CHEMICAL ENGINEERING TRANSACTIONS

VOL. 46, 2015

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Peiyu Ren, Yancang Li, Huiping Song

ISBN 978-88-95608-37-2; **ISSN** 2283-9216

DOI: 10.3303/CET1546207

A Study of the Subjective Expectation Model of Real Estate Price Fluctuation Employing Two-person Game Theory

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In the real estate market, the demanding of customers vary as their price expectations change. Conversely, changes in the needs of customers affect the price of real estate. This research proposes a model for expected alterations in real estate prices by analyzing variations using two-person game theory. The research also provides evidence for a psychological quantitative study of customer expectation.

1. Introduction

The real estate market fluctuates periodically. And on this basis, researchers make expectations about forthcoming changes. Li and Zhai (2009) separate these expectations into four categories: static, extrapolated, adaptive and rational expectations. With each category related to the percentage of information employed by each. They separate people's ability to make expectations through adaptation and bounded rationality: those who make the least adaption's make static expectations; those who make some adaptations make extrapolated expectations while those who show the greatest ability to adapt make adaptive expectations

The model for static expectations is similar to the well known "cobweb model". Such expectations are based on existing variables and circulate as a cobweb model (Xue, 1995). Metzler (1941) writes that extrapolated expectations occur when expectations about economic situations are not based solely on the levels of past economic variables, but also rely on the trends shown by these variables. The information behavioral agent used to make expectations is from the weighted ratio and vector sum of t-1 economic variables and t-2 economic variables. In a study of adaptive expectations, Cagan (1954) describes them as expectations which economic agents adjust in relation to mistakes in past decision making.

Yet, economic agents make expectation-based decisions relying on past experience alone in static, extrapolated and adaptive expectations. These are all, therefore, non-rational expectations, which results in useful information related to expectations not being fully used. Muth (1961) points out that economic agents employ past information only in their non-rational expectations. In so doing they exclude new information and knowledge of the effectiveness of government policy. Therefore, their decisions cannot be described as rational. He further argues that people's behavior is rational in economic society when they maximize their profits by making use of all available information resources: in other words, they make rational expectations. Rational expectation occurs when economic agents utilize every piece of information available in making the most accurate expectations possible about future economic conditions. This information includes economic variables such as price, profit and income. Nerlove and Fornari (1998) make some adjustment to the principle of rational expectation in their model of Quasi-rational expectations.

In contemporary research, Giovanni and Zheng (2007) put heterogeneity expectations into a dynamic model of real estate. They propose that heterogeneity expectations exist when people cannot make optimistic expectations because of an asymmetry in information between current economic conditions and their uncertainty about future incomes.

Hong and Stein (1999) found that the influence of positive news at time t will attract investment, increase customer demanding and boost prices for a long time because of the multiplier effect. However, the driving effect of their expectation shows that value above the long-term average will be paid by the customers in $t+i$. The driving effect of this expectation also shows us that customer expectations always lag behind the information at their disposal. This lag creates changes in supply and demand. Hong and Stein also separate the expectants into new observers and tracking traders, and construct an expectation for bonds on the basis

Please cite this article as: Shen D., Yang Z., 2015, A study of the subjective expectation model of real estate price fluctuation employing two-person game theory, Chemical Engineering Transactions, 46, 1237-1242 DOI:10.3303/CET1546207

of this separation. Even though everyone makes rational expectations, the result of these expectations can be irrational.

2. Two-person game theory

Fluctuations in real estate prices affect customer expectations, leading to further changes in real estate prices. Moreover, as game time accumulates, customers will summarize past experience in games and make adjusted responses in future games. This research considers using evolutionary game theory to analyze this problem.

2.1 Real estate prices increase in t+1

Consider employing both an asymmetric game replication dynamic and an evolutionary stable strategy at time *t* to make analyses (Xie, 2006). Suppose *x* proportion of players in group 1 expect real estate prices to increase and (*1-x*) expect prices to decrease. Also suppose *y* proportion of players in group 2 expect real estate prices to increase and (1-*y*) expect prices to decrease. Furthermore, assume *c* as the expected cost of the increased real estate prices. Thus the expected gain and loss in position 1 u_{1i} , u_{1d} and the group average gain $\overline{u_1}$ are:

$$u_{1i} = y \bullet a + (1-y) \bullet (a+c) \tag{1}$$

$$u_{1d} = y \bullet (-a-c) + (1-y) \bullet (-a-c) \tag{2}$$

$$\bar{u}_1 = x \bullet u_{1i} + (1-x) \bullet u_{1d} = x \bullet (a+c-cy) + (1-x) \bullet (-a-c) \tag{3}$$

The gain-lose matrix diagram in this situation is shown in Figure 2.1:

		Person two	
		Predicted rise	Predicted drop
Person one	Predicted rise	a,b	a+c,-b-c
	Predicted drop	-a-c,b+c	-a-c,-b-c

Figure 2.1: Dynamic equation one

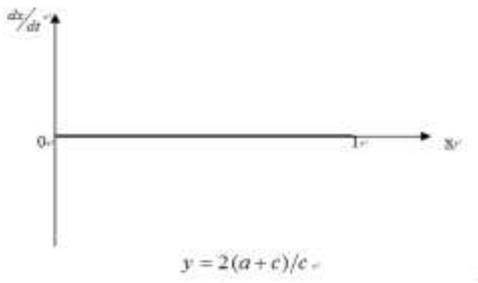
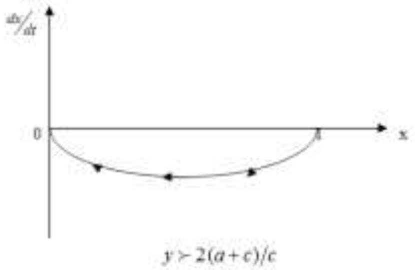


Figure 2.2: Phase one of evolutionary game one

Acquire replicator dynamics equation:

$$\frac{dx}{dt} = x(u_{1i} - \bar{u}_1) = x(1-x)[2(a+c) - cy] \tag{4}$$

Based on this dynamic equation, if $y = 2(a+c)/c$, dx/dt is always 0, all values of *x* are in a steady state; if $y \neq 2(a+c)/c$, then $x^* = 0$ and $x^* = 1$ will be the only two values to achieve a steady state; if $y \succ 2(a+c)/c$, $x^* = 0$ is ESS and if $y \prec 2(a+c)/c$, $x^* = 1$ is ESS if. The diagram below reflects the dynamic trend and stability of *x* in the 3 situations above:



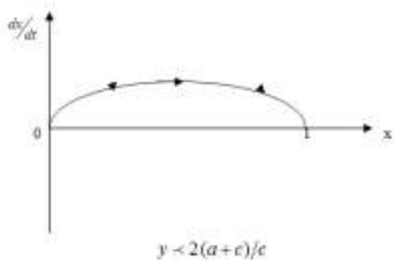


Figure 2.3: Phase two of evolutionary game one

Figure 2.4: Phase three of evolutionary game one

The expected gain and expected loss at position 2 u_{2i}, u_{2d} are:

$$u_{2i} = x \bullet b + (1-x) \bullet (b+c) \tag{5}$$

$$u_{2d} = x \bullet (-b-c) + (1-x) \bullet (-b-c) \tag{6}$$

$$\bar{u}_2 = y \bullet u_{2i} + (1-y) \bullet u_{2d} = y \bullet (b+c-cx) + (1-y) \bullet (-b-c) \tag{7}$$

Acquire replicator dynamics equation:

$$\frac{dy}{dt} = y(u_{2i} - \bar{u}_2) = y(1-y)[2(b+c) - cx] \tag{8}$$

Based on this dynamic equation, if $x = 2(b+c)/c$, dy/dt is always 0, all values of *y* achieved a steady state; when $x \neq 2(b+c)/c$, $y^* = 0$ and $y^* = 1$ will be the only two values to achieve a steady state. When $y \succ 2(a+c)/c$, $x^* = 0$ is ESS. When $y \prec 2(a+c)/c$, $x^* = 1$ is ESS. The diagram below reflects the dynamic trend and stability of *y* in the 3 situations above:

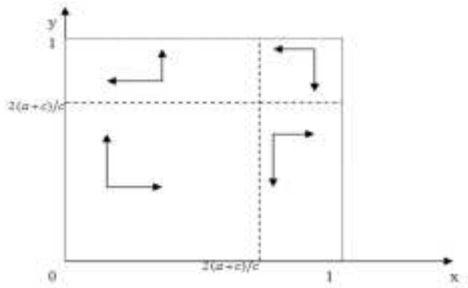


Figure 2.5: Steady strategy of game one

One can see that $x^* = 1, y^* = 0$ and $x^* = 0, y^* = 1$ is the advanced static strategy for this game. Because the static condition for *y* in the whole process is $x = 2(b+c)/c$, the static condition for *x* in the whole process is $y = 2(b+c)/c$. Thus the possibility of $x^* = 1, y^* = 0$ and $x^* = 0, y^* = 1$ is the same.

Because both conditions have the same probability the evolutionary stable strategy will mean that one group expects the price to be higher while the other group expects the price to be lower. Meanwhile, the conditions in which group A expects the price to be higher and group B expects the price to be lower have the same probability as the conditions in which group A expect the price to be lower and group B expect the price to be higher.

2.2 In the condition when the real estate price is lower at t+1
The gain-loss matrix diagram is shown in fig. 2.6:

		Person two	
		Predicted rise	Predicted fall
Person one	Predicted rise	-a-e,-b-e	-a-e, b+e
	Predicted fall	a+e,-b-e	a, b

Figure 2.6: Dynamic equation two

Assume that the expected cost to both players is *e* when the future real estate price will be lower. The profits from expecting the price to be higher and expecting the price to be lower are u_{1i} and u_{1i} and the group average profits $\overline{u_1}$ are:

$$u_{1i} = y \bullet (-a-e) + (1-y) \bullet (-a-e) \tag{9}$$

$$u_{1d} = y \bullet (a+e) + (1-y) \bullet a \tag{10}$$

$$\overline{u}_1 = x \bullet u_{1i} + (1-x) \bullet u_{1d} = (1-x) \bullet (a+ey) + (-a-e) \bullet x \tag{11}$$

Acquire replicator dynamics equation :

$$\frac{dx}{dt} = x(u_{1i} - \overline{u}_1) = x \bullet (x-1) \bullet (2a+e+ey) \tag{12}$$

According to this dynamic equation, *x* is always at a static condition if $y = 2a+e/-e$, dx/dt are always 0; $x^* = 0$ and $x^* = 1$ are the only two static condition if $y \neq 2a+e/-e$. $x^* = 1$ is ESS when $y \succ 2a+e/-e$, $x^* = 0$ is ESS when $y \prec 2a+e/-e$. The following diagram describes the dynamic trend and stability of *x* in the 3 situations mentioned above.

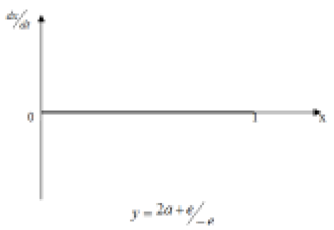


Figure 2.7: Phase one of evolutionary game two

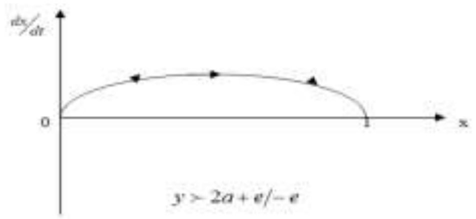


Figure 2.8: Phase two of evolutionary game two

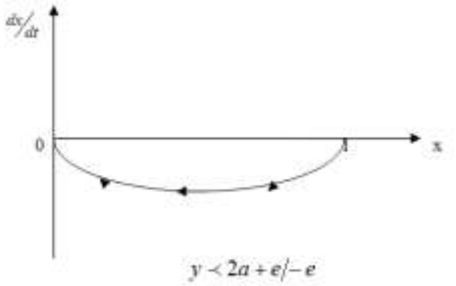


Figure 2.9: Phase three of evolutionary game two

In position 2, the profits from expecting the price to be higher and expecting the price to be lower are u_{2i}, u_{2d} and the group average profit is:

$$u_{2i} = x \bullet (-b-e) + (1-x) \bullet (-b-e) \tag{13}$$

$$u_{2d} = x \bullet (b+e) + (1-x) \bullet b \tag{14}$$

$$\bar{u}_2 = y \bullet u_{2i} + (1-y) \bullet u_{2d} = (1-y) \bullet (b+ex) + (-b-e) \bullet y \tag{15}$$

Acquire replicator dynamics equation：

$$\frac{dy}{dt} = y(u_{2i} - \bar{u}_2) = y \bullet (y-1)(2b+e+ex) \tag{16}$$

According to this dynamic equation, *y* is always at a static condition if $x = 2b + e/-e$, dy/dt are always 0; $y^* = 0$ and $y^* = 1$ are the only two static condition if $x \neq 2b + e/-e$. $y^* = 1$ is ESS when $x \succ 2b + e/-e$, $y^* = 0$ is ESS when $x \prec 2b + e/-e$. The following diagram describes the dynamic trend and stability of *y* in the 3 situations mentioned above.

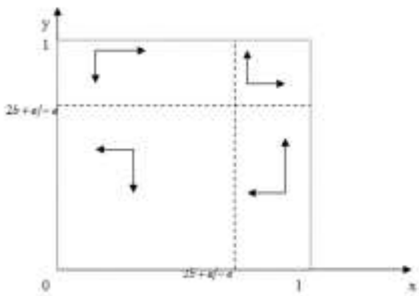


Figure 2.10: Steady strategy of game two

It can be seen from the diagram that $x^* = 0, y^* = 0$ and $x^* = 1, y^* = 1$ are this game's evolutionary stable strategy. The final static distribution depends on the value of $2b+e/-e$. The possibility of achieving $x^* = 1, y^* = 1$ stable is higher when $2b+e/-e \prec 0.5$. This means that the stable condition where both of the groups expect the real estate price to be higher is more likely before the decrease in the real estate price. The possibility of achieving $x^* = 0, y^* = 0$ stable is higher when $2b+e/-e \succ 0.5$. This means that the stable condition in which both of the groups expect the real estate price to be lower is more likely before the decrease in the real estate price. The possibility of achieving $x^* = 0, y^* = 0$ stable and $x^* = 1, y^* = 1$ is the same when $2b+e/-e = 0.5$. This means that the stable condition in which the groups expect the real estate price to be lower is as probable as when groups expect the real estate price to be higher before the decrease of the real estate price.

3. Summary

By summarizing the situations above, it can be seen that the possibility of the condition in which group 1 expects the price to be higher and group 2 expects the price to be lower has the same probability of occurring as the condition in which group 1 expects the price to be lower and group 2 expects the price to be higher before the real estate price increases. The probability of both groups expecting the price to be higher and the probability of both groups expecting the price to be lower depend on the value of $2b+e/{-e}$ before the real estate price decreases. This means that both of these groups have developed certain evolutionary distributions based on their asset conditions and expected costs in the long-term two-person game.

In fact, expectations of real estate price are often influenced by the expectations of others and therefore herd effect has some impact. When most people expect prices to be higher, individuals with limited rationality will expect the price to increase; conversely, individuals with limited rationality would expect the price to decrease when most people expect the price to be lower. The situation can be described by the following models: x correlates with y. If the value of x changes and tends to 1, y will move towards 1 but the extent would be smaller than x because it is affected by x; conversely, if y tends to be 1, x will move towards 1 in a smaller extent than y. Also, y will to some extent tend to 0 if x moves towards 0. This is also true for x when y moves towards 0.

This research analyzes the subjective expectation process of real estate prices using the two person game model. This research achieves two different evolutionary game stability strategies by analyzing post-experience expectations. In real estate price expectations of the two games, the decision maker is highly influenced by the others. In the future, more research should be conducted on real estate prices based on multi-player games.

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