

# Study on High-speed Train Operation Adjustment Based on Differential Evolution

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With the characteristics of numerous constraints, large density and high speed of train, High-speed train operation adjustment problem is a typical large-scale combinatorial optimization problem and also a kind of difficult problem of NP. In combination with the characteristic of China's high-speed railway, this paper builds the mathematical model of high-speed railway operation adjustment, and the model is solved and optimized with differential evolution algorithm. The result of the experiment shows that the Differential Evolution has very high rationality and feasibility in solving the high-speed train operation adjustment problem.

## 1. Introduction

Train operation adjustment refers to the situation that the train operation needs to be replanned to restore its orderly operation state since the train's actual operation state deviates from the expected value under the influence of many unexpected factors and emergencies. The searching of an optional train operation adjustment plan is very difficult and complicated due to the complicated network and dynamic changes of the railway transportation. Especially since the high-speed railway has many characteristics, such as the high density and speed of train, its operation is often influenced by many factors like bad weather, line equipment failure and the running of other vehicles. Different situations may involve different variables, and different variable has different influence on the train operation and their expressions of mathematical model can be so different that some variables may be related to the large-scale combinatorial optimization problem which is hard to find the optional solution in traditional mathematical method. In recent years, some new intelligent optimization algorithms like GA, PSO are applied to solve the train operation adjustment problem. However, problems like long searching time and premature are ubiquitous in these algorithm.

Differential Evolution (DE) isa new evolutionary computation technology (Storn P, Price K (1995) reported, Vesterstorn J, Thomsen R (2004) reported,) which was put forward by professor Storn and professor Price from Berkeley University in 1995. As a kind of bionic intelligent computation method, DE is considered to have been made great progress in algorithm structure because of its super performance (Duan Haibin et al (2011) reported, Liu Bo et al (2007) reported, Wu Lianghong et al (2007) reported) of less controlled parameters and simple optional process in solving complex optimization problem. However, since there are many problems like premature, low convergence rate and huge computation lying in this algorithm, a differential Evolution (IDE) is put forward in this paper. The simulation result shows that the IDE is superior to the standard differential evolution and can solve the high-speed train operation adjustment problem effectively.

## 2. The description of high-speed train operation adjustment problem and its mathematical model

### 2.1 Objective function

The train adjustment model of certain adjustment area can be defined as follows: Establishing that in the double operation line area, there are  $M$  stations and  $N$  trains among which  $N_1$  is the up train while  $N_2$  is the down train and  $N = N_1 + N_2$ ;  $A_{ij}^0$  and  $S_{ij}^0$  show the scheduled time when the  $i$  ( $i = 1, 2, \dots, N$ ) train arrive at and depart from the  $j$  ( $j = 1, 2, \dots, M$ ) station, while  $A_{ij}$  and  $S_{ij}$  show the actual time when the  $i$  train

arrive at and depart from the  $j$  station;  $\beta_j^s$  and  $\beta_j^a$  are the shortest interval time between the train's departure and arrival time, while  $\lambda_{ij}$  is the shortest dwell time of the  $i$  train in the  $j$  station;  $T_{j(j+1)}^i$  is the  $i$  train's minimum operation time between the  $j$  and the  $(j+1)$  station;  $\omega$  which is given according to the adjustment rule and expertise indicates the relative priority level of the train, and the bigger  $\omega$  is, the higher the train's priority is. Give a variable  $\tau_{ij}$  showed in formula (1) indicating whether the  $i$  will dwell in the  $j$  station or not.

$$\tau_{ij} = \begin{cases} 1 & S_{ij} \neq A_{ij} \\ 0 & S_{ij} = A_{ij} \end{cases} \quad (1)$$

Give a symbolic operation showed in formula

$$\text{sgn}(a, b) = \begin{cases} 1 & a > b \\ 0 & a \leq b \end{cases} \quad (2)$$

Thus we can get the mathematical model of high-speed train operation adjustment showed in formula (3):

$$f = \sum_{j=1}^M \sum_{i=1}^N (k_1 \omega_i (|A_{ij} - A_{ij}^0| + |S_{ij} - S_{ij}^0|) + k_2 \tau_{ij} \times \text{sgn}(A_{ij}, A_{ij}^0)) \quad (3)$$

In this formula, since  $\sum_{j=1}^M \sum_{i=1}^N \omega_i (|A_{ij} - A_{ij}^0| + |S_{ij} - S_{ij}^0|)$ , the delay time, and  $\sum_{j=1}^M \sum_{i=1}^N \tau_{ij} \times \text{sgn}(A_{ij}, A_{ij}^0)$ , the amount of

delay train, are different in dimension, we adopt  $k_1$  and  $k_2$  as the weighting coefficient to transfer the two different objects into an optional object, and the value of the coefficient should be taken according to the specific situation.

## 2.2 Constraint conditions of the high-speed train operation adjustment

In consideration of the practical situation of the high-speed train operation, the constraint conditions lying in the operation adjustment area are as follows

(1) The constraint condition of the train's earliest departure time is showed in formula (4)

$$S_{ij} \geq S_{ij}^0 \quad (4)$$

(2) The constraint condition of the train's operation time is showed in formula (5)

$$A_{i(j+1)} - S_{ij} \geq T_{j(j+1)}^i \quad (5)$$

(3) The constraint condition of the train's space interval is showed in formula (6)

$$A_{(i+1)j} - A_{ij} \geq \beta_j^a ; S_{(i+1)j} - S_{ij} \geq \beta_j^s \quad (6)$$

(4) The constraint condition of the train's overtaking ( $k$  goes behind  $i$ ) is showed in formula (7)

$$\omega_k > \omega_i ; A_{kj} - A_{ij} \geq \tau_{ij} \beta_j^a ; S_{ij} - A_{kj} \geq \tau_{ij} \beta_j^s \quad (7)$$

(5) The constraint condition of the train's station dwell time is showed in formula (8)

$$S_{ij} - A_{ij} \geq \tau_{ij} \lambda_{ij} \quad (8)$$

## 3. Standard differential evolution

Differential evolution is an algorithm that is based on population evolution, and it can be translated into the minimization problem solved as follows:

Thereinto,  $X = [x_1, x_2, \dots, x_{NP}]$  makes up the decision space,  $x_i(g) = [x_{1,g}, x_{2,g}, \dots, x_{d,g}]$ ,  $i = 1, 2, \dots, NP$ ,

$x_i^l \leq x_i \leq x_i^u$ ,  $x_i^u$ ,  $x_i^l$  are respectively the upper bound and lower bound of the decision space, and  $NP$  is

the size of decision space, and  $d$  is the dimension quantity, and  $g$  is the present evolution algebra. The fundamental principle of differential evolution is: firstly initialize the population, then conduct the operation of variation, crossover and selection to the individual of population and produce progeny population, and get the final result after repeated iteration. The concrete steps are as follows:

### (1) Initialization population

In the decision space  $X$ , the initialization vector that generated randomly is shown as formula (9):

$$x_i(0) = x_i^l + rand(0,1) * (x_i^u - x_i^l) \quad (9)$$

### (2) Mutation operation

After the multiplying of the difference vector of differential evolution with the scaling factor, the vector will have vector composition with the base vector. The typical mutation operator is formula (10)

$$v_i(g+1) = x_{r_1}(g) + F (x_{r_2}(g) - x_{r_3}(g)) \quad (10)$$

Thereinto,  $v_i(g+1)$  is the  $(g+1)$  th generation variation vector;  $F$  is the scaling factor;  $r_1, r_2, r_3$ , are integers that are different from each other;  $x_{r_1}(g)$  is the parent base vector in the  $g$  generation population, and  $(x_{r_2}(g) - x_{r_3}(g))$  are the parent difference vectors.

### (3) Crossover operation

Conduct crossover operation to each individual vector  $x_i(g)$  in the  $g$  generation population with variation individual  $v_i(g+1)$ , and there comes the new individual  $u_i(g+1)$ , so as to increase the diversity of the population individual, and the formula is as shown in formula (11):

Thereinto,  $CR$  is the crossover probability factor;  $x_{ij}(g)$  means the  $j$  dimension component of the  $i$  individual in the  $g$  generation population, and  $rand(j) \in [0,1]$  is the corresponding random number of the  $j$  dimension. The  $k$  is the corresponding coefficient of the  $i$  individual and it is always an integer that is selected randomly in list  $[1,2,\dots,D]$  and is used to ensure that there is at least one dimension component in the  $u_i(g+1)$  that is from the variation vector individual  $v_i(g+1)$ .

$$u_i(g+1) = \begin{cases} v_{ij}(g+1) & \text{if } (rand(j) \leq CR) \text{ or } j = k \\ x_{ij}(g+1) & \text{otherwise} \end{cases} \quad (11)$$

### (4) Selecting operation

The standard differential evolution takes the greedy selection strategy to conduct the test vector  $u_i(g+1)$  and the present population target vector  $x_i(g)$ ; by evaluating the fitness of both of them, select the superior individual for the search of next generation, and the formula is shown as formula (12).

$$x_i(g+1) = \begin{cases} u_i(g+1) & \text{if } f(u_i(g+1)) < f(x_i(g)) \\ x_i(g) & \text{otherwise} \end{cases} \quad (12)$$

Thereinto,  $f$  is the fitness function, and  $f(u_i(g+1))$  is the fitness value that is corresponded with the test individual  $u_i(g+1)$ .

## 4. Differential evolution algorithm and its solve

### 4.1 Design of code

The differential evolution takes the method of real number encoding, which for the purpose of simplicity, converts the train schedule into the integer minute system, and sets the time of one day to 1440 minutes, the experienced minute quantity from midnight 0 clock to a time present a moment as a certain moment, for example, the time 3:30 is converted into 210 min. The result after the algorithm iteration will be converted into integer value by the method of carrying rounding, for example, the iterated result 187.1 is converted into 188. According to each schedule, for the  $M$  stations in the adjustment zone and  $L$  train, we take  $2M \times N$  matrix for real number encoding, and  $X_{(k,i)}$  means the time when the  $i$  train arrives at and depart from the  $k$  station.



Table 2: Adjusted train operational plan

Train Number	Beijing South	Langfang		Tianjin South		Cangzhou West		Dezhou East		Jinan West
	From	To	From	To	From	To		From	To	From
G101	7:10	7:25	7:25	7:37	7:37	7:51	7:52	8:15	8:15	8:38
D331	7:15	7:31	7:31	7:44	7:46	8:07	8:07	8:30	8:32	9:21
G261	7:20	7:37	7:37	7:49	7:51	8:12	8:12	8:35	8:37	9:03
G57	7:25	7:44	7:44	7:59	8:01	8:22	8:22	8:45	8:47	9:11
G185	7:40	7:56	7:56	8:04	8:06	8:28	8:30	8:52	8:52	9:16
G263	7:55	8:12	8:12	8:25	8:25	8:44	8:44	9:05	9:05	9:27
G11	0:00	8:17	8:17	8:30	8:30	8:48	8:48	9:10	9:10	9:32
G107	8:08	8:25	8:25	8:39	8:39	8:59	9:01	9:28	9:30	9:54
G55	8:13	8:32	8:32	8:47	8:49	9:10	9:10	9:33	9:43	10:07
D315	8:18	8:38	8:38	8:54	9:14	9:38	9:38	10:05	10:12	10:39
D333	8:23	8:44	9:01	9:29	9:31	9:44	10:04	10:30	10:43	11:07
G31	8:30	8:48	8:48	9:03	9:03	9:24	9:24	9:48	9:48	10:02
G113	9:05	9:22	9:22	9:46	9:46	10:02	10:02	10:18	10:20	10:44
G115	9:16	9:37	9:39	10:03	10:03	10:19	10:19	10:36	10:38	11:02
G41	9:33	9:52	9:52	10:08	10:10	10:30	10:30	10:53	10:55	11:19
D317	9:38	9:59	10:01	10:17	10:17	10:42	10:56	11:26	—	—
G13	0:00	10:17	10:17	10:29	10:29	10:49	10:49	11:12	—	—
D335	10:30	10:51	10:53	11:10	—	—	—	—	—	—
G119	10:45	11:06	—	—	—	—	—	—	—	—

Note: the italics in the figure show the adjusted arrival time and departure time

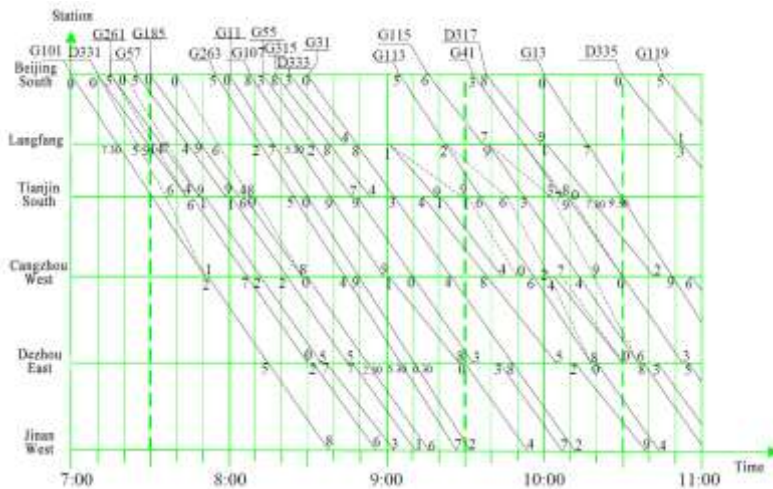


Figure 1: adjusted train operation diagram

6. Conclusions

This paper builds up a mathematical model to solve the high-speed train operation adjustment problem, and puts forward a new improved way to solve this problem effectively. The example demonstrates the feasibility and efficiency of the Differential Evolution in solving such problems triggered by itself like pre-mature, simple dimension.

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