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# Optimizing Spare Parts Inventory for Time-Varying Task

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As the requirement of task is dynamical, the usage of equipment will change with time and it will result in the fluctuating demand for spare parts all over the time. In order to improve equipment performance, the users need to forecast equipment failure frequency and demand for spare parts within the task period. Demand in a moment of task period is the result of cumulative effect of the failure spare parts during the repairing time. When the failure and repair time is exponential distribution, the current number of using equipment is the cumulative sum of equipment quantity. According to the non-stationary demand for spare parts, the method of inventory optimization based on non-stationary Poisson process is proposed. Considering the influence of dynamical task to spare parts, the function relationship between dynamical requirements for spare parts and time-varying task is founded. Besides analytical method which applies non-homogeneous Poisson process to calculate the required number of spare parts is obtained, and formula for the time-varying backorder number of spare parts is proposed. As to the optimization model the backorder number of spare parts is made as the optimization objective with the cost constraints. Real-time spare parts optimization calculation model for dynamical task demand is established with analysing the convexity of the objective function and the convex optimization methods is introduced. The best spare parts configuration is given in each moment according to the practical example.

## 1. Introduction

The working hours are constant under steady-state task conditions as well as the demand for spares, so the expected backorder that represents the system support ability won't change with time, that is, the time-varying factor is not considered into the optimization of inventory. However when the task intensity fluctuates the inventory is dynamically changing because of the randomness of task requirement. In order to make the inventory satisfies both the optimal ability of demand and economic affordability, it's necessary to dynamically control the configuration of spare parts.

Respecting the optimization of steady-state inventory, Eduardo(2009) and Lau(2004) study the optimization algorithm based on fixed support probability. As the optimization methods of inventory mentioned above mainly deal with the optimization of inventory under the steady-state demand, and the time-varying factor is not considered, which results in the disability of the optimization of inventory in each time interval. In regard to the calculation of time-varying backorder, Tongdan(2012) calculates the time-varying EBO of fixed time interval based on the process of Markov, especially considering the situation of one failed spare results in the whole system down. Lau(2006) due to the time-varying demand rate sets up the EOQ model of stochastic state, but its assumption is that the demand rate must be the linear time-varying increasing function.

According to the shortage of the above issue, this paper proposes the optimization of inventory considering the effect of dynamical task to the demand for spares based on the non-stationary Poisson process, sets up a function relationship between the dynamical demand for spares and time-varying task hours, draws the conclusion of mathematic method of calculating the spares demand based on the non - homogeneous Poisson process, and gives out the calculation formula of time-varying backorder. The optimization model sets time-varying EBO as optimization object, analyzes the convexity of the object function, introduces the convex optimal method, and sets up the calculation of time-varying inventory in regard to dynamical task with the constraint of cost.

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## 2. Model analysis

Equipment will generate spare parts demand when the items fail, and the supply time and maintenance time will both affect the operational readiness. In order to optimize the inventory, the first step is to forecast the demand rate on the basis of the reliability parameters and usage activities. As the reliability parameters is determined in the design phase, and use frequency is related to the task requirement, so the demand rate is fluctuating with the dynamical task. Regarding with the situation mentioned above, this paper proposes a new calculation model of spare part demand both considering the dynamical task and dynamical spare part program.

**2.1 Effects of dynamical task to the demand rate**

Demand rate is the number of spares generating from the failed items to the inventory as the equipment perform task, and the formula is:

$$\lambda_i(t)=\frac{UR(t)}{MTBF_i}\times QPA_i\times Nsys \tag{1}$$

Where $MTBF_i$ is mean time between failure of LRU *i*, $UR(t)$ is the daily usage of equipment, $QAP_i$ is the installation number of LRU *i* in the system, $Nsys$ is the number of system.

According to formula (1), when the task intensity is dynamically changing, the usage of equipment is time-varying as well. Assuming the initial demand for spares is 0, the demand is mutually independent in each disjoint time period. $\lambda_i(t)$ is intensity function and in a small time interval ( $t,t+\Delta t$ ) the probability of spare demand is $\varepsilon_i(t)$ of which $\varepsilon_i(t)$ is a tiny variable related to $\lambda_i(t)$ , that is, $\frac{\Delta\varepsilon_i}{\Delta t_i}\rightarrow 0$ when $\Delta t_i\rightarrow 0$ . Then according the definition of non-homogeneous Poisson Distribution, the demand of spares follows the non-homogeneous Poisson Distribution, and the parameter is:

$$Q_i=\Lambda_i(t_2)-\Lambda_i(t_1) \tag{2}$$

Where $\Lambda_i(t)=\int_0^t\lambda_i(s)ds$ . $\Lambda_i(t)$ is a mean function and represents the average failure number within the time period (0,t). When the task requirement changes with time and its lifetime follows the Exponential distribution, the failure number could approximate the Poisson distribution.The demand of spares is a non-homogeneous Poisson process with the intensity of $\lambda(t)$ , and the repair time is Exponential distribution. As the items in repairing are unavailable, the expected number of repairing spares can be considered as the expected number of failure spares. Assuming the repair time is Exponential distribution, whose average value is $1/\mu_i=TAT_i$ , the repairing spares is:

$$\xi_i(t)=\int_{t-TAT_i}^t\lambda_i(s)ds \tag{3}$$

Where $TAT_i$ is the repair time of LRU *i*.

Thus according to the above conclusion, the time-varying demand distribution is:

$$\Pr(DI_i(t)=k)=\frac{(\xi_i(t))^k}{k!}e^{-\Lambda_i(t)} \tag{4}$$

Where $DI_i(t)$ is the demand number of LRU i at time t.

**2.2 Time-varying EBO**

Spare backorder is the sum of lack of all parts per unit time, the mathematic function of EBO is:

$$EBO=\sum_{i=1}^{n}\sum_{k=s_i+1}^{\infty}(k-s_i)\times\Pr(DI_i=k) \tag{5}$$

Where $s_i$ is the stock of LRU *i*.

When the task changes with time, spare backorder will change as well. How to calculate the time-varying backorder is the key issue of spare optimization in the non-stationary.
According to the expected formula of discrete stochastic variable, *EBOi(s,t)* of LRU i is

$$
\begin{aligned}
&EBO_i(s,t) = \Pr(DI_i(t) = s_i(t)+1) + \\
&2\Pr(DI_i(t) = s_i(t)+2) + 3\Pr(DI_i(t) = s_i(t)+3) + \cdots \\
&= \sum_{k=s_i(t)+1}^{\infty} (k - s_i(t)) \Pr(DI_i(t) = k)
\end{aligned}
\tag{6}
$$

## 3. Optimal model and algorithm

### 3.1 Optimal model

The backorder number of spare parts is a significant indicator of that if the spare parts inventory can meet the ability of spare parts requirements. EBO is the demand number that can not satisfy the expectations of the supply of spare parts in some times, which is an important indicator of the spare parts guarantee. As long as we can not meet the needs, these will continue until one of the supplements or fault parts has repaired. The smaller the backorder number, the higher combat readiness rate of spare parts is. EBO can be seen as optimization objectives, the cost of spare parts is a constraint, and we consider the cost of the purchase of spare parts in this paper, $C_i$ is the cost of LRU i, and $\sum_{i=1}^{I} s_i c_i$ is the total cost of spare parts. The maintenance cost is limited within a certain range, then we build a constrained formula of maintenance costs, $C_m$ is maximum cost, and the constraint condition is task strength, we create a optimization model is as follows:

$$
\begin{aligned}
&\min \sum_{i=1}^{I} EBO_i(s_i, t \,|\, E(DI_i(t))) \\
&s.t. \sum_{i=1}^{I} c_i s_i \le C_m \\
&\quad FHP(t) = 24 \times UR(t) \times Nsys \\
&\quad 0 \le t \le T \\
&\quad t, s_i \in N, \forall i
\end{aligned}
\tag{7}
$$

$c_i$ is the unit price of LRU *i*, $FHP(t)$ is daily working hours, T is the total mission time.

The types of items are so many and the configuration is different from each other, so the combinations are large and it’s necessary to apply convex optimal method for the reason that it can guarantee the global optimization and efficiency.

Firstly analyze the convexity of $EBO(s)$

Definite h(s) a function with the discrete variable s. When first-order differential $\Delta h(s) = h(s+1) - h(s) \le 0$ and second-order difference $\Delta^2 h(s) = h(s+2) - 2h(s+1) + h(s) \ge 0$ , h(s) is convex function.

To prove that EBO is a convex function in any probability distribution and any variable s, the definition of EBO is introduced to convex function.

$$
\begin{aligned}
&\Delta EBO(s) = EBO(s+1) - EBO(s) \\
&= \Pr\{DI = s+2\} + 2\Pr\{DI = s+3\} + \cdots - \\
&\Pr\{DI = s+1\} - 2\Pr\{DI = s+2\} - 3\Pr\{DI = s+3\} - \cdots \\
&= -\Pr\{DI = s+1\} - \Pr\{DI = s+2\} - \Pr\{DI = s+3\} - \cdots \\
&\le 0
\end{aligned}
\tag{8}
$$

$$\begin{aligned}&\Delta^2 EBO(s) = \Pr\{DI = s+3\} + 2\Pr\{DI = s+4\} + \cdots - \\&2\Pr\{DI = s+2\} - 4\Pr\{DI = s+3\} - 6\Pr\{DI = s+4\} - \cdots + \\&\Pr\{DI = s+1\} + 2\Pr\{DI = s+2\} + 3\Pr\{DI = s+3\} + \cdots \\&= \Pr\{DI = s+1\} \geq 0\end{aligned} \quad (9)$$

It satisfies the requirement of convex function, so EBO is a convex function. Given c is the price of spare parts, then the expected marginal value $\{EBO(s-1) - EBO(s)\}/c$ is not increasing function. As the sum of convex function is still convex function, the inventory curve with the object of the sum of EBO EBO-C is a optimal curve.

**3.2 Optimal Algorithm**

Due to the wide variety of spares, the inventory configuration is different in different task. In order to quickly and efficiently calculate the optimal inventory it needs a valid algorithm. Convex optimal algorithm precisely meet these requirements. Here are convex optimization algorithm specific steps:

Step 1: initialize spare parts inventory

$$\sum_{i=1}^{I} S_i = 0, S_i \geq 0 \quad \forall i \quad (10)$$

Step2: push forward the time to $t = t' + \Delta t$ , where t 'indicates the last time, $\Delta t$ indicates the time interval and t represents the current moment

Step 3: calculate $EBO(S_0,...,S_i,...,S_M;t)$ and $EBO_i^*(t) = EBO(S_0,...,S_i+1,...,S_M;t)$ of LRU i at time t

Step4: compare each kind of spare part's $[EBO(s_k) - EBO(s_k+1)]/c_k$ , and find the kind of spare pare k that make the formula maximum.

Step5: increase the inventory of spare parts k, and add the cost of spare parts k to the total cost. set $EBO_i(S_0,...,S_i+1,...,S_M;t) = EBO_i^*(t)$

Step 6: If $EBO(S_0,...,S_i,...,S_M;t) \leq EBO_i^*(t)$ then return to step 2, otherwise stop.

According to the above steps the optimal inventory and smallest EBO can be acquired, where *EBO** is the smallest value at that time.

## 4. Case analysis

Make airplane as example, and optimize the initial ordering program of the group of aircraft. Usage scenario determines the group of aircraft operated with time-varying task, and each airplane consists of 10 types of LRU. In Figure 1 there are the daily usage rate $\lambda_i(t)$ under the whole task cycle. In Table 1 there are the parameters related to the initial spare parts, including reliability and maintenance design parameters and cost of spares. The optimal purpose is to obtain the configuration that makes EBO smallest under the situation of time-varying task and cost constraint.

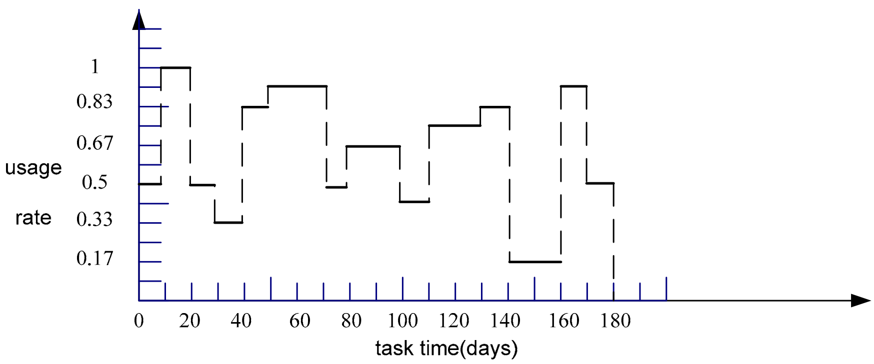


*Figure 1: The daily usage of airplane*

*Table 1: The input parameters of LRU*

| Item | Failure rate(year) | Price(thousand) | Repair time(day) | QPA | Cost(million) |
|---|---|---|---|---|---|
| LRU1 | 12 | 30 | 2 | 1 | 2 |
| LRU2 | 8 | 20 | 2 | 2 | |
| LRU3 | 14 | 40 | 2 | 1 | |
| LRU4 | 7 | 40 | 3 | 1 | |
| LRU5 | 8 | 50 | 4 | 1 | |
| LRU6 | 6 | 40 | 2 | 2 | |
| LRU7 | 7 | 50 | 2 | 1 | |
| LRU8 | 12 | 40 | 2 | 1 | |
| LRU9 | 10 | 30 | 3 | 1 | |
| LRU10 | 9 | 30 | 2 | 1 | |

According to formula (1)-(3) the average repairing number of LRU can be calculated. The repairing number of LRU in the whole life cycle is shown in Table 2.

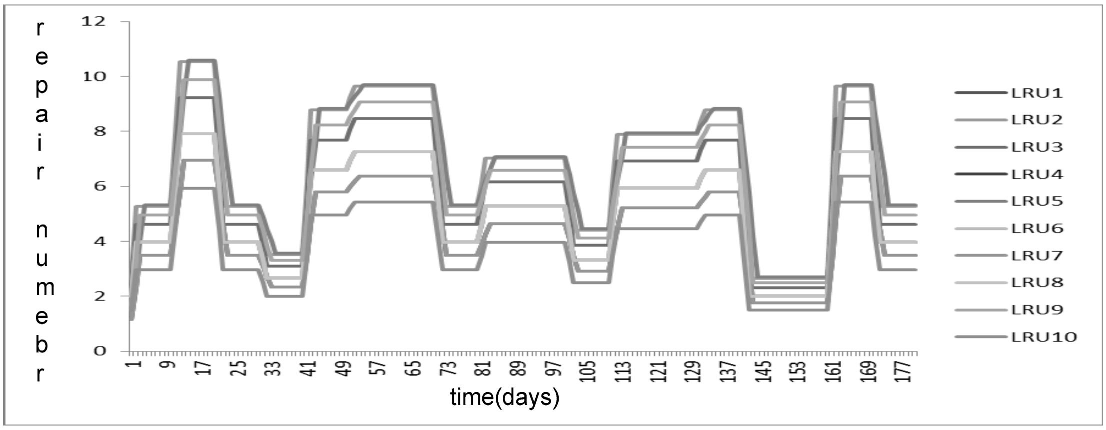


Figure 2: The repairing number of LRU

From the above figure, the repairing number of each LRU is changing with time in the whole task cycle, which results in the optimal inventory per time interval. So in order to get the proper inventory, it is necessary to optimize it making $\sum_{i=1}^{I} EBO_i(s_i, t \left| E(DI_i(t))\right.)$ largest per 30 days, and adopt the configuration at that time as the global optimal inventory schedule.

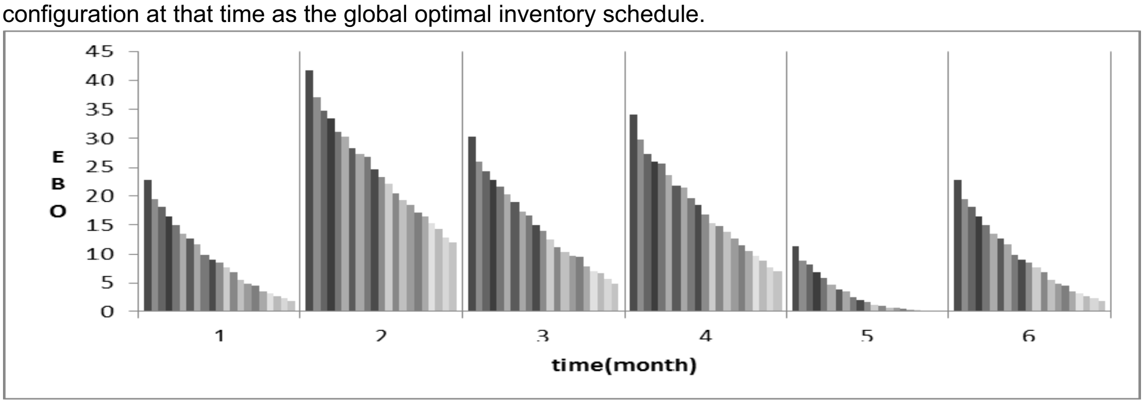


*Figure 3: The changes of EBO per month under the constraints of cost*

Figure 3 shows the optimal curve of EBO. The time interval is 30 days, and the cost constraint is 2 million dollars. From the fluctuation of task intensity, EBO changes significantly as well, and there even is a gap of 4-5 times in some months. So it's necessary to optimize the inventory in each interval to guarantee the best system efficacy under the cost constraint.

*Table 2: Optimal inventory of LRU per month*

| Inventory | Jan | Feb | Mar | Apr | May | Jun |
|---|---|---|---|---|---|---|
| LRU1 | 6 | 7 | 6 | 6 | 5 | 6 |
| LRU2 | 8 | 10 | 9 | 9 | 7 | 8 |
| LRU3 | 6 | 6 | 6 | 6 | 6 | 6 |
| LRU4 | 4 | 4 | 4 | 4 | 5 | 4 |
| LRU5 | 6 | 5 | 6 | 6 | 6 | 6 |
| LRU6 | 5 | 5 | 5 | 5 | 5 | 5 |
| LRU7 | 4 | 3 | 4 | 3 | 5 | 4 |
| LRU8 | 5 | 5 | 5 | 5 | 5 | 5 |
| LRU9 | 7 | 8 | 7 | 8 | 6 | 7 |
| LRU10 | 4 | 5 | 4 | 5 | 4 | 4 |

*Figure 2 shows the optimal inventory per month.*

## 5. Conclusion

This paper analyzes the time-varying variation of non-stationary spares demand based on the dynamical task, sets up function relationship between dynamical spare parts requirements and time-varying task, considering the dynamical task influence to spare parts requirements, applies the non-homogeneous Poisson distribution to propose a calculation model of backorder with non-stationary Poisson process. The convexity of object function is analyzed and the time-varying convex optimal method calculating the inventory is proposed as well as the typical case. This method can give out the optimal inventory according to the demand of each LRU, so there is a broad application prospects on the tradeoff decisions and optimal schedules of spares inventory. In future wore the following areas will be carried out: 1) further consider to introduce costs constraints of spare parts procurement batch costs, storage costs, as well as spare parts transportation; 2) apply simulation method to further verify the validity and accuracy of the optimization algorithm. 3) based on the dynamical task calculate the optimal inventory in each interval when it approaches 0 to control the inventory in real time.

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