

# Study on Supply Chain Buy-Back Contract Model under Risk-averse Condition

Zheng Liu<sup>\*a</sup>, Yuanjun Zhao<sup>b</sup>, Haichao He<sup>b</sup>, Min Zhou<sup>a</sup>

<sup>a</sup> School of Management, Shanghai University of Engineering Science, Shanghai 201620, China,

<sup>b</sup> School of Management, Donghua University, Shanghai 200051, China.

493185933@qq.com

In this paper, supposing a supply chain composed of a risk-neutral supplier and a risk-averse retailer, the buy-back contract with a risk-averse retailer is designed and modelled, the strategy of providing the buy-back contract to increase revenue and coordinate the supply chain is analyzed. It uses downside risk restriction to measure the degree of risk the retailer is eager to assume. It also analyses the return policy to coordinate the supply chain under risk-averse condition. Finally, a simulation shows the accuracy and effectiveness of the return policy.

## 1. Introduction

Supply chain coordination is the key issue of supply chain management, and the contract is an important way which is widely used to achieve supply chain coordination. In the supply chain, proper information and incentives is provided to optimize the supply chain system, maximize the profits of the supplier and retailer and share the income and risk by the members. The common contracts include wholesale price contract, revenue sharing contract, buy-back contract, quantity flexibility contract and so on, Buy-back contract is one of the most common supply chain coordination mechanisms.

Much of the research on supply chain contract has assumed that the agents in the supply chain are risk neutral. They maximize their respective expected profits or minimize the expected cost, these contracts are not apply to the supply chain consisting of risk-averse partners. Because of the uncertainty of the market environment, the partner of risk-averse will choose to avoid the risk. In most cases, supply chain contract mechanism cannot be implemented effectively when ignoring the impact of risk. So, how to coordinate the supply chain when consider the risk preference is increasingly becoming one of the focus issues of supply chain management.

Yoo et al. (2015) reported pricing and return policies under various supply contracts in a closed-loop supply chain in which a supplier has more bargaining power than a retailer. Ruiz-Benitez and Muriel (2014) showed higher profits and better coordination are achieved when buyer and vendor acting in a decentralized fashion do not consider any information about consumer returns. Kulkarni et al. (2015) considered that perfect coordination of partners will help them improve their profits considerably. Amin-Naseri and Khojasteh (2015) developed a price competition model under a demand uncertainty environment between two leader-follower supply chains that each of them consists of one risk-neutral manufacturer and one risk-averse retailer. Glock and Kim (2015) indicated that the type of competition is of major importance for the structure of the supply chain after the merger, and that under certain conditions, the merger may benefit all parties involved, i.e. the vendor, the retailers, and the consumers. Chernonog and Kogan (2014) found that the risk-averse retailer does not necessarily order less than the risk-neutral one and may introduce a bias by choosing a specific demand distribution. Kim and Park (2014) questioned if each kind of risk management contract can improve the utility of all supply chain members compared to the utility without any of those, and how the conditions to achieve such improvements are different. Budde and Minner (2014) investigated a newsvendor-type retailer sourcing problem under demand uncertainty who has the option to source from multiple suppliers. Yoo (2014) considered the supplier's different risk attitudes, whether risk averse or risk neutral and indicated the optimal conditions for a generous return policy setting without quality enhancement. Guler and Keskin (2013) analyzed coordination in a supply chain with random yield and random demand (SCRYRD). Chung and Erhun (2013)

studied the case in which the supplier needs to account for both old and young units. Arcelus et al. (2012) evaluated the pricing and ordering policies of a retailer, facing a price-dependent stochastic demand, within a newsvendor framework, under different degrees of risk tolerance and under a variety of optimizing objectives. Takahashi et al. (2011) calculated the optimal delivery time to alleviate the risk. Adida and DeMiguel (2011) showed, unlike in the symmetric chain, the asymmetric chain efficiency depends on product differentiation and risk aversion because of the interaction between these features and the asymmetry of manufacturers and retailers. Oezen et al. (2010) focused on cooperation among the retailers-the retailers coordinate their initial orders and can reallocate their orders in the warehouse after they receive more information about their demand and update their demand forecasts. Fisher and Raman (1996) reported much of the research on supply chain contract has assumed that the agents in the supply chain are risk neutral. This paper analyzes the impact of retailer's risk preference on the strategy of supply chain collaboration by considering downside risk factor, and coordinate the supply chain through buy-back contract when considering salvage of the unsold products.

## 2. Decision-making model of the decentralized supply chain in the risk-neutral case

### 2.1 Background description

In this paper, the supply chain consists of a risk-neutral supplier and a risk-averse retailer. The market demand is random and is subject to a known distribution. We first introduce the following notations:

$w$ : the per-unit wholesale price at which the supplier sell to the retailer

$c$ : the per-unit cost of the product

$b$ : the per-unit buy-back price of the unsold product from the retailer

$Q$ : the order quantity of the retailer

$p$ : the per-unit price at which the retailer sells the product

$r$ : the per-unit salvage of product handled by the supplier

$v$ : the per-unit salvage of product handled by the retailer

$\alpha$ : the target profit of the risk-averse retailer

$\beta$ : the probability of target profit of the risk-averse retailer

$D$ : the random demand of the consumer, the density function  $f(x)$  and distribution function  $F(x)$  are known.  $F(x)$  is continuous, differentiable and reversible in  $x$ , and assume  $w > c$ ,  $r > v$ .

### 2.2 Decentralized supply chain

In the centralized supply chain, whether the retailers are risk-averse or not, the market risk faced by the suppliers can fully be offset by buy-back contract. But in the decentralized supply chain, retailers and suppliers are independent from each other, they pursue their maximum benefits. Thus the retailer decides his optimal order with the following model:

$$\text{Max } E[\pi_{D(r)}^0(\min(Q, D))]$$

$$E(\pi_{D(r)}^0) = -wQ + \int_0^Q px f(x) dx + \int_0^Q v(Q-x) f(x) dx + \int_Q^\infty pQ f(x) dx \quad (1)$$

So the retailer's optimal order quantity is:

$$Q_{D(r)}^0 = F^{-1}\left(\frac{p-w}{p-v}\right) \quad (2)$$

## 3. Buy-back contract model for the supply chain in the risk-adverse case

### 3.1 Description of the risk-averse retailers

Risk-averse decision-making preferences are the most common tool to analyze the behavior of supply chain management decision; there are many ways to describe risk-averse preference. This section will use the concept of risk-aversion to measure the risk-averse extent of the retailer. The probability of the retailer's profit is less than or equal to his target profit, is defined as downside risk. Assuming that  $\alpha$  is retailer's target profit, the downside risk can be described as  $P\{\pi_r(Q, D) \leq \alpha\}$ .

The newsvendor wants to choose an order quantity  $Q$  so as to maximize his expected profit  $E[\pi_r(Q, D)]$ , while specifying that his actual profit should not fall below his target profit level of  $\alpha$  with a probability exceeding a specified  $\beta$ . Based on above, the model can be expressed as:

$$\max_{Q \geq 0} E[\pi_r(\min(Q, D))]$$

$$s.t. P\{\pi_r(\min(Q, D)) \leq \alpha\} \leq \beta$$

$$E[\pi_r] = -wQ + \int_0^Q px f(x) dx + \int_0^Q v(Q-x) f(x) dx + \int_Q^\infty pQ f(x) dx \quad (3)$$

### 3.2 The order decision of centralized supply chain

As in section 1.2, whether the retailers are not risk-averse, the market risk faced by the suppliers can fully be offset by buy-back. So the expected profit function is:

$$E[\pi_r(\min(Q, D))]$$

Max

$$E[\pi_r] = -cQ + \int_0^Q px f(x) dx + \int_Q^\infty pQ f(x) dx + \int_0^Q r(Q-x) f(x) dx \quad (4)$$

The optimal order quantity of the supply chain is:

$$Q_t = F^{-1}\left(\frac{p-c}{p-r}\right) \quad (5)$$

### 3.3 The order decision of decentralized supply chain

In the decentralized supply chain, there are risk aversion preferences for the retailer, the restriction is obtained by the conception of downside risk and the risk aversion parameter  $(\alpha, \beta)$ , thus the retailer's expected profit function is:

$$\max_{Q \geq 0} E[\pi_{D(r)}(\min(Q, D))]$$

$$s.t. P\{\pi_{D(r)}(\min(Q, D)) \leq \alpha\} \leq \beta$$

$$E[\pi_{D(r)}] = -wQ + \int_0^Q px f(x) dx + \int_Q^\infty pQ f(x) dx + \int_0^Q v(Q-x) f(x) dx \quad (6)$$

Assume the retailer's optimal order quantity is  $Q_{D(r)}$ , and the critical order quantity is  $Q_0 = \frac{\alpha}{p-w}$ , so

according to the difference of the interval, it can be classified as two cases:

(1) when  $Q_{D(r)} \leq Q_0$ ,

$\alpha = (p-w)Q_0 \geq (p-w)Q_{D(r)} \geq \max \pi_{D(r)}$  is available from  $Q_0 = \frac{\alpha}{p-w}$ , this  $P\{\pi_{D(r)}(Q, D) \leq \alpha\} = 1$ , so now the downside risk is 1, which is greater than any  $\beta$ , the retailer chooses no order because the downside risk doesn't satisfy the restriction of the target function.

(2) when  $Q_0 \leq Q_{D(r)}$ ,

The retailer's profile function is  $\pi_{D(r)} = p \min(Q, D) - wQ + V(Q)$ , in which

$$V(Q) = \begin{cases} v(Q-D) & Q \geq D \\ 0 & Q \leq D \end{cases}$$

$$\text{So } P(\pi_{D(r)}(Q, D) \leq \alpha) = P\left\{ \left[ \pi_{D(r)} \leq \alpha \right] \cup \left[ \pi_{D(r)} \leq \alpha \right] \right\},$$

and in which  $P[\pi_{D(r)} \leq \alpha] = P\{[p \min(Q, D) - wQ] \leq \alpha\} = P\{(p-w)Q \leq \alpha\} = 0$  and

$P[\pi_{D(r)} \leq \alpha] = P\{[p \min(Q, D) - wQ + v(Q - D)] \leq \alpha\} = P\{D \leq \frac{\alpha + (w-v)Q}{(p-v)}\}$ , Thus the retailer's

downside risk is  $F(\frac{\alpha + (w-v)Q}{(p-v)})$  when  $Q_0 \leq Q_{D(r)}$ , and risk increasing monotonously in Q.

When  $F(\frac{\alpha + (w-v)Q_{D(r)}}{p-v}) < \beta \leq 1$ : the optimal order quantity  $Q_{D(r)}$  satisfies the downside risk restriction, now the retailer's current optimal order quantity is the optimal order quantity of the decentralized supply chain, see below:

$$Q_D^\beta = Q_{D(r)}^0 = F^{-1}\left(\frac{p-w}{p-v}\right) \quad (7)$$

When  $F(Q_0) \leq \beta \leq F(\frac{\alpha + (w-v)Q_{D(r)}}{p-v}) \leq 1$ : the  $Q_{D(r)}$  cannot satisfy downside risk restriction of the target function.

When  $Q_0 \leq Q_{D(r)}$ , the downside risk  $F(\frac{\alpha + (w-v)Q}{p-v})$  increases monotonously in Q, and the retailer's expected profit function  $E(\pi_{D(r)})$  increases monotonously in Q in the left of the equation, so the retailer's optimal order quantity satisfies:

$F(\frac{\alpha + (w-v)Q}{p-v}) = \beta$ , and the quantity is

$$Q_D^\beta = \left(\frac{(p-v)F^{-1}(\beta) - \alpha}{w-v}\right) \quad (8)$$

### 3.4 The supply chain buy-back contract model with risk-averse condition

The ultimate goal of supply chain coordination is to maximize the expected profit of the decentralized supply chain; this makes the order quantity in the decentralized supply chain equal to the one in the centralized supply chain by buy-back contract. Assuming that buy-back price is b, the discussion according to the above conclusion is as follows:

(1) when  $F(\frac{\alpha + (w-v)Q_{D(r)}}{p-v}) < \beta \leq 1$  and  $Q_{D(r)} < Q_0$ , the quantity before implementing buy-back contract is

$Q_{D(r)} = F^{-1}\left(\frac{p-w}{p-v}\right)$ , the optimal order quantity after implementing contract is  $Q_I$ , so the buy-back price satisfies

$F^{-1}\left(\frac{p-w}{p-b}\right) = F^{-1}\left(\frac{p-c}{p-r}\right)$ , as a result, the buy-back price which can coordinate supply chain is

$$b = \frac{w(p-r) + p(r-c)}{p-c} \quad (9)$$

(2) When  $F(Q_0) \leq \beta \leq F(\frac{\alpha + (w-v)Q_{D(r)}}{p-v}) \leq 1$  and  $Q_0 < Q_{D(r)}$ , the quantity before implementing buy-back contract is

$Q_D^\beta = \left(\frac{(p-v)F^{-1}(\beta) - \alpha}{w-v}\right)$ , the optimal order quantity after implementing the contract is  $Q_I$ , so the buy-back price

satisfies  $\frac{(p-b)F^{-1}(\beta) - \alpha}{w-b} = Q_I = F^{-1}\left(\frac{p-c}{p-r}\right)$ , as a result, the buy-back price which can coordinate supply chain is

$$b = \frac{wQ_I + \alpha - pF^{-1}(\beta)}{Q_I - F^{-1}(\beta)} \quad (10)$$

## 4. Simulation application

Supposing that the parameters of a supply chain are as follows:  $p=2058$ ,  $c=1625$ ,  $w=1822$ ,  $v=600$  and  $r=900$  (Unit is supposed to be 1). The demand of the product follows normal distribution  $D \sim N(11900, 1700^2)$ .

Take the known condition into the equation (1)~(2), and get the optimal order quantity and maximum expected profit of the supply chain in the risk-neutral case. Table 1 shows the results.

*Table 1: The expectation profile function of the centralized supply chain under the risk-neutral condition*

		the optimal order quantity of the supplier Q	the maximum profit of the retailer E
	the centralized supply chain	1.1353e+004	4.4069e+006
risk- neutral	the decentralized supply chain	1.0222e+004	2.2007e+006

As showed in the table 1, in the risk-neutral conditions, the optimal results of the decentralized supply chain are lower than which of the centralized supply chain. In the decentralized supply chain, the decision based on maximizing the retailer's profit doesn't achieve the supply chain optimization. Therefore, contracts need to be taken to achieve supply chain optimization, but more details are not involved in this paper.

When the retailer's risk aversion parameter  $(\alpha, \beta) = (2000000, 0.05)$ , take the parameter into the buy-back contract model above, and get the buy-back price  $b=1.4268e+003$ , the retailer's optimal order quantity and expected profit can be obtained from the model built above.

*Table 2: The maximum expectation profit and the optimal order quantity of supply chain with the risk-averse condition*

		the optimal order quantity of the supplier Q	the maximum profit of the retailer E
	the centralized supply chain	1.1353e+004	4.4069e+006
risk-averse	the decentralized supply chain	9.9261e+003	1.9342e+006
	the decentralized supply chain after buy-back contract	1.1353e+004	2.4019e+006

As showed in the table 2. First, the optimal results of the centralized supply chain under the risk-averse condition are equal to those under the risk-neutral condition; this proves the correctness of the conclusions above. Second, the optimal results of the decentralized supply chain are lower than those of the centralized supply chain under the risk-averse condition. Last but not least, the retailer's optimal order quantity of the decentralized supply chain achieve which of the centralized supply chain through the implementation of buy-back contract and coordinate the supply chain.

## 5. Conclusion and future research

This paper studies a two-stage supply chain consist of a risk-neutral supplier and a risk-averse retailer, the buy-back contract model enables to coordinate the supply chain and prove that the model is effective to realize win-win goals.

Certainly there are still many limitations in this paper; it can be extended in the following areas. Firstly, there are only one supplier and one retailer in the supply chain, in fact the supply chain structure is often more complex. Secondly, this paper does not take into account capacity constraints of the retailer. In future research, capacity constraints and risk aversion preferences can be taken into account when designing the contract coordination.

## Acknowledgements

The authors thank the editor and reviewers for their insightful comments and suggestions. This work is financially supported by supported by the China Scholarship Council (201506630047), Supported by Innovation Program of Shanghai Municipal Education Commission (13ZS129), and Fundamental Research Funds for the Central Universities (CUSF-DH-D-2015065), Shanghai University Teacher Training Program (ZGCD15014) and Ph.D. Programs Foundation of Shanghai University of Engineering Science (2015).

## References

- Adida, E., & Demiguel, V. 2011. Supply chain competition with multiple manufacturers and retailers. *Operations Research*, 59(1), 156-172. DOI: 10.1287/opre.1100.0863.
- Amin-Naseri, M. R., & Khojasteh, M. A. 2015. Price competition between two leader–follower supply chains with risk-averse retailers under demand uncertainty. *International Journal of Advanced Manufacturing Technology*, 79, 1-17. DOI: 10.1007/s00170-014-6728-0.
- Arcelus, F. J., Kumar, S., & Srinivasan, G. 2012. Risk tolerance and a retailer's pricing and ordering policies within a newsvendor framework. *Omega*, 40(2), 188–198. DOI: 10.1016/j.omega.2011.05.007.
- Budde, M., & Minner, S. 2014. First- and second-price sealed-bid auctions applied to push and pull supply contracts. *European Journal of Operational Research*, 237(1), 370-382. DOI: 10.1016/j.ejor. 2014.03.007.
- Chernonog, T., & Kogan, K. 2014. The effect of risk aversion on a supply chain with postponed pricing. *Journal of the Operational Research Society*, 65(9), 1396-1411. DOI: 10.1057/jors.2013.85.
- Fisher, M., & Raman, A. 1996. Reducing the cost of demand uncertainty through accurate response to early sales. *Operations Research*. 44(1), 87-99. DOI: 10.1287/opre.44.1.87.
- Glock, C. H., & Kim, T. 2015. The effect of forward integration on a single-vendor-multi-retailer supply chain under retailer competition. *International Journal of Production Economics*, 167, 179-192. DOI: 10.1016/j.ijpe.2015.03.009.
- Güler, M. G., & Keskin, M. E. 2013. On coordination under random yield and random demand. *Expert Systems with Applications*, 40, 3688–3695. DOI: 10.1016/j.eswa.2012.12.073.
- Kim, K. K., & Park, K. S. 2014. Transferring and sharing exchange-rate risk in a risk-averse supply chain of a multinational firm. *European Journal of Operational Research*, 237(2), 634–648. DOI: 10.1016/j.ejor.2014.01.067.
- Kulkarni, S., Ponnaiyan, S., & Tarakci, H. 2015. Optimal ordering decisions under two returns policies. *International Journal of Production Research*, 53, 3720-3734(15). DOI: 10.1080/00207543.2014.988883.
- Ozen, U., Sobic, G., & Slikker, M. 2012. A collaborative decentralized distribution system with demand forecast updates. *European Journal of Operational Research*, 216(3), 573-583. DOI: 10.1016/j.ejor.2011.07.055
- Ruiz-Benitez, R., & Muriel, A. 2014. Consumer returns in a decentralized supply chain. *International Journal of Production Economics*, 147(1), 573-592. DOI: 10.1016/j.ijpe.2013.05.010.
- Takahashi, K., Aoi, T., Hirotsu, D., & Morikawa, K. 2011. Inventory control in a two-echelon dual-channel supply chain with setup of production and delivery. *International Journal of Production Economics*, 133(1), 403-415. DOI: 10.1016/j.ijpe.2010.04.019
- Yenho, T. C., & Feryal E. 2013. Designing supply contracts for perishable goods with two periods of shelf life. *International Journal of Production Economics*, 145(1), 53-67. DOI: 10.1080/0740817X.2012.654847
- Yoo, S. H. 2014. Pricing and return policy under various supply contracts in a closed-loop supply chain. *International Journal of Production Research*, 53(1), 106-126. DOI: 10.1080/00207543. 2014.932927.
- Yoo, S. H. 2014. Product quality and return policy in a supply chain under risk aversion of a supplier. *International Journal of Production Economics*, 154(4), 146–155. DOI: 10.1016/j.ijpe.2014.04.012.