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Actuator Health Prognosis for Designing LQR Control in Feedback Systems

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This paper deals with the control of actuators in feedback systems. Actuators are costly components for which health monitoring provides valuable information. Here, the control input law and its variations are assumed to deteriorate actuator until a degradation threshold leading the component to crash. With an observable degradation level according to a stochastic Gamma process, the actuator residual useful lifetime RUL prognosis is used to online reconfigure a Linear Quadratic Regulator control law. The main aim is to find a satisfactory trade-off between conflicting requirements i.e. system performances and actuator reliability.

1. Introduction

In feedback control systems, the actuators are of prime importance because they represent the physical link between the control law and the governed process. They are costly in many industrial cases and as a consequence need all attention during the process design step. The input control action $u(t)$ and its variations $\dot{u}(t)$ may be a source of stress leading to accelerate the actuator deterioration $D(t)$ (Pereira et al, 2010). The loss of winding insulation of an electrical motor due to a varying command voltage is an illustration of this. When a given degradation level L_f is reached, a short circuit occurs leading the component to crash.

Assuming an observable level of degradation, the residual useful lifetime RUL becomes valuable information. This latter is commonly used when designing maintenance policies (Wang, 2002). Here, RUL is integrated in controller design with the aim to find a satisfactory trade-off between system performances and actuator reliability. A RUL-based controller design becomes another mean for preserving health actuator and then to better plan maintenance action with money saving.

Irrespective of variable environmental conditions, in this paper an actuator is assumed degrading according to a stochastic process with two modes i.e. nominal and accelerated ones. The time of change of mode is random and depends on several factors e.g. the number of times the actuator has been stressed and its level of deterioration. While the first point is directly impacted by the kind of system mission as well as the strength of the control action, the latter tries to embody the sensitivity of actuator to stress. Moreover when degrading the actuator may be concerned with a loss of efficiency to fully implement $u(t)$.

This paper is organized as follows. Section 2 exposes the main features of a feedback system controlled with a LQR law standing for Linear Quadratic Regulator. It is also highlighted how the actuator degradation level impacts its effectiveness to fully implement $u(t)$. Then, Section 3 proposes a stochastic degradation model. This paper ends with numerical results after identifying system performance criteria in Section 5 and a RUL-based LQR algorithm in Section 6.

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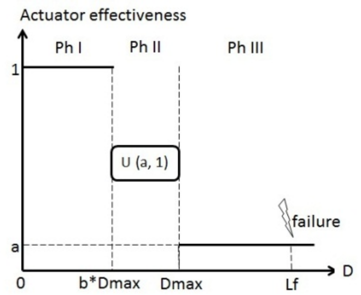
2. Feedback control system

2.1 Loss of effectiveness and degradation

A basic feedback control system is studied and defined with a linear time invariant state space representation such as

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) \end{cases} \tag{1}$$

A, B, C are constant matrices of suitable dimensions standing respectively for the state, the control and the output. $x(t)$ stands for the dynamic of the system, $u(t)$ for the control input and $y(t)$ is finally the output of the process. In the field of control systems, the term degradation is related to a loss of effectiveness to fully implement $u(t)$ when the actuator is in a faulty situation and not to an intrinsic physical deterioration. In such situation, equation (1) is rewritten as $\dot{x}(t) = A\,x(t) + B\Delta\,u(t)$ with $\Delta = \mathrm{I} - \mathrm{diag}\,(\delta_1 \dots \delta_r)$ the effectiveness matrix of positive or null coefficients (Khelassi et al, 2010). In this paper, the loss of effectiveness and the actuator degradation level $D(t)$ are clearly two different features of a feedback control system. Nonetheless, it is assumed a strong relationship between them as described in Figure 1.



The actuator effectiveness Δ decreases from 1 to a according to three different phases with $a \in [0,1]$. Let $Dmax$ a degradation threshold and $b \in [0,1]$. Phase *I* stands for a full effectiveness even if the actuator is experiencing an intrinsic degradation. Phase *II* is a transient step where the loss of gain Δ is uniformly distributed i.e. $\Delta \sim U(a, 1)$ and Phase *III* corresponds to a saturation of the actuator effectiveness. Δ is finally depending on time leading equation (1) to $\dot{x}(t) = A\,x(t) + B\Delta(t)\,u(t)$.

Figure 1: Actuator effectiveness model

2.2 Linear Quadratic Regulator LQR

There exists many ways for controlling a process variable (Yamaki, 2012). Among of them, LQR (Fodor, 2012) is commonly used in modern control. In few words, LQR provided linear feedback $u(t)$ by minimizing the following cost function

$$J\big(u(t)\big) = \int_{t_i}^{t_j} [x(t) - x^*(t)]^T Q\,[x(t) - x^*(t)] + u(t)^T R\;u(t)\,dt \quad \text{with } x^*(t) \text{ the desired state} \tag{2}$$

Matrices Q and R are weighting matrices. Q describes the importance given to minimize the tracking error and R the control effort. The feedback control minimizing equation (2) is $u(t) = -L\,x(t)^T + [C(-A + BL)^{-1}B]^{-1}y^*(t)$ with $L = R^{-1}B^TP$ and P the solution of the algebraic Riccati equation defined with $A^TP + PA - PBR^{-1}B^TS + Q = 0$. More details about Eq. (2) and guidelines for selecting both matrices Q and R are given in (Stevens, 2003).

3. Degradation model

3.1 Gamma process

It is generally agreed that the material fatigue has intrinsically a stochastic behaviour. For this reason, the actuator is assumed to degrade according to a stochastic process. The level of deterioration $D(t)$ monotonously increases over time as an accumulation of small positive increments and is modelled with a homogeneous Gamma process Ga with shape parameter $\alpha(t)$ and scale parameter β. The main features of such a process are

- $D(0) = 0$
- $D\big(t_{i+j}\big) - D(t_i) \sim Ga\;(\,\alpha(t_{i+j}) - \alpha(t_i)\,,\;\beta\,)$
- $D(t)$ has independent increments

Moreover, a random scalar variable Gamma distributed Y has for cumulative distribution function $F(y, \alpha(t), \beta) = \Gamma(\alpha(t), y/\beta) \;/\; \Gamma\big(\alpha(t)\big)$. $\Gamma(u)$, $\Gamma(u, v)$ stand for complete and incomplete gamma functions. The actuator failed when its level of deterioration is greater or equal to a given threshold L_f.

3.2 Stress model

As stated above, $u(t)$ and its variations $\dot{u}(t)$ may be a source of stress leading to accelerate the actuator deterioration $D(t)$. $u(t)$ acts as a covariate z impacting $D(t)$ in a proportional manner (Bagdonavicius, 2009) with $\alpha(t) = \alpha_{baseline}(t)\, e^{(\varphi_1 \mathbf{1}_{(z=1)} + \varphi_2 \mathbf{1}_{(z=2)} + \varphi_3 \mathbf{1}_{(z=3)})}$

In this paper, three level of stress are identified i.e. three possible states for z e.g. no stress, medium and high stress. φ_i is a regression parameter that punctually modifies the baseline when z belongs to state i. Here, this stress is considered as a shock that instantaneously impacts the deterioration speed α with a possible degradation level increasing.

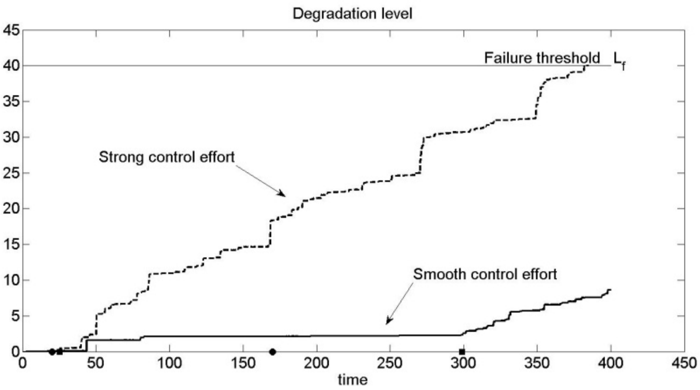


Figure 2: Stress model. T_s (Black disc) T_M (Black square)

The actuator is normally degrading in a nominal mode M_1 with parameters (α_1, β). After a given number of stress solicitations N_s, the actuator potentially may fall in an accelerated mode M_2 with parameters (α_2, β). The date of onset T_s corresponding to N_s solicitations depends on the kind of control law as well as on the type of mission. T_s will arrive sooner in time with a strong control effort $u(t)$ than with a smooth one. Let T_M the date related to the passage in mode M_2 with $T_M > T_S$. As a first approach if D_S is the level of degradation at T_s then D_M the level of degradation at T_M is defined such as $D_M = D_S\,(1 + \rho)$ with $0 \leq \rho \leq 1$. As the degradation process is monotonously increasing, the changeover to M_2 will arrive more quickly in the case of a strong control effort than in a smooth one that finally highlights the sensitivity to stress as illustrated in Figure 2.

4. Residual Useful Lifetime RUL

The residual useful lifetime is here connected to the time of failure estimate T_F. If the actuator is still working at time t_i with a measured level of degradation d_i then the conditional reliability R is expressed as follows

$$\begin{aligned} R(t|d_i) = R_i(t) &= P\big(D(t) \leq L_f \,\big|\, D(t_i) = d_i\big) \\ &= P\big(D(t) \leq L_f \cap D(t_i) \leq D_M \,\big|\, D(t_i) = d_i\big) + P\big(D(t) \leq L_f \cap D(t_i) > D_M \,\big|\, D(t_i) = d_i\big) \\ R_i(t) &= R_i^{\#1}(t) + R_i^{\#2}(t) \end{aligned} \tag{3}$$

$$\begin{aligned} R_i^{\#1}(t) &= P\big(D(t) \leq L_f \cap T_M < t \,\big|\, d_i\big) + P\big(D(t) \leq L_f \cap T_M \geq t \,\big|\, d_i\big) \\ &= P\big(D(t) \leq L_f \cap T_M < t \,\big|\, d_i\big) + P\big(D(t) \leq L_f \cap D(t) \leq D_M \,\big|\, d_i\big) \\ &= \int_{t_i}^{t} P(D(t) - D(u) \leq L_f - D_M)\, f_{T_M}(u|d_i)\, du + P(D(t) - D(t_i) \leq D_M - d_i) \\ &= \int_{t_i}^{t} F(L_f - D_M,\ \alpha_2(t-u),\ \beta)\, f_{T_M}(u|d_i)\, du + F(D_M - d_i,\ \alpha_1(t - t_i),\ \beta) \end{aligned}$$

$$\begin{aligned} f_{T_M}(u|d_i) &= \frac{d}{dt} P(T_M \leq u|d_i) = \frac{d}{dt} P(D(u) \geq D_M\ |d_i) \\ &= \frac{d}{dt}\left[1 - P(D(u) \leq D_M\ |d_i)\right] = -\frac{d}{dt} P(D(u) \leq D_M\ |d_i) = -\frac{d}{dt} P(D(u) - D(t_i) \leq D_M - d_i) \\ &= -\frac{d}{dt} F(D_M - d_i, \alpha_1(u - t_i), \beta) \end{aligned}$$

$$R_i^{\#2}(t) = P\big(D(t) \le L_f \;\cap\; D(t) > D_M \,\big|\, d_i\big) = P(D(t) - d_i \le L_f - d_i\,)$$
$$= F(L_f - d_i\,,\, \alpha_2(t - t_i)\,, \beta\,)$$

RUL is a stochastic variable with cumulative distribution function (3). Given a probability of failure q, RUL_i the residual useful lifetime at time instant t_i is then defined such as

$$RUL_i = t_{R_i(t) \ge 1-q} - t_i \tag{4}$$

Note that Eq (4) is a first approach of the residual useful lifetime because RUL_i does not account for stress impact that punctually modifies the Gamma process shape parameter $\alpha_{1,2}$.

5. System performances

During one finite time mission T_{span}, an actuator experiences N several phases Ph_i corresponding to different tracking setpoints. A setpoint is simply a target value that a control system must quickly attain. For example, a servo-motor has to track a predetermined setpoint to get a safety valve in a given position. In control theory three performance criteria are commonly used i.e. the static error position ε_p, the rise time t_r and the settling time t_{st} (Richards, 1993). For one phase Ph_i, such criteria are defined with

$\varepsilon_p^{Ph_i} = |\, y_*^{Ph_i} - y_\infty^{Ph_i} \,|$ $\quad t_r^{Ph_i} = \big(t_{up} \,/\, y(t_{up}) = 0.90\, y_\infty^{Ph_i}\big) - \big(t_{down} \,/\, y(t_{down}) = 0.10\, y_\infty^{Ph_i}\big)$

$t_{st}^{Ph_i} = t \;/\; \left|\frac{y(t) - y_\infty^{Ph_i}}{y_\infty^{Ph_i}}\right| \le 5\%$. Subscript ∞ only stands for system steady state and y_* for the desired tracking setpoint. In this paper, the actuator is assumed experiencing always the same mission with N phases. To obtain a representative behavior of the stochastic degradation process, the system life is simulated with N_{runs} runs. The performance criteria described above are then averaged such as

$$E_p = \frac{1}{N_{runs}\, N} \sum^{N_{runs}} \sum_i^N \varepsilon_p^{Ph_i} \qquad T_r = \frac{1}{N_{runs}\, N} \sum^{N_{runs}} \sum_i^N t_r^{Ph_i} \qquad T_{st} = \frac{1}{N_{runs}\, N} \sum^{N_{runs}} \sum_i^N t_{st}^{Ph_i} \tag{5}$$

6. RUL-based LQR algorithm

The aim is to online reconfigure the control law keeping a satisfactory trade-off between system performances and actuator reliability. Given an initial control design retained until T_S, the objective is to reshape this initial design trying to conserve as long as possible its main features in terms of average performances (5) and then to smartly degrade them for promoting the actuator health. This reconfiguration is obviously based on the value of the residual useful lifetime (4).

Given an alarm threshold RUL^* both matrices Q and R are updated in a proportional manner depending on RUL_i. If RUL_i is greater than RUL^* then the performances are rather preferred accounting nonetheless for actuator reliability. Conversely, if RUL_i is lower or equal than RUL^* then the actuator health is promoted at the expense of performances.

The sketch of the proposed LQR algorithm is summarized in Algorithm 1.

Algorithm 1: RUL-based LQR algorithm

Update Q , R matrices at time t_i (Q_i, R_i)	
Let $\delta \propto \| RUL_i - RUL^* \|$	
Switch ($RUL_i > RUL^*$)	
Case TRUE	Case FALSE
$Q_i \leftarrow Q_{i-1}$	$Q_i \leftarrow Q_{i-1} * (1-\delta)$
$R_i \leftarrow R_{i-1} * (1+\delta)$	$R_i \leftarrow R_{i-1} * (1+\delta)$
End Switch	

7. Numerical results - Discussions

A DC-motor controlled in position is here studied with the following matrices and simulation data (Table 1). The mission profile consists for the actuator to track three different position setpoints.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -10/9 & 1/9 \end{bmatrix} \tag{6}$$

Table 1: Simulation data

T_{span}	α_1	α_2	β	N_{runs}	N_s	φ_i	ρ	L_f	RUL^*
$400s$	0.01	0.1	1	100	20	$\{0,1,2\}$	10%	40	$200s$

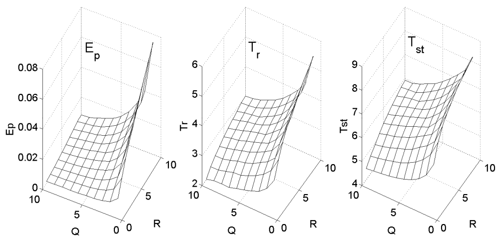


Figure 3: System performances

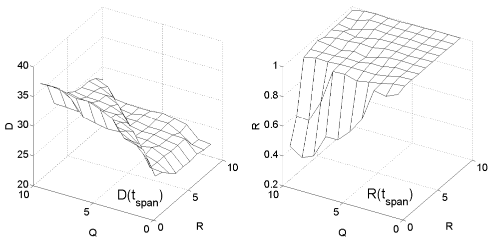


Figure 4: Actuator health

7.1 Initial design

In addition to system performances defined above i.e. (E_p, T_r, T_{st}), the actuator health is here considered as the value of wear and also as the value of reliability assessed at the end of mission $(D(T_{span}), R(T_{span}))$. Several weighting values in a range $\in [1, 10]$ are checked for both identity matrices (Q, R). In this first experiment, the actuator is assumed not experiencing any loss of efficiency spending the entire mission in Ph_I (see Figure 1). It can be noticed in Figures 3 and 4 the impact of such values on system performances and actuator health. In particular, the set $(10,1)$ acting as a strong control effort reveals the best performances at the expense of actuator health. These results are consistent with i) the LQR law and ii) with the proposed degradation model. High energy is spent in $u(t)$ per unit of time which solicits heavily the actuator. The number of stress solicitations N_s is quickly reached over time causing the actuator to rapidly fall in accelerated mode M_2.

7.2 RUL-based LQR control

The second experiment focuses on this initial set $(10,1)$. The aim is to keep as long as possible its performances by reconfiguring the LQR law aided with the algorithm described in Section 6. The results summarized in Table 2 (first two columns) indicate that the actuator health can be improved by moderately decreasing the performances. Until the moment T_s corresponding to N_s stress solicitations, the performances of the initial design are kept. Then, once T_s is reached the proposed RUL-based algorithm is initiated. The force of LQR law slightly decreases which results in less stress the actuator. The potential degradation increments due to stress have less effect with as a possible consequence to delay the changeover to M_2.

Table 2: LQR performances & Actuator health. $(Q = 10, R = 1)$

With RUL_i	Without RUL_i	With RUL_i	Without RUL_i
No loss of effectiveness		Loss of effectiveness $a = 0.5$ $b = 0.5$ $D_{max} = 15$	
$E_p = 0.0021$	$E_p = 0.0007$	$E_p = 0.0107$	$E_p = 0.0012$
$T_r = 2.3186$	$T_r = 1.8000$	$T_r = 2.9296$	$T_r = 1.9657$
$T_{st} = 4.8445$	$T_{st} = 4.0970$	$T_{st} = 5.7588$	$T_{st} = 4.3748$
$D(T_{span}) = 34.7836$	$D(T_{span}) = 35.9021$	$D(T_{span}) = 34.0809$	$D(T_{span}) = 35.2840$
$R(T_{span}) = 0.54$	$R(T_{span}) = 0.40$	$R(T_{span}) = 0.70$	$R(T_{span}) = 0.50$

The third and last experiment deals with the impact of loss of effectiveness. The values of (a, b, D_{max}) are chosen such as the actuator can experience the three phases $Ph_{I,II,III}$ during one mission. Table 2 (last two columns) illustrates that this loss of effectiveness for the actuator to fully implement $u(t)$ results obviously in lower system performances but also has a positive impact on its health. With a loss of gain, the actuator may early experience Ph_{II} (then Ph_{III}) which results in delaying the date T_s. Consequently, the passage in accelerated mode M_2 is also delayed. This loss of effectiveness weakens LQR law and therefore the resulted stress. Its effects combined with a RUL-based control law allow the increasing of actuator health while preserving satisfactory system performances. Figures 5 and 6 illustrate how the initial LQR control is online RUL-based reshaped and the corresponding impact on $u(t)$.

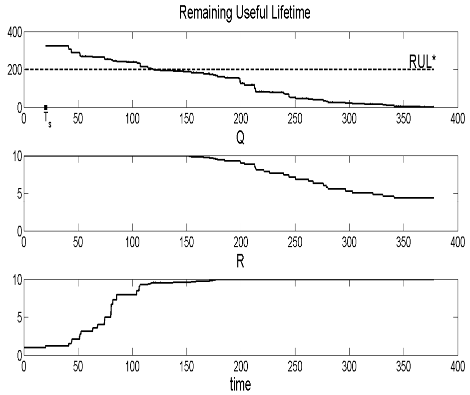


Figure 5: RUL-based LQR law

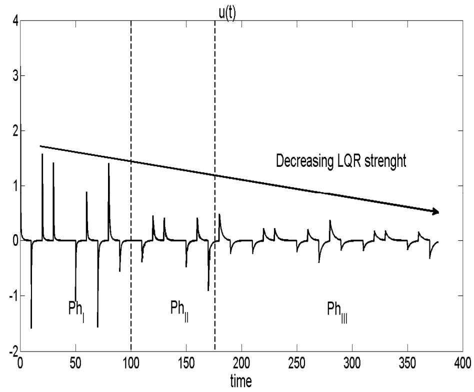


Figure 6: Input control $u(t)$

8. Conclusion

The objective of this paper is mainly to propose a modeling framework integrating the deterministic behavior of a feedback LQR controlled system and a stochastic degradation process. Both behaviors are closely interrelated due to the input law $u(t)$ and its variations. Satisfying system performances as well as actuator health over the entire mission may be a difficult task for a control designer. Here, a basic RUL-based LQR algorithm is proposed to find a satisfactory trade-off. The future work is i) to refine RUL_i relation (4) taking more into consideration the potential degradation increments due to stress and ii) to improve this algorithm by optimizing the alarm threshold RUL^* .

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