

A Mixed Integer Nonlinear Model for Closed-Loop Supply Chains with Incentive and Proximity based Product Returns

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Electronics firms in different locations are being required to collect used products due to environmental and health hazards. In order to meet environmental requirements, firms carry out collection activities and provide incentive offers to attract product returns. A mixed integer non-linear programming model for a Closed-Loop Supply Chain including decisions for collection activities, incentive offers and recovery options is formulated and validated. Quantity is modeled as a function of incentive offers and distance between the collection centers (distribution/retail centers) and consumers. The quality of product returns follows an arbitrary probability distribution, which is shifted by incentive level. Quality of product returns dictates the possible recovery options, which these can undergo. The model is subjected to scenario analysis. This is comprised of conditions wherein rebate, or discount incentives is preferred and when low or high incentive levels are favored. High stockout cost to secondary consumers encouraged the model to perform more cash rebate activities to stimulate more product returns to satisfy secondary consumer demand. In another situation, high cost of activities while having high stockout cost to secondary consumers induced the model to carryout discount activities as this would generate sales rather than the cash rebate which simply incentivizes the participation in the takeback program.

1. Introduction

Electronics firms in different locations are being required to collect used products due to environmental and health hazards, in order to meet requirements, firms carry out collection activities and provide incentive offers to attract product returns. These product returns are then sorted to decide on the appropriate recovery options to use (Donmez and Turkay, 2013). These could in the form of refurbishing, remanufacturing, cannibalizing and controlled disposal. Incentives have been shown as the most significant factor influencing consumer participation in product take-back programs (Balisado, 2013). The importance of this relationship was highlighted by Aras and Aksen (2007), stating that items collected without incentives are impractical for recovery. Firms therefore must decide on proper take-back programs and incentive levels to meet legislated collection requirements and obtain items of adequate quality for recovery while maximizing the profit potential of recovery. The type of incentive to offer: discount or cash rebate, is another relevant decision to consider. This is because of the difference in consumer reaction to the two types. The discount incentives increase demand for new products, but typically yields lower consumer participation than cash rebate (Lundin, 2012). On the other hand, previous literature on incentive-quality relationship includes that of Matter et al. (2015), who conducted a case study, which focused on the effect of institutionalizing tax and rebate incentives on solid waste recycling schemes. The findings of this research suggest that the quality of recycled products was unreliable without the implementation of these tax incentives. Govindan et al. (2015) further state that complexity increases in managing closed loop supply chains (CLSC) since demand and product returns are likewise inherently uncertain. Yamzon et al. (2016) demonstrated the effects of introducing different types of incentives to the quantity and quality of product returns in a CLSC. The distance between consumers and the product take-back program has been shown to be the second most significant factor in influencing consumer participation. Through a Design of Experiments (DOE) conducted by Balisado (2013), it was identified that the location of the collection centers was the second most significant factor in consumer participation. Therefore, the location of product take-back programs is also a relevant decision, which should be incorporated in a CLSC model. This study considers these decisions simultaneously with the allocation of collected items to each recovery option. This is because

recovery options are highly dependent on the quantity and quality of product returns available. To date, no other study has been made which incorporates incentive types, continuous incentive-quality relationship, and proximity-quantity relationship for product takeback in a closed-loop supply chain model.

2. Model Development

The following section presents the model development for the CLSC understudy. A model for the network is formulated using Mixed Integer Non-Linear Programming (MINLP). The model aims to make investment, operational and marketing decisions that maximize the profit of the firm while meeting targets in product collection, investment budget and virgin raw material usage. The indices used along with the relevant parameters and decision variables are initially presented in Table 1 and Table 2. These are followed by a discussion on the network requirements for the CLSC.

Table 1: Indices

Notation	Definition	Notation	Definition
i	Potential distributor or retailer facilities	l	Available activities which offer discount incentives
j	Fixed primary consumers and secondary consumers	m	Available activities which offer cash rebate incentives
k	Type of recovery options to be performed	o	Available activities which do not offer incentives
t	Time periods in the planning horizon	r	Raw Material or component types

Table 2: Parameters and decision variables

Notation	Definition
d_{jt}	Demand from customer j on time period t
$f(n_{jlt}^a)$	Quantity of product returns from consumer j from activity l as a function of discounts and distance offered on time period t
$f(n_{jmt}^b)$	Quantity of product returns from consumer j from activity m as a function of cash rebates and distance offered on time period t
f_o^c	Quantity of product returns from activity o
q_k^u	Upper limit of the acceptable quality level for recovery option k
q_k^l	Lower limit of the acceptable quality level for recovery option k
$Qual_l(q/d)$	Probability distribution function of acquiring product returns of quality q using activity l offering a discount of d
$Qual_m(q/i)$	Probability distribution function of acquiring product returns of quality q using activity m offering a cash incentive of i
x_{ikt}	Quantity of products k transferred from supplier to distributor i in period t
y_{ijkt}	Quantity of products k transferred from distributor i to customer j in period t
p_{kt}	Quantity of products that have undergone recovery k in period t
c_{kt}	Capacity for recovery option k in period t
u_{ijkt}	Quantity of products that could be recovered under option k from consumer j returned to distributor i in period t
w_{ikt}	Quantity of products that could be recovered under option k returned from distributor i to the supplier/manufacturer in period t
n_{jlt}^a	Discount offered for customer j in activity l on period t
n_{jmt}^b	Cash Rebate offered for customer j in activity m on period t
b_{it}	1, if distributor i is operational on time-period t and 0, otherwise

2.1. Downstream flow of products

The inventory of new and refurbished products stored in each distribution or retailer (d/r) facility for each period is defined by Eq(1). This amount is equal to the amount of products held from the previous period, plus the products that arrived in the current period less the products sent out in the current period. Eq(2) ensures that items moving out from the supplier/manufacturer (s/m) facility are limited by the amount of items manufactured or refurbished on the given period. Meanwhile, constraint Eq(3) provides the production and recovery capacity limits. The production, refurbish, remanufacture and cannibalize quantities in each period should be less than or equal to the corresponding capacity. Consequently, Eq(4) shows that the total assigned capacity for each activity is dependent on the capacity expansion that could be performed in each period.

$$I_{ikt} = I_{ikt-1} + x_{ikt} - \sum_j y_{ijkt} \quad \forall i, k, t \quad (1)$$

$$\sum_i x_{ikt} \leq \sum_k p_{kt} \quad \forall k, t \quad (2)$$

$$p_{kt} \leq C_{kt} \quad \forall k, t \quad (3)$$

$$e_{kt} + C_{kt} = C_{kt+1} \quad \forall k, t \quad (4)$$

The succeeding constraints Eq(5) and Eq(6) apply the demand of primary and/or secondary consumers. The total deliveries plus the shortage must be greater than or equal to the demand, while the total deliveries must be less than or equal to the demand. Note that the variable q_{jlt}^a is added to the demand for new products, in Eq(5) and Eq(6), if discount activities are considered because it is assumed that consumers who participate in the aforementioned purchase new products upon returning their used items.

$$\sum_i \sum_k y_{ijkt} + s_{jt} \geq d_{jt} + q_{jlt}^a \quad \forall j, t \quad (5)$$

$$\sum_i \sum_k y_{ijkt} \leq d_{jt} + q_{jlt}^a \quad \forall j, t \quad (6)$$

2.2. Upstream flow of products

Constraints Eq(7) - Eq(9) define the quantity of product returns collected by selected activities. This is equal to the binary variable for selecting activities multiplied by the corresponding quantity function for product returns.

$$Q_{jlt}^a = g_{jlt}^a f(n_{jlt}^a) \quad \forall j, l, t \quad (7)$$

$$Q_{jmt}^b = g_{jmt}^b f(n_{jmt}^b) \quad \forall j, m, t \quad (8)$$

$$Q_{jot}^c = g_{jot}^c f^c \quad \forall j, o, t \quad (9)$$

In Eq(7) and Eq(8), the binary variables g_{jlt}^a and g_{jmt}^b can be excluded if the corresponding quantity function, $f(n_{jlt}^a)$ or $f(n_{jmt}^b)$, have values of 0 at incentive levels of 0. This is because incentive levels automatically have values of 0 if the corresponding activity is not chosen. However, the binary variables must be included if the functions do not have values of 0 at incentive levels of 0 in order to ensure that no product returns come from activities which are not selected. The constraint Eq(10) enforces product collection targets τ_t . The total quantity of product returns collected by the different activities must be greater than or equal to the product collection target in each period.

$$\sum_j (\sum_l Q_{jlt}^a + \sum_m Q_{jmt}^b + \sum_o Q_{jot}^c) \geq \tau_t \quad \forall t \quad (10)$$

Constraint Eq(11) computes for the quantity of product returns collected which can undergo each recovery option. The quantity of refurbishable product returns collected is the sum of the collection of each activity type multiplied by the probability of collecting refurbishable product returns from each activity.

$$R_{jkt} = \sum_l (Q_{jlt}^a \cdot \text{prob}_{jkt}^a) + \sum_m (Q_{jmt}^b \cdot \text{prob}_{jkt}^b) + \sum_o (Q_{jot}^c \cdot \text{prob}_{jkt}^c) \quad \forall j, k, t \quad (11)$$

Non-linearity arises from the above constraint because the variables for quantity, such as q_{jlt}^a , and the probability variables for quality are multiplied to each other. Both of these variables are functions of incentive levels. Eq(12) and Eq(13) compute the probability of acquiring recoverable product returns from consumer j with activities l or m on time t . This is the integral of the quality function for discount activities from the imposed quality limits. For instance, the limits are from the minimum quality level (q_k^l) to maximum quality level (q_k^u) given a corresponding discount level.

$$\text{prob}_{jkt}^a = \int_{q_k^l}^{q_k^u} \text{Qual}_l(q/d = n_{jlt}^a) dq \quad \forall j, l, t \quad (12)$$

$$\text{prob}_{jkt}^b = \int_{q_k^l}^{q_k^u} \text{Qual}_m(q/i = n_{jmt}^b) dq \quad \forall j, m, t \quad (13)$$

Eq(14) and Eq(15) define the quantity of product returns moving from the facilities. In Eq(15), the total amount delivered to each d/r facility from a primary consumer is set to be equal to the quantity collected for each recovery option.

$$\sum_i u_{ijkt} \leq V_{jkt} \quad \forall j, k, t \quad (14)$$

$$R_{ikt} = \sum_j u_{ijkt} + R_{ik(t-1)} - w_{ikt} \quad \forall i, k, t \quad (15)$$

Eq(16) computes the product returns available for each recovery option. This constraint links the downstream and upstream flow of products through the variable p_{kt} . Note that for each recovery option, the amount received p_{kt} refers to product returns which are strictly in the quality level of the respective recovery options. For example,

remanufacturing could utilize returned products that are remanufacturable and could likewise accommodate those to be refurbished. This is in line with the prescribed quality level discussed in the previous section. As a result, refurbishable product returns would be added to the quantity of remanufacturable product returns, while the quantity which has been chosen for refurbishing would be subtracted. Similarly, for other recovery options, the remaining product returns for higher order recovery options are added.

$$p_{kt} = \sum_{k=k}^{k_{max}} \sum_i w_{ikt} - \sum_{k=k+1}^{k_{max}} p_{k,t} \quad \forall k, t \quad (16)$$

Eq(17) to Eq(19) define logical requirements for the d/r facilities and incentives. Eq(17) ensures product returns are not delivered to closed d/r facilities. While Eq(18) and Eq(19) ensure that incentive levels are 0 for activities which are not selected.

$$\sum_k x_{ikt} + \sum_j \sum_k u_{ijkt} \leq Mb_{it} \quad \forall i, t \quad (17)$$

$$n_{jlt}^a \leq Mg_{jlt}^a \quad \forall j, l, t \quad (18)$$

$$n_{jmt}^b \leq Mg_{jmt}^b \quad \forall j, m, t \quad (19)$$

Eq(20) computes the consumption of raw materials or components by remanufacturing, refurbishing and production of new products. This must be less than the target consumption of virgin raw materials plus the amount which is recovered from cannibalizing. Eq(21) ensures the investments in capacity and in opening distribution facilities are within the investment budget for each time period.

$$\sum_{k=remanufacture}^{k_{max}} p_{kt} RC_k \leq RT_{rt} + p_{k=cannibalize, t} RC_{k=cannibalize} \quad \forall t \quad (20)$$

$$\sum_k EC_k e_{kt} + \sum_i FC_i b_{it} \leq IB_t \quad \forall t \quad (21)$$

The objective of the CLSC optimization model in this study is to maximize profit which is defined as the difference between the sales revenue (from the primary and secondary consumers) and fixed and variable costs from both upstream and downstream movement of products. Eq(22) to Eq(25) define the different components of the aforementioned profit function. Sales revenue is dependent on the amount of products that ship out from the d/r to the customers. The total fixed cost is composed of the fixed costs obtained from undertaking the different product take back activities, and the fixed costs of operating the d/r and s/m facilities. Variable costs include the transportation costs from the s/m and d/r facilities', the recovery costs for each option, expansion cost for the recovery options, shortage penalties for failure to fulfil all customer orders, holding costs for the inventory of recovered and returned products, and the incentive costs brought about by each product take back activities.

$$Max \sum_t profit_t = Sales Revenue_t - Fixed Cost_t - Variable Cost_t \quad (22)$$

$$Sales Revenue_t = \sum_k SP_k \sum_i \sum_j y_{ijkt} \quad \forall t \quad (23)$$

$$Fixed Cost_t = AC^a \sum_j \sum_l g_{jlt}^a + AC^b \sum_j \sum_m g_{jmt}^b + AC^c \sum_j \sum_o g_{jot}^c + FC^1 z_t + \sum_i FC^1_i b_{it} \quad \forall t \quad (24)$$

$$Variable Cost_t = \sum_i \sum_k TC_{ik}^1 x_{ikt} + \sum_i \sum_j \sum_k TC_{ijk}^2 y_{ijkt} + \sum_k RC_k p_{kt} + \sum_k EC_k e_{kt} + \sum_j SC_j s_{jt} + \sum_k HC_k \sum_i (I_{ikt} + V_{ikt}) + \sum_k IC_k^a \sum_j \sum_l Q_{jlt}^a + \sum_k IC_k^b \sum_j \sum_m Q_{jmt}^b \quad \forall t \quad (25)$$

3. Model Validation

The model was validated through COUENNE, a nonlinear solver in General Algebraic Modeling System (GAMS). The model was relaxed such that quantities of products and product returns are not required to have integer values. The model was validated using a logarithmic function for quantity of product returns and an exponential distribution for quality of product returns. Hypothetical values were used for the validation.

3.1. Quantity and quality function parameters

The relationship between the incentive levels offered and the quantity of product returns is modeled by an arbitrary function. For the model validation, a logarithmic function is assigned for quantity of product returns as in Eq(26). The left side consisted of the quantity of product returns. A constant was placed in the equation to ensure that an activity should be selected such that a base amount of product returns is obtained.

$$quantity = constant + \frac{coefficient}{distance} * \log_{base}(incentive level + shift) \quad (26)$$

A logarithmic function allows the incremental quantity received to diminish as incentive levels increase, such that additional incentives at high incentive levels are not as effective as in low incentive levels. This represents a scenario wherein there is a limit to the quantity that could be collected, regardless of any further increase of incentive offer. This could be associated with the actual number of products being used by consumers or the number of consumers willing to return their products. The "constant" term denotes the amount of product returns generated should no effort on the part of the firm to generate product returns be present. The function also

models distance to affect this incremental quantity, as a greater distance between the distributor/retailer center and the consumer would lessen the incremental quantity received for a higher incentive level. Therefore, the “distance” term in the equation models the inversely proportional nature of the proximity of the collection center to the consumer. The “incentive” variable is put inside the logarithm term to make sure that as the incentive levels per activity increase, there is also an increment added to the total quantity of product returns. Finally, a “shift” term is added to the incentive level to ensure that the value inside the logarithm term would not be zero, which would render the equation invalid. Meanwhile, the quality of product returns is treated as a variable, which follows a distribution function. An exponential distribution is used. This is a conservative distribution, which assumes that more low-quality product returns would always be collected than high quality product returns. The cumulative exponential distribution is used by the model to compute probabilities of collecting product returns in each quality range. This follows the form shown in Eq(27).

$$F(q) = 1 - e^{-\frac{\text{incentive denominator}}{\text{incentive level} * \beta} q} \quad (27)$$

Wherein β is the mean quality level and q is the quality level for which the cumulative probability is being taken. In order to increase quality of product returns as incentive increases, the mean quality is multiplied by the incentive level divided and by an incentive denominator which controls the effect of the incentive on quality. In this way, the mean quality of product returns increases as incentive level increases while maintaining the properties of an exponential distribution. The plots for the quantity functions are shown in Figure 1. The quantity eventually plateaus, making offering higher incentives less effective. When the distance between consumer and distributor/retailer center is greater, lesser quantity of product returns is obtained, across product takeback activities. The figure on the right shows the base quality function, which occurs at an incentive level of 0, and the curve at an incentive level of 80. The movement of the curve shows a higher probability of collecting high quality product returns at an increased incentive level. On the other hand, there is a higher probability of collecting low quality product returns at lower incentive levels.

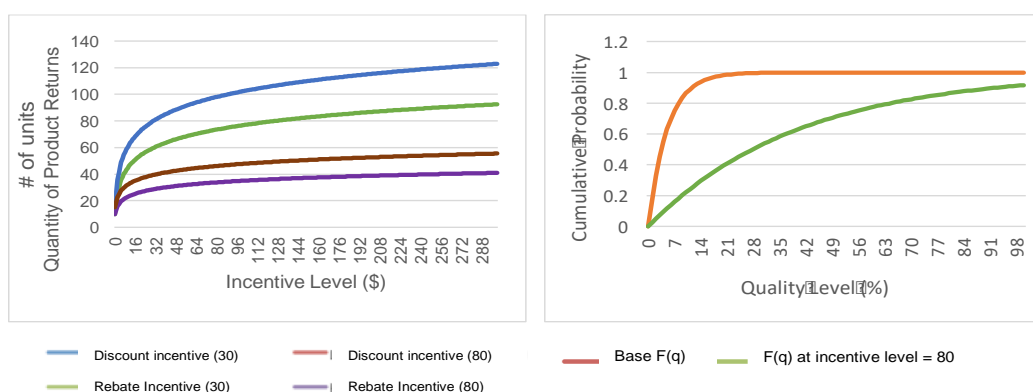


Figure 1: Quantity function plots

3.2 Analysis of results

The model is run for four time periods, the first period being an initialization period. The results are summarized in Figure 2. Combined sales revenue for new products sold to primary consumers consisted of products coming from new product production and remanufacturing recovery option, and refurbished products, which were sold to secondary consumers. Aside from the Sales Revenue obtained, it was also observed that the production and recovery costs constitute the largest portion of costs followed by cash rebate cost. Since the source of revenue comes from the amount of products sold to both primary and secondary consumers, the model would therefore prioritize allocating cost in the production of new products and recovery via refurbishing of products to be sold to secondary consumers. Also, it follows that cash rebate should also constitute a considerable amount of cost percentage because this recovery option produces a higher quantity of product returns. Another insight that can be gained is that the total cost exhibited a gradual increase, which reflected the effect of having an investment budget per period. Since Time-period 1 is an initialization period, it does not contain any item flows between operational distributor/retailer facilities. In time-period 2, forward and reverse item flows begin to take place. One cash rebate activity is selected and one discount activity. Forward product flows are only directed through one distributor/retailer facility, although the other distributor/retailer facility is also opened in order to carry out product take-back activities. Refurbishing and remanufacturing recovery options are utilized. The third-time period produced similar results to the second-time period, except that two cash rebate activities are carried out

instead of a mix of both types. Both refurbishing and remanufacturing recovery options are utilized. The fourth period produced similarly selected two cash rebate activities, but chose not to satisfy secondary consumer demand. Instead, remanufacturing was carried out on product returns.

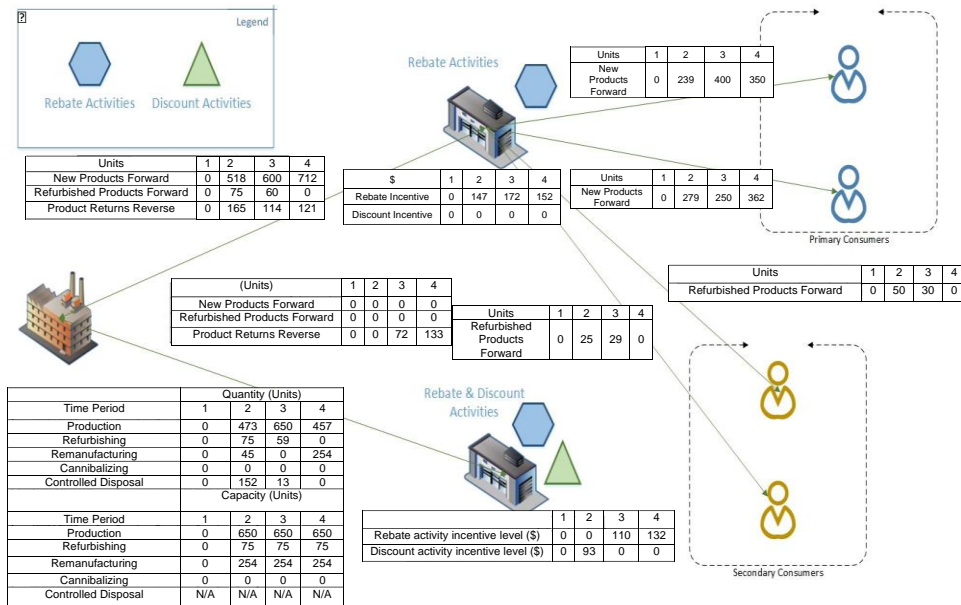


Figure 2: Model results

4. Conclusions

A multi-period mathematical model that incorporates decisions on incentive levels, take-back program type and location for closed-loop supply chains was proposed in the study. The presented model can properly decide on standard CLSC decisions. Furthermore, Monte Carlo simulation was performed and the results revealed that the model produced significantly lower (p -value < 0.0001) objective function values under uncertain demand than in the equivalent deterministic problem. This implies that the solution delivered by the model is not able to achieve the objective function value promised under uncertain demand. This observation was addressed using the TORO procedure, which produced solutions that dominated the deterministic solution in terms of mean and standard deviation of objective values under uncertain demand. The procedure allows the decision maker to select among non-dominated solutions according to risk-appetite.

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