (Zhu, 2015), the problem is proposed that whether the theorem also has application in other aspects. The inequality proof problem of 2 is drawn off. That is to prove when $x>0$,

$$\frac{x}{1+x} < \ln(1+x) < x \tag{4}$$

(This question has a certain degree of difficulty, pay attention to timely guidance).

2.5 Methods, Means and Matters Needing Attention

Teaching Methods: Heuristic teaching, Example teaching, Analogy Teaching

Teaching Means: The combination of classroom teaching and the use of multimedia courseware. Matters Needing Attention: The similarities and differences of condition and conclusion between Rolle's theorem and Lagrange theorem and the relationship between them; Rolle's theorem is a special case of the Lagrangian theorem; Lagrange theorem is generalization of Rolle's Theorem;

note the mean value point of Rolle theorem and Lagrange mean value theorem is a point in the open interval, rather than an arbitrary point within the interval or a designated point, in other words, the two mean value theorem only "qualitatively" pointed out the existence of the median point, rather than "quantitatively" indicated the specific value of the median point;

By learning the application of Rolle and Lagrange mean value theorem in this section as well as the application of each chapter in the future, repeatedly experience the significance and function of these theorems in calculus (Pan, 2014).

2.6. Process

Teaching content and teaching strategy:

Review: 1) Rolle theorem If the function $f(x)$ is satisfied: (1) be continuous on the closed interval [a,b], (2) be guided in the open interval (a,b),(3) be equal to the value of the function at the point of the interval, that is $f(a)=f(b)$.Then there is at least one point $\xi(a<\xi<b)$, in the interior (a,b), so that the derivative of the function $f(x)$ at that point is equal to zero, that is $f(\zeta)=0$.

2) Geometric meaning: There is the tangent no perpendicular to the x axis at every point on the closed interval [*a,b*], and the connection of the two ends of the line and parallel to the *x* axis of the curve *f*(*x*), there is at least a point C, making its tangent parallel to the *x* axis.

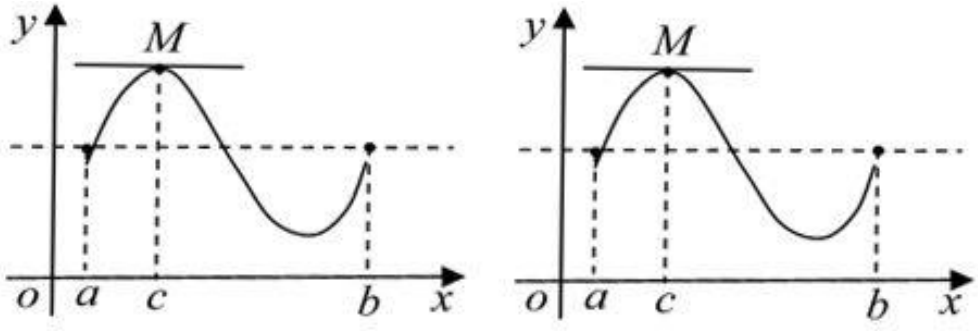

Figure 2: Geometric sketch of Rolle theorem

Lagrange mean value theorem: (1) in practical application, the condition (3) of Rolle's Theorem cannot be satisfied, so its application is limited. If the condition (3) is removed, it is the Lagrange mean value theorem to be introduced.
Lagrange mean value theorem: If the function *f(x)* is satisfied: (1) continuous on the closed interval *[a,b]*, (2) be guided in the open interval *(a,b)*, Then there is at least one point $\xi(a<\xi<b)$, in the interior*(a,b)*, so that $f(b)-f(a)=f'(\xi)(b-a)$. Before proving the Lagrange mean value theorem, the condition and the conclusion of the theorem are first known from the geometric point of view.
Geometric meaning: The above (1) type can be deformed into

$$f'(\xi)=\frac{f(b)-f(a)}{b-a} \tag{5}$$

The slope of the right end of the equation is for the string AB, So there is no discontinuity in the interval [*a,b*] and each of its upper point is not perpendicular to the x axis of the tangent line, there is at least one point C, Making the tangent of the C point parallel to the string AB. When *f(a)=f(b)*, the Lagrange theorem becomes Rolle theorem, Rolle theorem is a special case of the Lagrange mean value theorem and Lagrange mean value theorem is generalization of Rolle's Theorem, the following part is to prove Lagrange mean value theorem with Rolle theorem.
The following part is trying to prove the Lagrange mean value theorem by application of Rolle theorem, which is inspired by the special relationship between Lagrange mean value theorem and Rolle theorem.
Analysis: To prove that
$f(b)-f(a)=f'(\xi)(b-a)$ form, Is to permit the establishment of $f'(\xi)=f(b)-f(a)(b-a)$,That is to prove the establishment of

$$f'(\xi)-\frac{f(b)-f(a)}{b-a}=0. \tag{6}$$

If the left of this equation is a function, For example, F (x) derivatives in point words, and then it becomes $F(\xi)=0$. And this form is the same to the form of conclusion of the theorem of Rolle. Therefore, it is necessary to find such a function f(x), if it can meet the conditions of Rolle's Theorem, we can prove Lagrange mean value theorem by using Rolle's Theorem. Then what kind of function is f(x). At this time, guide the students to observe the left of $f(\xi)-((f(b)-f(a))/(b-a))=0$, from the first item $f(\xi)$, naturally we thought of function $f(x)$, and by second item

$$\frac{f(b)-f(a)}{b-a} \tag{7}$$

to guide students to think of function

$$\frac{f(b)-f(a)}{b-a}x, \tag{8}$$

In this way, the auxiliary function *F(x)=f(x)-((f(b)-f(a))/(b-a))•x* can be constructed. For function *F(x)* to first determine whether to meet the conditions of Rolle theorem, after verification, Function *F* (*x*) in the interval [*a*,*b*] does meet the conditions of Rolle theorem, then we can apply the Rolle theorem to prove the Lagrange mean value theorem.

Prove: Constructing auxiliary function *F(x)=f(x)-((f(b)-f(a))/(b-a))•x*. We know that F(x) is continuous on the closed interval [*a*,*b*], F(x) can be guided in the open range (*a*,*b*), and *F(a)=(bf(a)-af(b)/(b-a)=F(b)*. By Rolle's theorem, There is at least one point in the inside (*a*,*b*), so that *F(ζ)=0*, That is *f(ζ)-((f(b)-f(a))/(b-a))=0*, so then *f(b)-f(a)= f′(ζ)(b-a)*.

Theorem certificate.

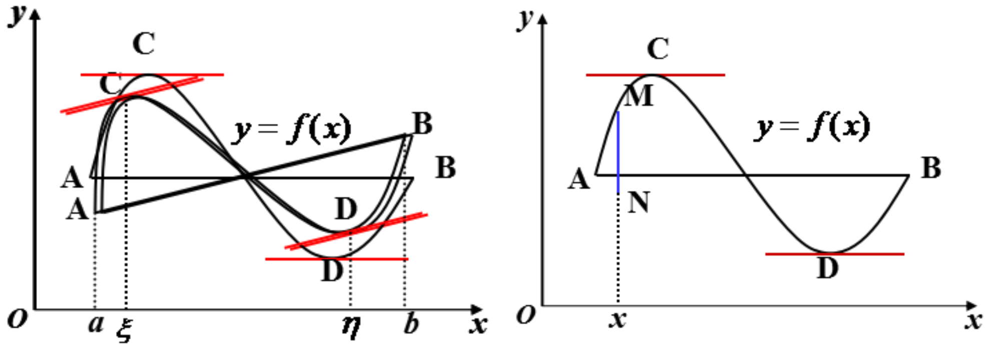

Figure 3: Comparison of geometric relations of differential mean value theorem

Notes: 1) Constructing the auxiliary function method is a very important method to prove the mathematical proposition. The above is a method different from the teaching material proving the Lagrange mean value theorem by the constructing function. In the process, we first analyzed the conclusion and formed the auxiliary function *F(x)=f(x)-((f(b)-f(a))/(b-a))•x*. By using the Rolle theorem, Lagrange mean value theorem is proved.

2) For at least one point in the theorem, the (1) type was established, which indicates that the mean value point ξ must exist, but there may only be one, and there may be more than one, that is, the existence of the value point ζ may not be the only, and the value ζ is not specific.

3) In the formula (1), if *f(a)= f(b)*, then *f′(ζ)=0*. Then the Lagrange mean value theorem becomes Rolle theorem, so here is the further explanation: Rolle's Theorem is a special case of the Lagrange mean value theorem and Lagrange mean value theorem is generalization of Rolle's Theorem

Application of Lagrange mean value theorem: 1) can be used to prove the equation; 2) can be used to prove inequality.

Example 1 Prove that if the derivative of the function *f(x)* in the interval I constants zero, then *f(x)* is a constant in the range I.

Proof: Take any $x_1, x_2 \in I$, might as well set $x_1 < x_2$, By condition *f*(*x*) is continuous on [x_1,x_2], and is capable of guiding in (x_1,x_2), then there is at least one point $\zeta (a<\zeta<b)$ make the equation

$$f(x_1) - f(x_2) = f'(\xi)(x_2 - x_1) \ . \qquad (9)$$

Establish. from *f(x)=0*, then *f(ζ)=0*, $f(x_2)= f(x_1)$ is from the upper. From the arbitrary nature of x_1, x_2, *f(x)* is a constant On I.

The above is the application of Lagrange mean value theorem in equality proof, in fact, Lagrange mean value theorem can also be used to prove inequality. How to use the Lagrange mean value theorem to prove it? (Ask questions) to guide the students to think.

Example 2 Prove when $x>0$, $(x/(1+x)) < Ln(1+x) < x$.

Thinking and Deepening and Broadening the Teaching Content in this Section. Thinking questions：If the two conditions in the theorem are not satisfied, is the conclusion of the theorem still established?

3. Conclusion

This paper, through research on the differential mean value theorem of teaching content, optimizing teaching design, application in the classroom teaching, gives the classroom teaching practice in the specific practices, thereby improving the effect of classroom teaching. The followings is reflection problem: whether the students have grasped the key points? As for the use of the multimedia courseware, whether the students recognize its assistance and positive effect? Whether the students accept the teaching strategies and the content of the teaching? Do the students have any other ways to construct the auxiliary function?

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