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Slice Method for Rock Slope Stability Analysis Based on the Hoek–Brown Criterion

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Hoek-Brown criterion is widely used to predict rock strength and analyze stability. Convert the H-B formula to find the relationship between the normal stress σ_n and shear stress τ of the sliding surface. Combined with the H-B criterion, gives a formula F_s to solve the safety factor. The research shows that the F_s is associated with H-B constants m_i, geological strength index GSI, disturbance factor D, slice parameter α_i , ration of normal stress and saturated single axis compressive strength Sr_i. Through building rock slope model, to analyze the safety factor Fs at different ranges of parameters. Compared with the results of Swedish slice method, Janbu method, Bishop method and M-P method, it displayed the new algorithm’s rationality and limitations .Finally verifying the applicability of the new algorithm’s through the engineering example.

1. Introduction

The stability analysis of rock slope has been the key and hot point in engineering research. The traditional methods such as Swedish slice, Jan-bu, Bishop and M-P based on M-C criterion. The cohesion *c* and the internal friction angle *φ* are used to calculate the safety factor value. They have been matured in theory, and widely used in project practice. But the M-C criterion can’t reflect the influence of the nonlinear failure and engineering geology for rock masses. In 1980, E. Hoek and E. T. Brown proposed the H-B criterion to supplement the deficiency of the M-C criterion (HOEK E, BROWN E T (1980)). The H–B strength criterion can describe the destruction and deformation characteristics of rock mass with good accuracy, reflect the nonlinear characteristics of rock destroyed, and well present the surface stress state, blasting damage and stress relieving effect on strength. The relevant parameters concerning H–B criterion are difficult to be determined and the corresponding applicability for jointed rock mass is poor, as *GSI* involved is difficult to be determined. Because of this, through it is widely used in rock mass strength research, but it can’t be used in the stability analysis of Rock Slope directly. Many scholars have done a lot of research on this subject, strength reduction method based on the Hoek–Brown Criterion was used to analysis of slope stability(Lin Hang et al. (2007); Wu Shunchuan et al. (2006)). But most of scholars explore the relationship between the cohesion *c* and the internal friction angle *φ* with the H–B parameters m_i, a, s and the geological strength index *GSI*. The H-B criterion is not used to solve slope stability analysis question directly. The H–B strength criterion was used to analysis of slope stability combined with computer technology(Carranza-Torres C (2004); Shen J et al. (2012)). This article will use the H-B criterion to analysis slope stability combined with Matlab and offer a reference of application for the H-B criterion directly.

2. Formula of Fs under the Hoek-Brown criterion

E. Hoek and E. T. Brown (HOEK E, BROWN E T (1980)). proposed The H-B strength criterion in 1980. The equations are expressed as follows:

$$\sigma_1=\sigma_3+\sigma_c(\frac{m_b}{\sigma_c}\sigma_3+1)^{0.5} \tag{1}$$

Where σ_1 is the maximum principal stresses, σ_3 is the minimum principal stresses, σ_c is the uniaxial compressive strength of the intact rock, m_b is the H–B constant of the intact rock.

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E. Hoek and E. T. Brown(HOEK E (1992); HOEK E et al. (1998)) improved the H–B strength criterion in 1992. The equations are expressed as follows:

$$\sigma_1=\sigma_3+\sigma_c(\frac{m_b}{\sigma_c}\sigma_3+s)^a \tag{2}$$

Where m_b, s and a are the H–B parameters that depend on the degree of fracturing of the rock mass and can be estimated from the Geological Strength Index (GSI) (HOEK E et al. (2002)), given by

$$m_b=m_i\exp(\frac{GSI-100}{28-14D}) \tag{3}$$

$$s=\exp(\frac{GSI-100}{9-3D}) \tag{4}$$

$$a=\frac{1}{2}+\frac{1}{6}(e^{-\frac{GSI}{15}}-e^{-\frac{20}{3}}) \tag{5}$$

Derivate of the formula (2), the new equations are expressed as follows:

$$\frac{d\sigma_1}{d\sigma_3}=1+am_b(m_b\frac{\sigma_3}{\sigma_{ci}}+s)^{a-1} \tag{6}$$

It should be noted that the expressions (7) and (8) are valid for any non-linear failure envelope. In the case of the generalized Hoek-Brown failure criterion (2), after replacing the functions σ_1 and $d\sigma_1/d\sigma_3$ in equations (7) and (8), we get

$$\frac{\sigma_n}{\sigma_{ci}}=\frac{\sigma_3}{\sigma_{ci}}+\frac{am_b(m_b\frac{\sigma_3}{\sigma_{ci}}+s)^a}{2+am_b(m_b\frac{\sigma_3}{\sigma_{ci}}+s)^{a-1}} \tag{9}$$

$$\frac{\tau}{\sigma_{ci}}=(\frac{m_b\sigma_3}{\sigma_{ci}}+s)^a\frac{\sqrt{1+am_b(m_b\frac{\sigma_3}{\sigma_{ci}}+s)^{a-1}}}{2+am_b(m_b\frac{\sigma_3}{\sigma_{ci}}+s)^{a-1}} \tag{10}$$

In order to calculate shear stress τ, for the given values of the input parameters m_b, s, a, σ_{ci} and σ_n, Eq. (9) is solved iteratively to calculate the σ_3 value. Having obtained σ_3, Eq. (10) can be used to calculate shear stress τ. τ/σ_{ci} is a function of σ_n/σ_{ci}, m_b, GSI and D. Therefore, τ/σ_{ci} can be expressed as follows:

$$\frac{\tau}{\sigma_{ci}}=f_4(\frac{\sigma_n}{\sigma_{ci}},m_i,GSI,D) \tag{11}$$

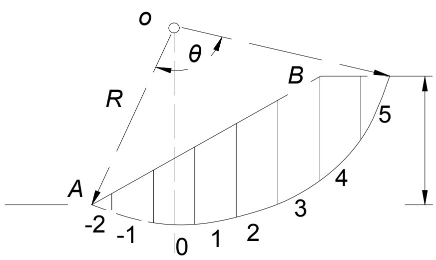

Fig.1 Slip surface of rock slope

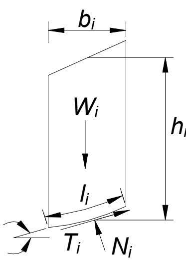

Fig.2 Stresses on a given slice

The factor of safety (F_S) will be defined as the ratio of the moment $M_s=\sum W_iR\sin\alpha_i$, associated with forces that tend to stabilize the slope, and the moment $M_w=\sum T_iR$, associated with forces that tend to destabilize the slope.

$$F_s=\frac{M_R}{M_s}=\frac{\sum T_iR}{\sum W_iR\sin\alpha_i} \tag{12}$$

Where T_i and W_i are the forces acting at an arbitrary slice *(i)*, and R and $R\sin\alpha_i$ are the corresponding distances between the forces and the center of the failure surface —see Fig.1 and Fig.2.

For the arbitrary slice *(i)* represented in Fig.1 and Fig.2, the forces T_i and W_i can be expressed in terms of the shear stress $\tau^{(i)}$ and normal stresses $\sigma_n^{(i)}$ acting on the base of the slice (i, on the failure surface) as follows,

$T_i = \tau_i * l_i$ and $N_i = W_i \cos\alpha_i = \sigma_n^{(i)} l_i$ The factor of safety (F_S) can be equally written as

$$F_s = \frac{\sum \tau^{(i)} l_i}{\sum \frac{\sigma_n^{(i)} l_i}{\cos\alpha_i} \sin\alpha_i} = \frac{\sum \tau^{(i)} \cos\alpha_i}{\sum \sigma_n^{(i)} \sin\alpha_i} \tag{13}$$

Replacing equations (11) into equation (13), the factor of safety (F_S) can be equally written as

$$F_s = \frac{\sum f_4(\frac{\sigma_n^{(i)}}{\sigma_{ci}}, m_i, GSI, D)\cos\alpha_i}{\sum \frac{\sigma_n^{(i)}}{\sigma_{ci}} \sin\alpha_i} \tag{14}$$

The equations (14) show that the factor of safety (F_S) is a function associated with $\sigma_n^{(i)}/\sigma_{ci}$, α_i, m_i, GSI and D. At the same time, $\sigma_n^{(i)}$ is associated with γ, h_i and α_i as follows,

$$\sigma_n^{(i)} / \sigma_{ci} = \gamma h_i \cos\alpha_i / \sigma_{ci} = Sr_i \tag{15}$$

Thus, replacing equations (15) into equation (14), the factor of safety (F_S) can be equally written as

$$F_s = \frac{\sum f_4(Sr_i \cos\alpha_i, m_i, GSI, D)\cos\alpha_i}{\sum SR_i \sin\alpha_i} = f_5(Sr_i, \alpha_i, GSI, D, m_i) \tag{16}$$

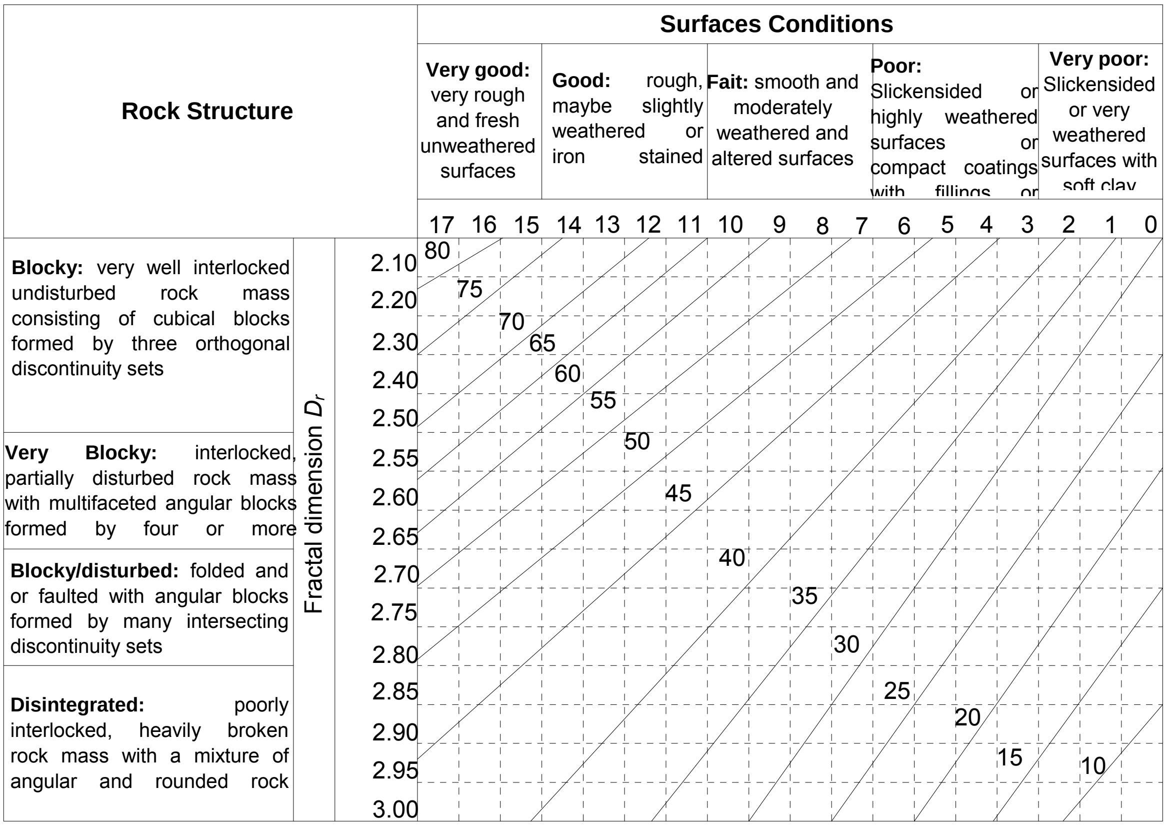

Figure 3: Geological strength index (GSI) in Hoek-Brown criterion

3. Example and comparison

3.1 The value of rock parameter:

The equations (16) show that the values of *GSI* and m_i are the key elements that must be considered when analysis slope stability with the H-B criterion. It is clear that the rock mass strength parameters are sensitive to the GSI value by E. Hoek and E. T. Brown. The lack of parameters to describe surface conditions of the discontinuities and the rock mass structure prevents to obtain a more precise value of *GSI*. Sonmez introduced rock structural hierarchy and rock surface condition rating to achieve quantitative *GSI* system (SONMEZ H, ULUSAY R (1999)). Su Yong-hua introduced rock mass block index and weathering index to depict characteristics of rock mass, realizing the quantification of *GSI* (Su Yong-hua et al. (2009)). In the aspects of quantitative *GSI* system, Cai and Kaiser et al proposed a method based on block size and joint condition factor J_C, while Hu Sheng-ming developed an approach relying on joint count J_v and joint condition factor J_C (Hu Sheng-ming, Hu Xiu-wen (2011)). These *GSI* quantitative methods have enriched the theory of *GSI* concerning rock classification and improved the precision of *GSI* value. However, the abovementioned methods have serious limitations, as they cannot widen the application of H–B strength criterion in the jointed rock mass with apparent anisotropy, and cannot reflect the influence of rock mass discontinuity on the rock mass strength. Meanwhile, *GSI* cannot present three-dimensional characteristics of jointed rock mass, and is not capable of reflecting the spatial distribution of rock mass discontinuity, which affects the estimation of *GSI*. For these reasons, the authors suggest two terms namely, 'fractal dimension, D_r' based on distribution of rock masses discontinuities and 'surface condition rating, *SCR*', estimated from the input parameters. It is a more precise *GSI* value from intersection point of fractal dimension and *SCR* ratings when the modified *GSI* chart is used in Fig.3.

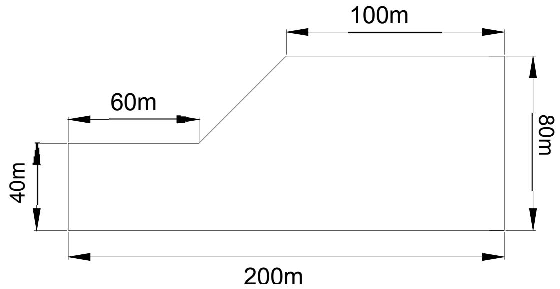

Fig.4 Model size of rock slope (unit: m)

3.2 Values of *GSI* impact on the safety factor *Fs* :

3.2.1 Verification model

According to the model index, the distance from slope toe to left side is 1.5 times long than the height of slope, the distance from slope top to right side is 2.5 times long than slope height, the slope angle is 45°, slope height is 40 meter in Fig 4. The slope consisting of highly fractured rock masses with the following input parameters.

Table 1:The values of F_s under different GSI and m_i

GSI	10	30	50	70	100
F_s	1.162	1.435	2.130	2.951	9.091

Table 2: The values of F_s under different m_i

m_i	5	10	15	20	25	30
F_s	1.237	1.718	2.103	2.337	2.619	2.817

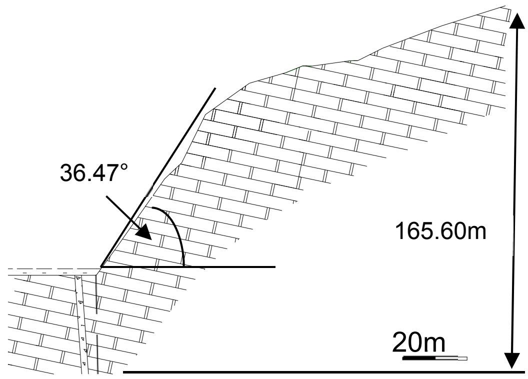

Fig.7 Simplified CAD drawing of Bridge

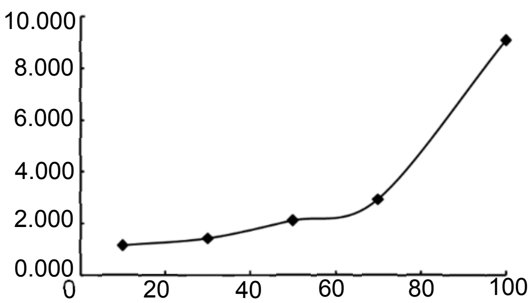

Fig.5 Relationships between GSI and F_s

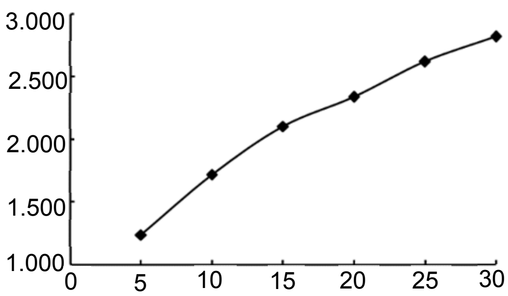

Fig.6 Relationships between m_i and F_s

The equation (16) shows that the values of σ_{ci}, m_i and *GSI* are necessary factor for the safety factor F_s. Table 1 shows that the values of the safety factor F_s change as the values of *GSI* changing with the following input parameters: D = 0, σ_{ci} = 10MPa, m_i = 10. It shows that the variation of the safety factor F_s increases gradually in a small range when *GSI*<60 and increases rapidly when *GSI*>60 in Fig.5. Actually, the better interlocked undisturbed rock masses are, the higher values of *GSI* are. At the same time, the less development structure and surfaces conditions of rock are, the higher the values of safety factor F_s are. Table 2 shows that the values of the safety factor F_s change as the values of m_i changing with the following input parameters: D = 0, σ_{ci} = 10MPa, *GSI* = 30. It shows that the variation of the safety factor F_s increases gradually as m_i increased in Fig.6. The values of m_i have the great influence to the stability of rock in a certain range, but beyond this range, the influence tends to stagnation.

4. Engineering example

A slope from Wujiang river build a bridge located at Guiyang province. The necessary data were collected by authors from this bridge. The slope with the height of 165.60m and the angle of 36.47° has a sequence consisting of compact limestone. There are two penetrating joints of the slope which have dip angle with the following parameters: J1: 0°–20°∠80–85°; J2:70°–90°∠80–85°. Two joints are "X" shape and cut rock with the angle of 70°. And second fault activity had occurred in the region before. Many joints and cracks were developed nearby the "X" shape and formed many discontinuities for rock masses. It shows that the rock slope of the bridge draw by *CAD* in Fig.7.

Table 3: Calculation parameters

Slice number	Calculate parameter					
	h_i	l_i	α_i	Sr_i	σ_3/σ_{ci}	τ/σ_{ci}
1	7.5912	10.3170	-4.8788	0.0068	0.0040	0.0175
2	19.0600	7.3835	16.4730	0.0163	0.0103	0.0301
3	26.8180	7.3835	16.4730	0.0230	0.0149	0.0379
4	35.7900	12.4910	27.4900	0.0283	0.0187	0.0403
5	43.5810	7.9583	32.8890	0.0327	0.0218	0.0419
6	47.7710	5.9337	32.8890	0.0358	0.0241	0.0446
7	54.4550	15.6050	37.8560	0.0384	0.0259	0.0438
8	63.0130	10.5740	42.0400	0.0418	0.0284	0.0436
9	65.5760	8.2310	44.6210	0.0417	0.0284	0.0418
10	63.7750	8.6263	46.3820	0.0393	0.0266	0.0389
11	61.7760	8.6263	46.3820	0.0380	0.0257	0.0381
12	58.4200	15.7960	49.5590	0.0338	0.0227	0.0332
13	50.8770	17.5610	52.0600	0.0279	0.0184	0.0277
14	41.8500	12.3100	54.2570	0.0218	0.0141	0.0223
15	35.1350	8.9323	55.6460	0.0177	0.0113	0.0188
16	29.1080	9.4411	57.2890	0.0140	0.0088	0.0154
17	22.7910	9.0233	59.7570	0.0102	0.0062	0.0116
18	15.2390	10.5440	59.7560	0.0069	0.0041	0.0090
19	6.5650	10.0700	61.8360	0.0028	0.0015	0.0048
20	1.1385	2.7461	61.8370	0.0005	0.0001	0.0020

The value of *GSI* is estimate to 30 by table 1 and the value of m_i is 10. The value of D is equal 0 because of no excavation. The value of *GSI*, m_i and D were used to calculate the H–B parameters m_b, s and a. The results were m_b =0.8208, s = 0.0004 and a = 0.5223. The rock slope consisting of highly fractured rock masses with the following input parameters: σ_{ci} =28MPa, γ = 27 KN/m3, φ = 32°and c = 330 Kpa.

The model with the height of 230 m and the width of 310 m and the slope height of 170 m was used to analysis slope stability with the help of Slope/W module under SIGMA/W in Geostudio 2004. Number of slices is 20, water pressure and disturbance is ignored. The most dangerous slip surface from the rock slope model is calculated by the Geostudio2004. Table 3 shows parameters for slices in the model which used by the Geostudio 2004. The value of Sr_i is calculated by the value of γ, h_i and σ_{ci}. The value of σ_3/σ_{ci} is calculated by equations (9) for each slice. The value of τ/σ_{ci} is calculated by equations (10) for each slice. Finally, the value of the safety factor F_s was calculated by equations (13) and equations (14):

$$F_s = \frac{\sum(\tau / \sigma_{ci})\cos\alpha_i}{\sum SR_i \sin\alpha_i} = \frac{0.4205}{0.3072} = 1.3833$$

At the same time, the values of the safety factor F_s were 1.408, 1.396, 1.430 and 1.436 by the method of slice, Jan-bu, Bishop and M-P. Compared with all results, the value calculated by the new method is smaller

5. Conclusions

The H-B strength criterion is widely used in underground and slope engineering. But it is seldom used to research slope stability because of the H–B parameters are difficult to determine and the applicability is poor. The exact value of the *GSI* is very difficultly obtained by E. Hoek and E. T. Brown. It is a more precise *GSI* value from intersection point of fractal dimension based on distribution of rock masses discontinuities and *SCR* ratings when the modified *GSI* chart is used in Fig.3. It is found that the factor of safety (F_S) can be equally written as equations (16).

The fractal dimension of rock discontinuities is a function which combined with rock strength, the formation of environmental and engineering geological characteristics. It is feasible that the three-dimensional fractal dimension of rock discontinuities is used to describe rock structures.

The result of the safety factor F_s from the H-B criterion compared with the result from the method of M-C, Jan-bu, Bishop and M-P shows that the error is existed in different methods. The value of the safety factor F_s is 1.3833and is smaller than the results of the other methods is 1.408, 1.396, 1.430 and 1.436.

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