CHEMICAL ENGINEERING TRANSACTIONS
VOL. 33, 2013
A publication of AIDIC
The Italian Association of Chemical Engineering
Online at: www.aidic.it/cet
Guest Editors: Enrico Zio, Piero Baraldi

ISBN 978-88-95608-24-2; **ISSN** 1974-9791
DOI: 10.3303/CET1333180

Stochastic Definition of State-Space Equation for Particle Filtering Algorithms

Matteo Corbetta*, Claudio Sbarufatti, Andrea Manes, Marco Giglio

Politecnico di Milano, Dipartimento di Meccanica, Via La Masa, 1 – 20156 Milan – Italy
matteo.corbetta@polimi.it

Particle Filtering is a nonlinear and non-Gaussian model-based Bayesian Filtering algorithm based on Monte Carlo Sampling techniques. This filtering methodology can be used to increase the reliability and the availability of the monitored system combining the measurements on the system itself and the analytical model of the observed phenomena. Inside this context, the basic idea of Particle Filtering is the estimation of the system degradation through a series of weighted particles simulating the dynamic evolution of a time process. Two different mathematical models are needed in order to implement it: firstly, the stochastic observation equation linking the measures (and their uncertainties) with the current state of the system, and secondly the stochastic degradation equation linking the present state to the prior state, or the Dynamic State Space (DSS) model. The DSS model is usually based on deterministic parameters plus an artificial noise added through Monte Carlo Sampling in order to produce a stochastic process. The simple deterministic model with added noise is not able to account for all the uncertainties occurring in a real environment in many cases, producing poor results. Thus, the definition of a Stochastic Dynamic State Space (SDSS) model is proposed here. The SDSS merge the deterministic equation of the observed phenomenon with the statistical definition of the parameters available in literature (or mathematically extrapolated from historical data). It is inserted in a Particle Filtering algorithm and applied to crack growth estimation in metallic structures. The results of the algorithm with the Stochastic Dynamic State Space model are compared with a Particle Filtering based on traditional DSS in terms of crack length estimation and remaining lifetime evaluation performances.

1. Introduction

All the model-based filtering methods are based on a Dynamic State Space (DSS) model governing the time-evolution of the system. In a Bayesian formulation, the DSS specifies the conditional density of the state given the previous state (Haug, 2005). This DSS model is founded on the mathematical equations describing the observed phenomena, in which an artificial noise is added in order to produce a stochastic process. Notwithstanding the simple deterministic equations with added noise can be effective for the system state estimation, they can produce poor results in prediction problems. As a matter of fact, a lot of mathematical models have parameters known in statistical terms, because of the uncertainties about the real phenomenon (as fatigue crack growth, creep degradation, damping of dynamical systems etc.) and the intrinsic uncertainty of regression methods. As a consequence, the implementation of the mathematical laws with deterministic parameters can lead to improper conclusion about the remaining lifetime of the system, a crucial issue in a maintenance perspective. So a Stochastic Dynamic State Space (SDSS) model for Particle Filtering (PF) algorithm is proposed here.
Literature about Particle Filtering is extensive and another paper will be useless, so the authors decide to omit in-depth mathematical dissertation of the algorithm. The interested reader can refer to Arulampalam et al. (2002) for further information. The paper is organized as follow: a summary of the PF concept will be given inside the section 2. The thorough explanation of the proposed Stochastic DSS model will be given in the section 3 whereas results and conclusions are discussed in sections 4 and 5.

Please cite this article as: Corbetta M., Sbarufatti C., Manes A., Giglio M., 2013, Stochastic definition of state-space equation for particle filtering algorithms, Chemical Engineering Transactions, 33, 1075-1080 DOI: 10.3303/CET1333180

2. Basic of Particle Filtering methods

Let you consider a DSS model f_k describing the evolution of a general system state $\boldsymbol{x}_k$ and suppose that the DSS model satisfies the hypothesis of the Markov processes of order one. The evolution equation can be expressed by the general function visible in (1), where the subscript k indicates the general k^{th} discrete time step. If measurements $\boldsymbol{z}$ of the (unknown) state $\boldsymbol{x}$ are available, it is also possible to write the observation equation linking the measures $\boldsymbol{z}_k$ with the system state $\boldsymbol{x}_k$ (2).

$$\boldsymbol{x}_k = f_k(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) \tag{1}$$

$$\boldsymbol{z}_k = h_k(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{2}$$

The term $\boldsymbol{v}_{k-1}$ in (1) represents the noise added to the deterministic equation to produce a stochastic process, while $\boldsymbol{n}_k$ in (2) represents the noise on the measures (generally unknown). The objective is the evaluation of a conditioned Probability Density Function (PDF) of the system state given the vector of the noisy observations $p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k})$. It is possible to solve this problem with two steps: the prediction and the update (Cadini et al., 2009). The prediction of the random variable $\boldsymbol{x}$ at the k^{th} time step is expressed by the Chapman Kolmogorov equation (3). The updating of this prediction is made as soon as a new observation $\boldsymbol{z}_k$ becomes available through Bayes' rule (4).

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}, \boldsymbol{z}_{0:k-1}) p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{0:k-1}) d\boldsymbol{x}_{k-1} \tag{3}$$

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{0:k-1})} \tag{4}$$

Where $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is the likelihood of the observation, $p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k-1})$ is the posterior PDF at the previous step (becoming the prior) and $p(\boldsymbol{z}_k|\boldsymbol{z}_{0:k-1})$ is a normalizing constant. Unfortunately, the analytical resolution of (3) and (4) is not possible in the most of the engineering problems. Numerical methods based on Monte Carlo Sampling as the Sequential Importance Sampling (SIS) can be used as an alternative.

2.1 Sequential Importance Sampling / Resampling

Assuming a known posterior PDF $p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k})$, it can be approximated by N_s particles $\boldsymbol{x}_{i,k}$ sampled by the same PDF (5).

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) \approx \hat{p}(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) = \frac{1}{N_s} \sum_{i=1}^{N_s} \delta(\boldsymbol{x}_k - \boldsymbol{x}_{i,k}) \tag{5}$$

However, sampling by the true posterior PDF is usually impossible, but the Importance Sampling technique can overcome the problem. Suppose to have a general PDF from which it is simple to draw samples (or particles) $\boldsymbol{x}_{i,k}$, the exact posterior target distribution can be written as a function of this PDF called Importance Density Function (IDF) $\pi(\boldsymbol{x}_k|\boldsymbol{z}_{0:k})$ Eq (6). The posterior can be approximated by a set of N_s normalized weights $w_{i,k}$ associated to the particles $\boldsymbol{x}_{i,k}$ Eq (7).

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) = \int \frac{p(\boldsymbol{x}_{i,k}|\boldsymbol{z}_{0:k})}{\pi(\boldsymbol{x}_{i,k}|\boldsymbol{z}_{0:k})} \pi(\boldsymbol{x}_{i,k}|\boldsymbol{z}_{0:k}) \delta(\boldsymbol{x}_k - \boldsymbol{x}_{i,k}) d\boldsymbol{x}_{i,k} \tag{6}$$

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) \approx \hat{p}(\boldsymbol{x}_k|\boldsymbol{z}_{0:k}) = \sum_{i=1}^{N_s} w_{i,k} \delta(\boldsymbol{x}_k - \boldsymbol{x}_{i,k}) \tag{7}$$

The core of the problem is to define the particle weights $w_{i,k}$. It is possible to demonstrate the weights at k^{th} step depending on these at the $(k-1)^{th}$ step, the likelihood of the observation, the transition PDF $p(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1})$ and the IDF, as visible in (8). The normalization of the weights is expressed in Eq (9) (Cadini et al., 2009).

$$w_{i,k}^* = \frac{w_{i,k-1} p(\boldsymbol{z}_k|\boldsymbol{x}_{i,k}) p(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1})}{\pi(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1}, \boldsymbol{z}_{0:k})} \tag{8}$$

$$w_{i,k} = \frac{w_{i,k}^*}{\sum_{i=1}^{N_s} w_{i,k}^*} \tag{9}$$

It's easy to notice the dependence of the weights at k^{th} time to the previous ones. So, the estimation of the posterior PDF is a sequential updating of the prior, available at time 0. A well-known problem of SIS is the concentration of the weights into a small set of particles $\boldsymbol{x}_{i,k}$ producing poor results in the PDF estimation. Thus different Resampling techniques have been developed over the last years. Resampling consists in a new particle sampling $\boldsymbol{x}_{i,k}$ and the re-setting of the weights. It can be made when the number of the effective particles underneath a certain threshold N_{th} or at each time iteration. In this work, the Systematic Resampling technique is used. See Arulampalam et al. (2002) for more information about Resampling.

3. Dynamic State Space model

As mentioned in the above section, the DSS model relates the state $\boldsymbol{x}_{k-1}$ at previous time step with the state $\boldsymbol{x}_k$. The state is not univocally identified, but it has an associated conditional PDF because of the random noise. This associated PDF is named Transition PDF $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and it describes the probability of $\boldsymbol{x}_k$ given the previous value $\boldsymbol{x}_{k-1}$. Concerning mathematical law describing the state evolution, it depends on some parameters $\boldsymbol{\vartheta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_n]$ having some physic senses or empirically extrapolated. If the dependence of the state $\boldsymbol{x}_k$ from the parameter vector is highlighted, the Transition PDF becomes $p(\boldsymbol{x}_k(\boldsymbol{\vartheta})\,|\boldsymbol{x}_{k-1}(\boldsymbol{\vartheta}))$. Inside the standard PF applications available in literature, mathematical parameters have always been set as deterministic ones. So all the possible state samples are governed by the same deterministic trajectory altered by random noise $\boldsymbol{v}$. According to these considerations, it is simple to infer the definition of the model parameter and the artificial noise become crucial issue for the performance of the algorithm.

3.1 Noise definition

The artificial random noise is not in-depth studied inside the Particle Filter literature. Notwithstanding the mathematical treatment of the artificial noise is easy and it is not interesting from a theoretical viewpoint, it becomes very important for the applicability of PFs to real cases. Considering a noise summed to (10) and a noise multiply by (11) the deterministic evolution of whatever system $g(\boldsymbol{x})$.

$$\boldsymbol{x}_k = f_k(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) = g_k(\boldsymbol{x}_{k-1}) + \boldsymbol{v}_{k-1} \tag{10}$$

$$\boldsymbol{x}_k = f_k(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) = g_k(\boldsymbol{x}_{k-1}) \cdot \boldsymbol{v}_{k-1} \tag{11}$$

The expected value of the random variable $\boldsymbol{x}_k$ must to be equal to the system state described by the deterministic part of the DSS only, hence $E(\boldsymbol{x}_k) = \overline{g_k}(\boldsymbol{x}_{k-1})$. This implies the expected value of the noise has to be zero in the Eq (10) and one in the Eq (11). Otherwise, the DSS produces a biased evolution of the system with respect to the deterministic equation. Focusing on noise variance, it should be defined according to previous experience, the uncertainty of the measurement system and the uncertainty produced by the real operating conditions. The noise has not a single correct definition and it depends on the application. In section 4 an example of noise added to deterministic equation is pointed out.

3.2 Stochastic definition of model parameters

If the law parameters have a deterministic definition inside the PF algorithm (as in the standard PF application), the artificial noise is the only element producing a stochastic process. A new approach is proposed in this work. Empirical laws are usually known with a related uncertainty due to possible errors inside the measurement systems, the shortage of available data, the uncertainty related to the regression procedures or the intrinsic uncertainty of the real environment. So, many statistical descriptions of model parameters are available in literature for a wide range of engineering problems. It is possible to use the parameter PDFs to produce a swarm of possible state evolution, in which every sample of the system state has a particular sample of the parameter PDFs.

Let you consider the parameter vector $\boldsymbol{\vartheta} \in \mathbb{R}^n$. Every element of the vector is a random variable describing one parameter of the DSS equation. Now, the j^{th} parameter can be defined $\vartheta_j \sim pdf\left(\mu_{\vartheta j}, \sigma^2_{\vartheta j}\right) \;\; \forall j = 1 \div n$. In addition, it is possible to define the covariance matrix of $\boldsymbol{\vartheta}$, $COV(\boldsymbol{\vartheta}) \in \mathbb{R}^{n \times n}$. The extra-diagonal elements are different from zero only if a correlation among the different parameters ϑ_j is present. So, the parameter vector $\boldsymbol{\vartheta}$ can be completely defined through a multivariate PDF with expected value and covariance matrix expressed in Eq (12) and Eq (13), respectively.

$$E(\boldsymbol{\vartheta}) = \mu_{\boldsymbol{\vartheta}} = [\mu_{\vartheta_1}, \mu_{\vartheta_2}, \dots, \mu_{\vartheta_n}] \tag{12}$$

$$COV(\boldsymbol{\vartheta}) = \begin{bmatrix} \sigma^2_{\vartheta_1} & \sigma_{\vartheta_1,\vartheta_2} & \cdots & \sigma_{\vartheta_1,\vartheta_n} \\ \sigma_{\vartheta_2,\vartheta_1} & \sigma^2_{\vartheta_2} & & \vdots \\ \vdots & & \ddots & \sigma_{\vartheta_{n-1},\vartheta_n} \\ \sigma_{\vartheta_n,\vartheta_1} & \dots & \sigma_{\vartheta_n,\vartheta_{n-1}} & \sigma^2_{\vartheta_n} \end{bmatrix} \tag{13}$$

The pseudo-code of PF algorithm with the stochastic definition of the model parameter is the following:

1. Starting from step 0 (assuming known the quantities $\mathbf{x}_{-1}$):
   - Initialize $\mathrm{N_s}$ particles $\mathbf{x}_{\mathrm{i},0} \sim \mathrm{IDF}\left(\mathrm{E}(\mathbf{x}_{\mathrm{i},0}), \mathrm{V}(\mathbf{x}_{\mathrm{i},0})\right)$ and their weights $\mathrm{w}_{\mathrm{i},0} = \frac{1}{\mathrm{N_s}} \;\forall\, \mathrm{i} = 1 : \mathrm{N_s}$
   - Draw $\mathrm{N_s}$ samples from multivariate PDF of parameters $\boldsymbol{\vartheta}_\mathrm{i} \sim \mathrm{MVPDF}(\mu_{\boldsymbol{\vartheta}}, \mathrm{COV}(\boldsymbol{\vartheta}))$
   - Assign at each particle one parameter sample $\mathbf{x}_{\mathrm{i},0} \rightarrow \mathbf{x}_{\mathrm{i},0}(\boldsymbol{\vartheta}_\mathrm{i})$ and the weights $\mathrm{w}_{\mathrm{i},0} = \mathrm{p}(\boldsymbol{\vartheta}_\mathrm{i})$, $\forall\, \mathrm{i} = 1 : \mathrm{N_s}$
2. Run PF algorithm:

- Sequential estimation of $\mathrm{p}(\mathbf{x}_\mathrm{k}|\mathbf{z}_{0:\mathrm{k}})$ according to normalized weights (equation 7, 8 and 9).

3. If resampling is required at general k^{th} step:
   - $\mathrm{N_s}$ particle resampling $\mathbf{x}_{\mathrm{i,k}}$ according to some resampling method
   - Re-draw $\mathrm{N_s}$ samples from multivariate PDF of parameters $\boldsymbol{\vartheta}_\mathrm{i}\sim\mathrm{MVPDF}\left(\mu_{\boldsymbol{\vartheta}}, \mathrm{COV}(\boldsymbol{\vartheta})\right)$
   - Re-assign parameter samples at each particle $\mathbf{x}_{\mathrm{i},0} \rightarrow \mathbf{x}_{\mathrm{i},0}(\boldsymbol{\vartheta}_\mathrm{i})$ and the weights $\mathrm{w}_{\mathrm{i},0} = \mathrm{p}(\boldsymbol{\vartheta}_\mathrm{i})$, $\forall\, \mathrm{i} = 1 : \mathrm{N_s}$
   - Go to step 2.

Since the parameter $\boldsymbol{\vartheta}_i$ remains the same along the particle life and the IDF and the Transition PDF remain the same during the SIS procedure, the hypotheses of Markov processes are satisfied.

4. Application to a simulated fatigue crack growth problem

Consider a simulated aluminium 2024 plate, within a 1.8 mm centre-crack inside, subjected to constant amplitude load. The algorithm is used to estimate the crack length starting from sequential noisy observation and to estimate the Residual Useful Life (RUL) of the plate, considering a failure crack length of 140 mm. The DSS model of Fatigue Crack Growth (FCG) (14) is built through the crack growth velocity expressed by NASGRO equation (15). The authors omit other information about Fracture Mechanics and FCG for concision. See Broek (1989), Giglio and Manes (2008) Giglio et al. (2010), and NASA J.S. centre (2002) for additional material.

$$a_k = a_{k-1} + 2\Delta N \frac{da}{dN} v_{k-1} \tag{14}$$

$$\frac{da}{dN} = C\left(\frac{1-f}{1-R}\Delta K\right)^m \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q} \tag{15}$$

Virkler et al. (1978) have proved the intrinsic uncertainties of the empirical parameters of the FCG curve due to the countless uncertainties of the real environment. Roughly speaking, this implies that a crack propagation can observe a FCG equation in which the parameters are slightly different with respect to the average values. Then, it is possible to build a DSS with stochastic definition of empirical parameters. In this work, the parameters C, m and $\Delta K_{th,0}$ are statistically defined, while the other ones are taken fixed. Nonetheless, many complications can be added inside the model. The distributions of C and m are defined according to Virkler's data (Virkler et al., 1978), keeping constant the coefficient of variation $cv = \sigma/\mu$. The uncertainty of $\Delta K_{th,0}$ is extrapolated by a simple analysis of the uncertainty of the ΔK_{th} at $R = -1$ noticed by Holper et al., (2003) and directly applied to ΔK_{th} at $R = 0$. Virkler has reported $\log(C)$ and m as normally distributed. Whereas Beretta and Villa (2008) demonstrate the correlation between $\Delta K_{th,0}$ and the couple of parameters C, m can be neglected without loss in prediction performance. Hence the vector parameter $\boldsymbol{\vartheta}$ assumes the form $\boldsymbol{\vartheta} = \left[\log(C),\ m,\ \Delta K_{th,0}\right]$ with $\mu_{\boldsymbol{\vartheta}} = [-26.7631,\ 3.2,\ 3.2571]$ according to NASGRO software, and covariance matrix expressed in Eq (16) (still observing Virkler's data). The noise v_k is defined as a log-Normal variable $v_k = e^{\omega_k}$ since the crack length can only increase over time (so ω_k is normally distributed). According to the definition of the noise in the section 3, the expected value of the crack length has to be equal to the value of the deterministic equation; with this mandatory recommendation, the expected value and the variance of the variable ω_k have to observe the relation in Eq (17). The mean and the variance of ω_k are set equal to -0.25 and 0.5, respectively. The measurement system is simulated with an uncertainty of $\pm 1\ mm$ on the real crack length, while the measures are provided to the algorithm every 2000 load cycles.

$$COV(\boldsymbol{\vartheta}) = \begin{bmatrix} \sigma^2_{\log(C)} & \sigma_{\log(C),m} & 0 \\ \sigma_{m,\log(C)} & \sigma^2_m & 0 \\ 0 & 0 & \sigma^2_{\Delta K_{th,0}} \end{bmatrix} = \begin{bmatrix} 0.9966 & -0.1853 & 0 \\ -0.1853 & 0.0346 & 0 \\ 0 & 0 & 0.658 \end{bmatrix} \tag{16}$$

$$\mu_{\omega_k} = -\frac{\sigma^2_{\omega_k}}{2} \tag{17}$$

4.1 Results

The algorithm starts with 1.8 mm crack length and it stops when the crack reaches 70 mm. The simulated crack is built with a couple of parameters arbitrary chosen inside the parameter bivariate PDF ($C = 4.2884e-12, m = 3.1072$). It is purposely chosen different from the average parameters ($C = 2.383e-12, m = 3.2$) to underline the performance of the algorithm. This case refers to resampling under the threshold (effective particles less than 60 % of the total ones). Figure 1 shows the difference between the prediction of the state evolution of Standard PF (cases (a) and (c)) with respect to the Particle Filter with SDSS (cases (b) and (d)). It is clear the SDSS produces an enlargement of the possible evolution of the

crack length because of the adoption of stochastic parameters. The performances about the conditioned PDF $p(a_k|z_{0:k})$ are the same for the two techniques because the defined noise v_k is enough to account for all the possible values a_k between two different time step $k-1$ and k. Then, the definition of the noise is very important to avoid markedly imprecisions about the state estimation. The expected value of the RUL is almost the same with the two different algorithms, as visible in Figure 2. This correspondence of the average RUL with deterministic and Stochastic DSS is due to the estimation of the system state a_k, similar for both DSS models. So the only advantage of the SDSS is to account for the parameter uncertainties describing the phenomenon. Figure 2 emphasizes this particular consideration based on the RUL graph. As a matter of fact, the PF with SDSS includes the target RUL inside its σ-band in almost all the PF operation. It hardly ever happens in the standard PF.

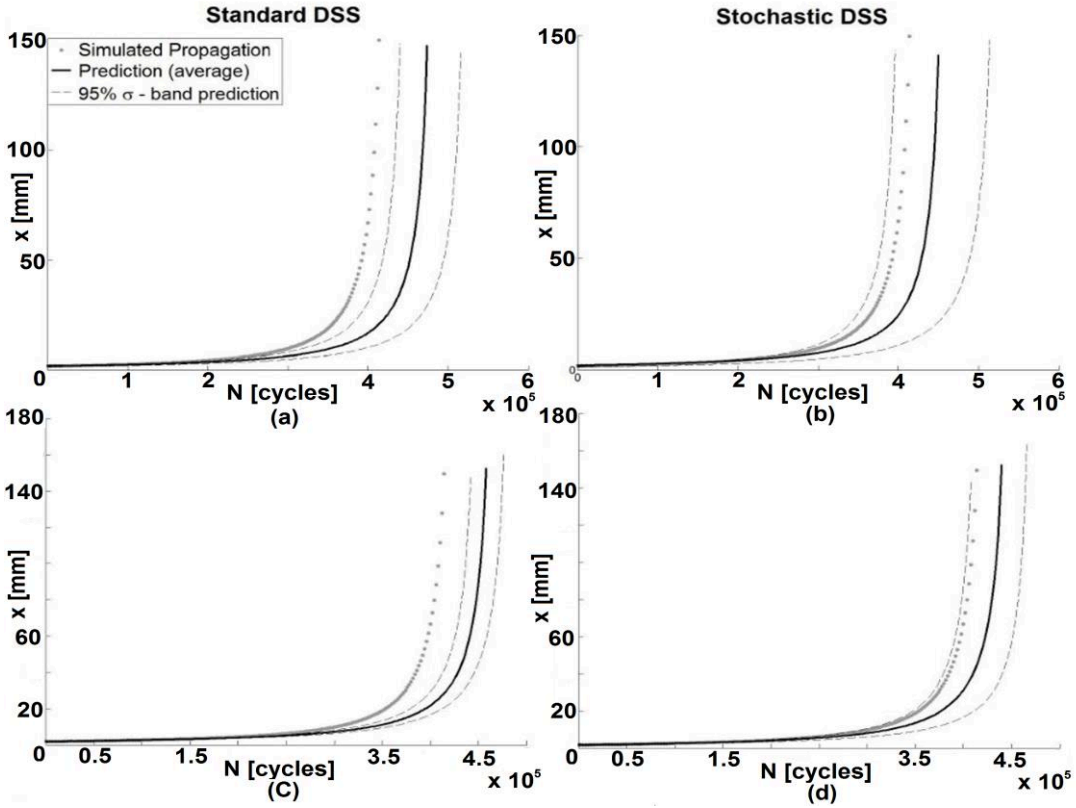

Figure 1: Comparison of the PF algorithm with standard DSS (a,c) and Stochastic DSS (b,d). The wider σ-band of the SDSS is clearly visible after 10 measures (b) and after 50 measures (d)

5. Conclusions

A methodology for Stochastic DSS modelling aimed at Bayesian Filtering has been proposed here. This technique slightly improves the prediction performance of the algorithm with an enlargement of the failure PDF caused by a wider range of possible crack trajectories. The methodology can be expanded to other problems, provided that the Hypothesis of Markov Processes is observed and the parameter uncertainties of a particular phenomenon are known. Additionally, this method can be very useful for lifetime evaluation of real components subjected to crack propagation, Giglio and Manes (2006 for crack propagation on aluminium nomex panel, 2008 for fatigue crack propagation on helicopter panel, and 2011 for fatigue crack propagation after ballistic damage), Colombo et al. (2007) and Viganò et al. (2012) for crack propagation on helicopter components. Moreover, it can be inserted inside advanced maintenance framework such as Condition Based Maintenance (CBM) or Predictive Maintenance (PM). In point of fact, markedly gaps between real crack growths with respect to the simulations are possible (Sbarufatti et al., 2012) and in a real-time maintenance perspective, all the possible trajectories of the crack evolution have to accounted for. For example, this method can be put inside On-line Structural Health Monitoring systems for aeronautical structures (Sbarufatti et al., 2012). A very interesting improvement of this methodology is the possibility to update the parameter PDFs according to the measurements z_k in time in order to reduce the prior uncertainty and centre the DSS model on the real (observed) evolution of the damage.

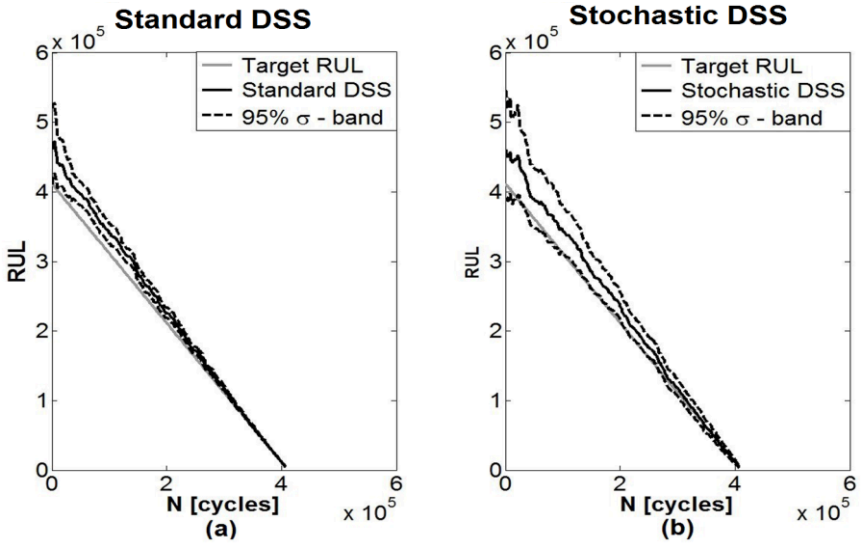

Figure 2: RUL evolution (grey line) of the simulated propagation and related estimation of PF (black lines) in the standard DSS case (a) and in the Stochastic DSS case (b)

References

Arulampalam, M. S., Maskell, S., Gordon, N. Clapp, T., 2002, A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking, IEEE Transactions on signal processing, 50, No. 2.

Beretta, S., Villa, A., 2008, A RV approach for the analysis of Fatigue Crack Growth with NASGRO equation, ASRANET Maritime Conferences, Athens, Greece.

Broek, D., 1988, The practical Use of Fracture Mechanics, Kluwer Academic Publishers, Dordrecht, Netherlands.

Cadini, F., Zio, E., Avram, D., 2009, Monte Carlo-based filtering for fatigue crack growth estimation, Probabilistic Engineering Mechanics, 24, 367 – 373.

Colombo D., Giglio M., Manes A., 2007, 3D fatigue crack propagation analysis of a helicopter component, International Journal of Materials & Product Technology, 30(1/2/3), 107-123.

Giglio M., Manes A., Fossati M., Mariani U., Giani C., 2010, Comparison of fatigue crack propagation behaviour of Al 2024 and Al - Li 8090 Helicopter fuselage panels, Journal of Testing and evaluation, 38, No 1, 73 – 82.

Giglio M., Manes A., 2006, Crack propagation on aluminum nomex panels: experimental and numerical approach, The Arabian Journal for Science and Engineering, 31 2C, 1 – 16.

Giglio M., Manes A., 2008, Crack propagation on helicopter panel: Experimental test and analysis, Engineering Fracture Mechanics, 75, 866 – 879.

Giglio M., Manes A., 2011, Terminal ballistic effect on the crack growth assessment of a helicopter rotor drive, Engineering Fracture Mechanics, 78(8), 1542 – 1554.

Haug, A.J., 2005, A Tutorial on Bayesian Estimation and Tracking Techniques Applicable to Nonlinear and Non-Gaussian Processes, MITRE Technical Report, MTR 05W0000004, MITRE Corporate Headquarters McLean, Virginia, USA.

Holper, B., Mayer, H., Vasudevan, A. K., Stanzl-Tschegg, S. E., 2003, Near threshold fatigue crack growth in aluminum alloys at low and ultrasonic frequency: Influences of specimen thickness, strain rate, slip behaviour and air humidity, International Journal of Fatigue, 25, 397 – 411.

NASA J. S. Center and Southwest Research Insitute, 2002, NASGRO Reference Manual, Version 4.02. SwRI®, San Antonio, Texas, USA.

Sbarufatti, C., Corbetta, M., Manes, A., Giglio, M., 2012, Finite Element based Bayesian Particle Filtering for the estimation of crack damage evolution on metallic panels, European Conference of Prognostic and Health Management, Dresden, Germany.

Viganò F., Manes A., Giglio M., 2012, Numerical simulation of the slant fracture of a helicopter's rotor hub with a ductile damage failure criteria, Fatigue and Fracture of Engineering Materials and Structures, 35, Issue 4, 317 – 327.

Virkler, D.A., Hillberry, B. M., Goel, P. K., 1978, The statistical nature of fatigue crack propagation, Technical Report Air Force Flight Dynamics Laboratory, TR-78-43.

Zio, E., Peloni, G., 2011, Particle Filtering prognostic estimation of the remaining useful life of nonlinear components, Reliability Engineering and System Safety 96, 403 – 409.