

Robust Model Predictive Control of a Plate Heat Exchanger

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Advanced process control includes optimization-based tools that are recently widely implemented in industry to maximize economical effectiveness and to minimize environmental impact. Robust model predictive control (MPC) is one of these strategies and it combines benefits of model predictive control and robust control approaches. This study investigates improvement of control performance and increase of energy savings using the soft-constrained robust MPC with integral action for a laboratory plate heat exchanger. Soft constraints on control inputs keep the heat exchanger in required operation conditions and enable to use the feasible range of manipulated variable effectively with decreasing of energy cost. Integral action of the predictive controller ensures offset-free reference tracking. Simulation results obtained using the newly designed robust predictive controller with soft constraints and integral action confirm improved control response and increased energy savings in comparison with the results reached using the predictive controller with hard constraints and without active soft constraints.

1. Introduction

Industrial production is going to be highly affected by the wide implementation of advanced technologies in the Industry 4.0 concept. Nowadays, the advanced optimization-based tools are used in industry to maximize economical effectiveness (Lucia et al., 2014) and to minimize environmental impact (Fan et al., 2018). In the paper by Walmsley et al. (2017), the fossil fuel utilization and the associated emissions were reduced by implementing the total site heat integration to appropriately integrate mechanical and thermal vapour recompression with multi-effect evaporators at older Kraft mills. Vojtesek et al. (2017) described a chemical reactor by a nonlinear lumped-parameter model and they proposed and implemented the hybrid adaptive control strategy for control of this reactor. The main benefit of the developed strategy was that the controller changed its parameters with respect to the actual state of the system identified during the control process. This resulted in improvement of control performance and in decreasing energy needed for control. Wan et al. (2017) introduced the novel approach improving control performance and reducing respond time of the economic model predictive control. Control and optimization were designed separately, i.e., the respond time was reduced in control zone, and the economic benefits were improved in optimization zone. Ahmetović et al. (2017) combined the non-linear-programming-based model of the multiple-effect evaporation systems and the heat exchanger network model for optimization and heat integration of the overall system. Properties of the derived mixed-integer non-linear-programming-based model were analysed to find the optimal operation conditions. Klaučo and Kvasnica (2017) implemented the optimization-based reference governor control to improve safety and economic performance of a boiler-turbine system that was controlled by a set of interconnected PI controllers. The plate heat exchangers are extensively implemented in energy industries, such as district heating systems, absorption chilling systems, and electricity production systems (Wang et al., 2018). Efficient control of these plants is a key problem in improving the dynamic response and ensuring the stability of the control system. Wang et al. (2018) derived the state space model of the plate heat exchanger, and designed the two-degree-of-freedom loop-shaping H_∞ controller to improve the dynamic performance of the plate heat exchanger. The case study validated the model accuracy and confirmed the improved control performance in comparison with the well-tuned PI controller. The challenge of control design for such energy intensive industrial plants as heat exchangers is their complex non-linear and asymmetric behaviour affected by various structured or non-

structured uncertainties. Fratzak et al. (2016) formulated and validated a simplified dynamical model for practical applications of the plate heat exchangers. Structural and numerical stability of the derived model were investigated and the sensitivity analysis and comparison with finite difference approximation were proposed. Matušů and Pekař (2017) investigated the closed-loop system robust stability analysis of the uncertain heat exchangers using various graphical-based strategies. The linear matrix inequalities (LMIs) enable formulating the complex control design in the tractable form of the convex optimization problem. Antonov and Helsen (2016) proposed the novel method of the LMI-based robustness analysis in the model predictive control (MPC) framework for the thermal system.

This paper extends results of the research focused on advanced controller design for the plants widely used in chemical, petrochemical, pharmaceutical, and food industries (Oravec et al., 2016) and the research focused on LMI-based formulation of the soft-constraints (Oravec et al., 2017). This study investigates improvement of control performance and increase of energy savings using the soft-constrained robust MPC with integral action for a laboratory plate heat exchanger. Robust MPC is also used to optimize the control performance subject to the uncertain parameters. The advantage of the robust MPC in comparison with the conventional PID control is that the robust model predictive controller generates the control actions taking into account constraints on input and output variables as well as various uncertainties in the controlled system. This is not the case of the conventional PID control as the conventional PID controllers do not evaluate the control actions subject to any constraints on input and output variables and to uncertainties. But the advantage of PID control is simpler design compared to the robust MPC. The robust MPC design is based on a solution of the convex optimization problem that has the form of semidefinite programming formulated via a set of linear matrix inequalities (Bakošová et al., 2017).

The main contribution of this paper is the design and analysis of the soft-constrained robust MPC with integral action and application of this control strategy for the plate heat exchanger. Soft constraints on control inputs and system outputs keep the heat exchanger in required operation conditions and enable to use the feasible range of the manipulated variable effectively with decreasing of energy cost. Integral action of the predictive controller ensures offset free reference tracking. Simulation results obtained using the newly designed robust predictive controller with soft constraints and integral action confirmed improved control response and increased energy savings in comparison with the results reached using the predictive controller with hard constraints and without active soft constraints.

2. Plate heat exchanger

The laboratory plant includes a laboratory plate heat exchanger with holding tube (Figure 1, device I), two retention tanks for cold fluid (Figure 1, devices II) and a circulation heater for preparing heating fluid (Figure 1, device III). The principles of the plate heat exchangers and their constructions are described, e.g., in the book Klemeš et al. (2015), and the technical details of the considered plant are in the manual Armfield (2007). The cold fluid is preheated in the circulation heater to the fixed temperature $T_{\text{hot}} = 58 \text{ }^{\circ}\text{C}$. The cold fluid from one of retention tanks and the hot fluid from the circulation heater are fed to the plate heat exchanger, where the cold fluid is heated to the required temperature. Two peristaltic pumps (Figure 1, devices IV, V) feed cold and hot fluid to the plate heat exchanger. In our experiments, water was used as both, the heated and the heating fluids.

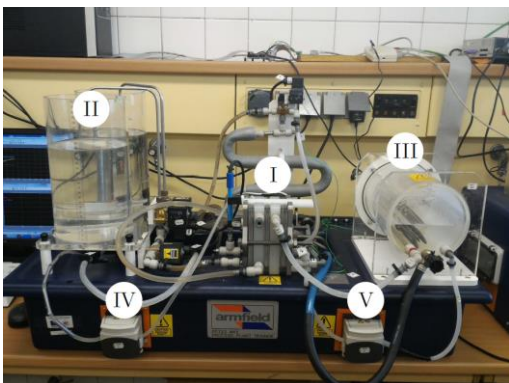


Figure 1: Armfield PCT23, plate heat exchanger (I), retention tanks for cold fluid (II), circulation heater (III), peristaltic pump for cold fluid dosing (IV), and peristaltic pump for hot fluid dosing (V)

The controlled process was the plate heat exchanger and the single-input single-output (SISO) control configuration was considered. Volumetric flow rate of the hot fluid q was the manipulated input variable. The peristaltic pump was the actuator. The input voltage U to the peristaltic pump manipulated speed of the rotor rotations and through the speed, it manipulated the flow rate of the heating fluid. As the manipulated variable q was calculated in the control algorithm, it was converted into the voltage U using a calibration curve. Temperature of the heated fluid T in the outlet stream was the controlled output.

The step-responses of the heat exchanger were measured to identify the model parameters. Due to the non-linear behaviour of the process, several step changes of the system input were carried out. As the system behaviour was asymmetric, the step-changes in both directions were realized, i.e., positive and negative increments of input volumetric flow rate q were done. In total, eight step responses were measured. As the laboratory heat exchanger had a complex behaviour affected by various uncertain parameters, the mathematical model had the form of a transfer function with interval uncertainties. To design robust MPC, the mathematical model of the controlled process was needed in the form of a discrete-time state space system. To identify the parameters of the mathematical model in the discrete time domain, the sampling time $t_s = 1$ s was used.

3. Soft-constrained robust MPC design with integral action

For the soft-constrained robust MPC design with integral action, the heat exchanger model was formulated as the linear discrete-time state-space system with the polytopic uncertainty, and it had the form:

$$x(k+1) = A_v x(k) + B_v u(k), \quad y(k) = C_v x(k), \quad x(0) = x_0, \quad (1)$$

where k is the sample of discrete time, $x(k)$ is the vector of system states, $u(k)$ is the vector of manipulated input variables, and $y(k)$ is the vector of system output variables. The matrices A_v , B_v , C_v have appropriate dimensions. The polytopic uncertainty is considered in the model of the controlled system and the family of uncertain systems in Eq(1) is described as follows

$$\mathbb{A} = \text{convhull}([A_v, B_v, C_v], \forall v = 1, \dots, 4) \quad (2)$$

where \mathbb{A} is the convex hull of the system vertices. The minimum and maximum values of the matrices in Eq(2) are summarized in Table 1. These parameters were experimentally identified and the details are in the paper Oravec et al. (2016).

Table 1: Minimum and maximum parameters of the uncertain system model

| Vertex matrix | A_v | B_v | C_v |
|---------------|--------|--------|--------|
| Minimum | 0.8767 | 0.9370 | 0.0500 |
| Maximum | 0.9556 | 0.9776 | 0.1447 |

To ensure offset free reference tracking and to design robust MPC with integral action, the system in Eqs(1)-(2) was extended as follows:

$$\hat{x}(k+1) = \hat{A}_v \hat{x}(k) + \hat{B}_v u(k), \quad y(k) = \hat{C}_v \hat{x}(k), \quad \hat{x}(0) = \hat{x}_0, \quad (3)$$

where the matrices of the extended system are given by:

$$\hat{A}_v = \begin{bmatrix} A_v & 0 \\ -t_s C_v & I \end{bmatrix}, \hat{B}_v = \begin{bmatrix} B_v \\ 0 \end{bmatrix}, \hat{C}_v = [C_v \quad 0], \quad (4)$$

and the vector of states is extended subject to the integral action as

$$\hat{x}(k) = \begin{bmatrix} x(k) \\ \sum_{i=0}^k x(i) \end{bmatrix}. \quad (5)$$

The goal of robust MPC with integral action is to compute a gain matrix of the linear state feedback control law

$$u(k) = F(k) \hat{x}(k) \quad (6)$$

so, that the closed-loop system is robustly stable. Simultaneously, the quadratic cost function

$$J = \sum_{k=0}^N \left(x(k)^T Q_p x(k) + \left(\sum_{i=0}^k x(i) \right)^T Q_I \left(\sum_{i=0}^k x(i) \right) + u(k)^T R u(k) \right), \quad (7)$$

is minimised. Here, Q_p , Q_i , R are the weighting matrices of the proportional part, integration part, and manipulated variables, respectively. The weighting matrices need to be tuned subject to the main requirements on the closed-loop control performance.

Moreover, closed-loop control trajectories must respect the symmetric constraints on manipulated variables and control variables

$$-u_{\text{hard}} \leq u(k) \leq u_{\text{hard}}, \quad -y_{\text{hard}} \leq y(k) \leq y_{\text{hard}}, \quad (8)$$

and they should fulfil also the soft constraints

$$-u_{\text{soft}} \leq u(k) \leq u_{\text{soft}}, \quad -y_{\text{soft}} \leq y(k) \leq y_{\text{soft}}. \quad (9)$$

Then, the convex optimization problem in the form of the semidefinite programming (SDP) is solved to meet the control requirements:

$$\min(\gamma + Q_{\text{soft},u}^T s_u + Q_{\text{soft},y}^T s_y) \quad (10)$$

$$\begin{bmatrix} 1 & * \\ x(k) & X \end{bmatrix} \succeq 0, \quad \begin{bmatrix} X & * & * & * \\ \hat{A}_v X + \hat{B}_v Y & X & * & * \\ \hat{Q}^{1/2} X & 0 & \gamma I & * \\ R^{1/2} Y & 0 & 0 & \gamma I \end{bmatrix} \succeq 0, \quad (11)$$

$$\begin{bmatrix} X & * \\ Y & U_{\text{hard}} \end{bmatrix} \succeq 0, \quad \begin{bmatrix} X & * \\ C[\hat{A}_v X + \hat{B}_v Y] & Y_{\text{hard}} \end{bmatrix} \succeq 0, \quad (12)$$

$$\begin{bmatrix} X & * \\ E_u Y & U_{\text{soft}}(s_u) \end{bmatrix} \succeq 0, \quad \begin{bmatrix} X & * \\ E_y C[A_v X + B_v Y] & Y_{\text{soft}}(s_y) \end{bmatrix} \succeq 0, \quad (13)$$

where X is the symmetric positive definite weighted inverse Lyapunov matrix, Y is the auxiliary controller tuning matrix, U_{hard} , Y_{hard} , U_{soft} , Y_{soft} are the matrices representing the hard and soft constraints, respectively. Matrices

$\hat{Q} = \begin{bmatrix} Q_p & 0 \\ 0 & Q_i \end{bmatrix}$, R are the weighting matrices of system states and manipulated variables from Eq(7)

Reference source not found.), $Q_{\text{soft},u}^T$, $Q_{\text{soft},y}^T$ are the weighting matrices of soft-constrained manipulated and control variables. The vectors s_u , s_y are the optimization slack variables. Matrices E_u , E_y have ones and zeros on principal diagonals and zeros elsewhere and they serve to indicate the soft-constrained manipulated variables and controlled variables. Symbol * denotes the symmetric structure of linear matrix inequalities (LMIs) in Eqs(11)-(13). Using the feasible solution of SDP in Eqs(10)-(13), one can construct the gain matrix of the control law in Eq(6) as follows:

$$F = YX^{-1}. \quad (14)$$

4. Results and discussion

Soft-constrained robust model-based predictive controller with integral action was implemented by a computer with CPU i7 3.4 GHz and 8 GB RAM. Soft-constrained robust MPC was designed using MATLAB/Simulink R2017b environment and MUP toolbox (Oravec et al., 2016). The SDP in Eqs(10)-(13) was formulated using toolbox YALMIP (Löfberg, 2004), and solved using a solver MOSEK (Mosek, 2017). Simulation time $t_{\text{sim}} = 100$ s was considered for simulations of closed-loop control, i.e., 100 control steps for the sampling time $t_s = 1$ s.

Two control scenarios were considered to investigate the benefits of soft-constrained robust MPC with integral action. Scenario I considered robust MPC with integral action designed without soft constraints, i.e., just hard-constraints were assumed. Scenario II implemented both, soft constraints and hard constraints. The hard constraints on control inputs and controlled outputs were set to keep the heat exchanger in suitable operation conditions. The soft constraints on controlled outputs were set to push the controlled temperature into the close neighbourhood of the reference value $T_{\text{ref}} = 45$ °C as well as to keep the actuator, i.e., the peristaltic pump dosing heating fluid into the plate heat exchanger, in optimal operation conditions. The detail setup of the considered input and output constraints is summarized in Table 2. The initial temperature of the heated fluid was $T_0 = 25$ °C. The weighting matrices of the quadratic cost function in Eq(7) formulated for robust MPC were: $Q_p = 10$, $Q_i = 10$, $R = 100$, $Q_{\text{soft},u} = 1 \times 10^5$, $Q_{\text{soft},y} = 1 \times 10^3$. The results are presented in Figure 2 and Table 3. Figure 2 depicts the closed-loop control trajectories in both control scenarios. Figure 2 a) shows the closed-loop performances of the controlled output of the plate heat exchanger, and Figure 2 b) presents the associated manipulated inputs, i.e., optimized control trajectories of the hot fluid volumetric flow rate. Only two trajectories

are presented for each control scenario, and these trajectories represent limit behaviour of the controlled process corresponding to the minimum and maximum values of the interval uncertainties of model parameters. In Figure 2, the dashed trajectories represent the Scenario I, i.e., the control performance of robust MPC with integral action and without active soft-constraints, i.e. only with hard constraints.

Table 2: Symmetric hard and soft constraints used for the plate heat exchanger control

| Scenario | Control input | Value [ml/s] | Controlled output | Value [°C] |
|---------------------------|----------------|--------------|-------------------|------------|
| Inactive soft constraints | $q_{hard,min}$ | 3.7 | $T_{hard,min}$ | 5.0 |
| | $q_{hard,max}$ | 11.3 | $T_{hard,max}$ | 65.0 |
| Soft constraints | $q_{soft,min}$ | 6.0 | $T_{soft,min}$ | 41.2 |
| | $q_{soft,max}$ | 9.0 | $T_{soft,max}$ | 48.8 |

Table 3: Comparison of two control scenarios using analytical quality criteria

| Scenario | Uncertain parameter | Overshoot [%] | Quadratic Cost Function Value [$\times 10^6$] | Total Consumption of Heating Fluid [m^3] |
|---------------------------|---------------------|---------------|---|--|
| Inactive soft constraints | Minimum | 59 | 5.212 | 0.817 |
| | Maximum | 89 | 9.266 | 0.910 |
| Active soft constraints | Minimum | 37 | 4.672 | 0.814 |
| | Maximum | 68 | 7.976 | 0.907 |

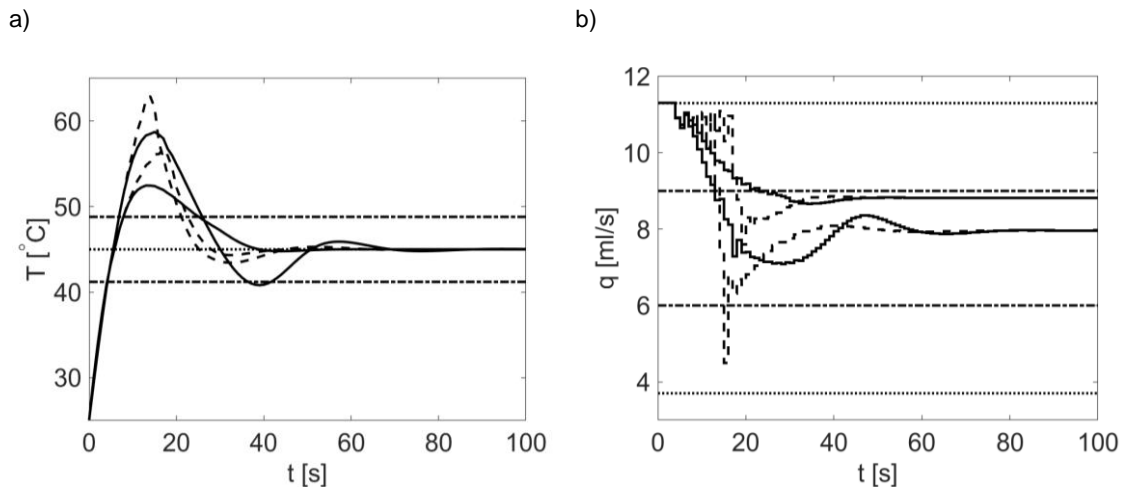


Figure 2: Control responses of the plate heat exchanger: (a) controlled outputs – hard-constrained (dashed), soft-constrained (solid), soft constraints (dash-dotted), reference (dotted), (b) manipulated input – hard-constrained (dashed), soft-constrained (solid), soft constraints (dash-dotted), hard constraints (dotted)

The hard constraints must be ensured during control and are represented by dotted lines in Figure 2 b). As can be seen from Figure 2 b), the manipulated input stays always within the hard constraints. The control response is faster, but the price for it is the larger overshoot and the higher heating fluid consumption (Table 3). The solid trajectories in Figure 2 were ensured by the robust MPC with integral action and active soft-constraints (Scenario II). The soft constraints can be violated whenever necessary and are denoted by dash-dotted lines. The control response is slower, but the overshoots are smaller and the heating fluid consumption is lower (Table 3). Moreover, the control inputs were pushed into the soft-constrained region. In this scenario, the hard constraints were also active.

The control performance was compared also by numerical values of analytical criteria, such as the overshoot, the value of quadratic cost function in Eq(7), and the total consumption of heating fluid, see Table 3. The soft-constrained robust MPC ensured lower values of all observed criteria for both cases, minimum and maximum values of uncertain parameters.

5. Conclusions

Improvement of the closed-loop control performance of the plate heat exchanger using the novel soft-constrained-based robust MPC strategy with soft constraints and integral action is demonstrated in the paper. Soft-constrained-based control strategy keeps the control inputs and outputs in the required operation conditions. Integral action of the controller ensures the offset free control responses. Simulation results confirmed the improved control performance in both, controlled output trajectories and manipulated input trajectories. The controlled temperature shows reduced overshoots, and optimized manipulated inputs lead to reduced energy consumption measured by heating fluid consumption. Further research will be focused on the laboratory implementation of the designed control strategy.

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