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Solution Strategies for the Design and Planning of Supply Chains and Embedded Plants

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In this work we present solution strategies for the task of designing supply chains with the explicit consideration of the detailed plant performance of the embedded facilities. Taking as a basis an mixed integer linear programming (MILP) model introduced in a previous work, we propose three solution strategies that exploit the underlying mathematical structure. The strategies are based on bi-level decomposition algorithm and Lagrangean decomposition method. Also, we propose a hybrid approach that takes advantage of both well known strategies, improving the solution obtained by Lagrangean methods. Numerical results show that the bi-level approach works more efficiently than Lagrangean and hybrid decomposition, and lead to significant CPU savings when compared to the full space MILP for large scale problems.

1. Introduction

Decisions made in supply chain management (SCM) have been traditionally divided in three basic levels according to their temporal and spatial scale: strategic, tactical and operational. Several authors have recognized the importance of integrating decision levels in SCM as an effective manner to increase the overall profit, but very few contributions have been made in this field.

In the last years, however, there have been some attempts to combine decisions in supply chain (SC) optimization. In particular, Corsano and Montagna (2011) presented a mixed integer linear programming (MILP) model for the simultaneous optimization of SC and plant design. In that work, decisions regarding the SC network optimization, such as nodes selection and materials distribution, are together considered with multiproduct batch plant design decisions in order to attain a more integrated perspective of the SC design problem. However, the integration of decision making levels in SCM further increases the complexity of the modelling approach, as additional variables and constraints need to be defined to represent decisions of different nature within a single model. Hence, one of the main challenges of integrating decisions in SCM is the efficient solution of large-scale problems. It is therefore not surprising that the majority of published works dealing with decision levels integration in SC optimization have resorted to decomposition methodologies for obtaining optimal or near optimal solutions in short CPU times.

Van den Heever et al. (2001) proposed a specialized heuristic algorithm based on the concept of Lagrangean decomposition for the long-term design and planning of offshore hydrocarbon field infrastructures with complex economic objectives. Jackson and Grossmann (2003) presented two different decomposition schemes to solve a multisite production planning and distribution model that were based on Lagrangean decomposition. You and Grossmann (2008) proposed a decomposition algorithm based on Lagrangean relaxation for solving the integrated stochastic inventory management and supply chain network design.

Bi-level decomposition was also applied to large-scale optimization problems related to integrated SC decisions. Iyer and Grossmann (1998) solved a MILP model for determining the optimal selection and expansion of processes over a long-range planning horizon using a rigorous bi-level decomposition algorithm. Guillén-Gosálbez et al. (2010) addressed the design of hydrogen supply chains for vehicle and proposed a bi-level algorithm to expedite the search for the optimal Pareto solutions.

Despite these algorithmic developments, the use of decomposition strategies in the integrated design of SCs along with their embedded facilities has been quite scarce. Because of this, current full space approaches can only tackle problems that consider only a limited number of plants, depots and clients.

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In this work we fill this research gap by presenting several customized decomposition algorithms for this problem. The MILP model for the design of SCs presented in Corsano and Montagna (2011) that includes equations to model the performance of the batch plants of the network is taken as a basis to develop our algorithmic framework. Such a spatial integration of decision-making levels leads to a complex formulation that is hard to solve in reasonable computational time. This MILP becomes even more complex as the number of plants increases, mainly because of the presence of complicating constraints that are required to model the plant performance precisely. Hence, the main contribution of this work is the development of three tailored algorithms inspired on bi-level and Lagrangean decomposition schemes that exploit the problem structure, making it possible to tackle large-scale problems encountered in practice. Numerical results show that our approaches outperform standard branch and cut codes applied to the full space MILP.

2. Problem statement

In this work, we consider a generic SC like the one shown in Figure 1.

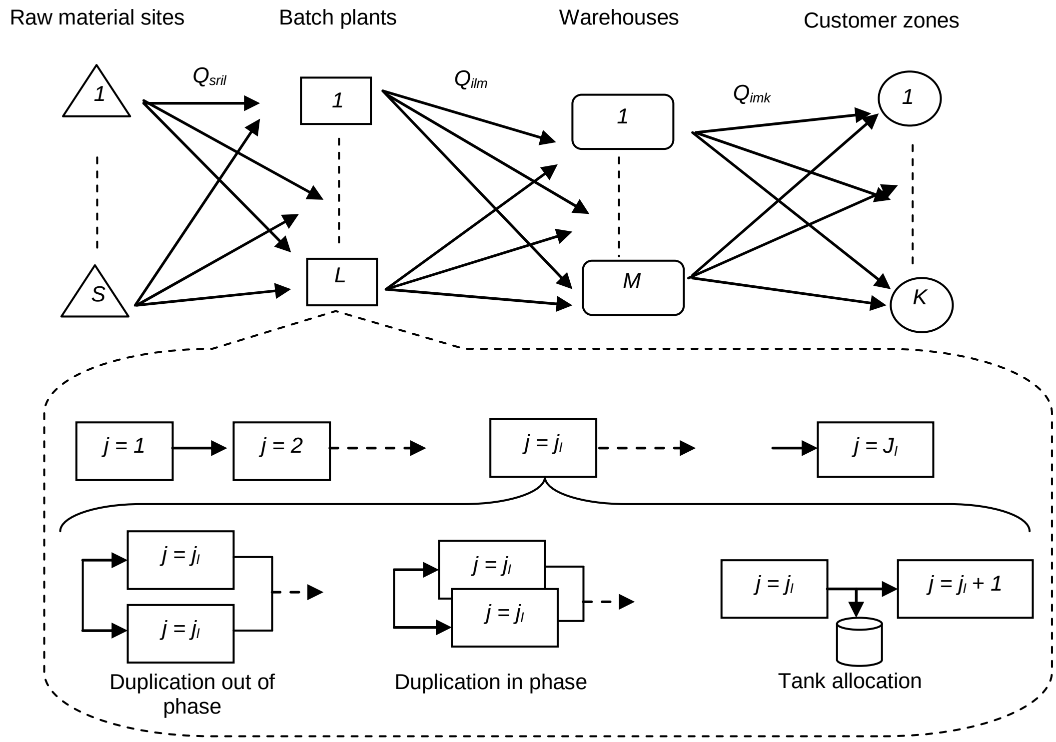

Figure 1: Integrated SC and multiproduct batch plant design

Based on this representation, we formally state next the problem of interest. Given is a set of raw materials sites $s \in S$. Each raw material site *s*, has one or more types of raw materials *r*, $r \in R$, with limited capacity Q_{sr}^{UP}, to be delivered to plants $l \in L$, which operate during time horizon H_l. Each multiproduct plant has a set of batch stages $j \in J_l$, for producing a set of products $i \in I$.

For each multiproduct batch plant, we consider in phase and out of phase unit duplication. The allocation of intermediate storage tanks between two batch stages is also considered. These can be allocated in $|J_l| - 1$ positions in plant *l*, where position *j* is defined between batch stages *j* and *j* + 1. A zero-wait (ZW) transfer policy is adopted between consecutive batch stages.

According to the usual unit procurement policy, a set SV_{jl} of P_{jl} discrete unit sizes, $SV_{jl} = \{VF_{jl1}, VF_{jl2}, ..., VF_{jlP_{jl}}\}$ is available for stage *j* in plant *l*. Similarly, a set of G_{jl} discrete sizes for storage tanks $STF_{jl} = \{VTF_{jl1}, VTF_{jl2}, ..., VTF_{jlG_{jl}}\}$ is available for position *j* in plant *l*.

Final products are delivered from plant *l* to several warehouses $m \in M$, each of them with a limited capacity Q_m^{UP}. Products are then transported from warehouse *m* to different customer zones $k \in K$, in order to satisfy a known product demand D_{ik}. Assuming that cost parameters associated to plants and warehouses installation, investment, production, distribution, raw materials and operation are known, the problem consists of determining simultaneously:

- The SC topology (nodes allocation).
- The SC planning (production rates and flows among nodes).
- The multiproduct batch plan design (plant configuration and unit sizes).

in order to fulfill the product demands with minimum cost. The cost function considers installation, investment, production, operation, and transportation costs.

3. Mathematical model

In this work the decomposition strategies are developed taking as a basis the MILP formulation addressed by Corsano and Montagna (2011), which is not presented here for space reasons. This model represents the full space problem and it will be referred as (FP) in the rest of the paper.

4. Solution strategies

4.1. Bi-level decomposition

Bi-level algorithms decompose the original full space model into two subproblems at different hierarchical levels between which they iterate until a termination criterion is satisfied. Our proposed bi-level algorithm solves a lower bounding master problem (LBP), which is a relaxation of the full space problem (FP), to obtain a lower bound on the cost. This relaxation is constructed by dropping the integrality requirement of the binary variables that model the selection of products to be produced in each plant and the selection of unit and storage tank sizes. Because of these modifications, problem (LBP) is less complex in combinatorial terms than the original model (FP). The master problem provides as output a SC configuration that is optimal in the relaxed search space, but not necessarily in the full space model. Hence, the configuration obtained from (LBP), i.e. allocated plants and warehouses, is fixed in the upper bounding problem (UBP), to obtain an upper bound on the total cost of the network and determine at the same time the values of those variables that were relaxed in (LBP). As only a subset of plants and warehouses is selected for solving (UBP), this model contains fewer integer variables and is not as combinatorially complex as problem (FP).

Problems (LBP) and (UBP) are then solved iteratively until a termination criterion is reached. In every iteration a new integer cut is added to (LBP) in order to exclude from the search space SC configurations already explored in previous iterations. As iterations proceed, the difference between the best lower and upper bounds (i.e., optimality gap) decreases. Two termination criteria that tend to work well in practice are to stop either when the difference between the lower and upper bounds falls below a desired tolerance or when a maximum number of iterations is reached.

4.2. Lagrangean decomposition

This method relies on constructing a relaxation of (FP) (i.e., Lagrangean dual problem), obtained by dualizing (dropping) some "complicating" constraints of the model. There are different ways to construct the relaxed problem according to the type of constraints that are dropped. In the context of SCM, the two main approaches are spatial and temporal decomposition. In this work, spatial decomposition is applied.

Specifically, the mass balances between production plants and warehouses are dualized and the relaxed problem is then decomposed into two subproblems: (P1) for plants and (P2) for warehouses.

After solving the Lagrangean dual for some values of the Lagrangean multipliers, we obtain a lower bound on the total cost. We then need to generate an upper bound and a feasible solution to the original problem. To this end, we fix some variables of the original model according to the output of the dual model and then solve it in a reduced domain so as to yield a valid upper bound.

After obtaining an upper bound, we can generate new values for the Lagrangean multipliers and repeat the overall procedure until a termination criterion is satisfied.

4.3. Hybrid strategy

A third strategy is proposed that combines the basic ideas of both Lagrangean and bi-level decomposition algorithms. The number of binary variables of (P1) solved at each iteration of the Lagrangean decomposition is similar to the number of binary variables of (FP), and so is the computational burden. To expedite the solution of (P1), we propose the following modifications:

- Subproblem (P1) is solved relaxing some of its binary variables, namely those that model the size of the units and tanks. This leads to lower computational burdens.
- The following integer cuts are added to (P1) in each iteration of the algorithm in order to avoid repeated solutions:

$$\sum_{l \in WL_1^{iter}} ex_l - \sum_{l \in WL_0^{iter}} ex_l \leq \left| WL_1^{iter} \right| - 1 \quad (1)$$

where ex_l represents the binary variable for plant selection and:

$WL_1^{iter} = \{ l : ex_l = 1 \text{ in the optimal solution of } (\text{P1}) \text{ at iteration } iter \}$

$WL_0^{iter} = \{ l : ex_l = 0 \text{ in the optimal solution of } (\text{P1}) \text{ at iteration } iter \}$.

Since the number of binary variables is reduced in subproblem (P1), the computational burden is lowered. The production plants selected by (P1) are then fixed in the reduced model (FP). In order to avoid solutions explored in previous iterations, we add integer cuts to (P1), so it is guaranteed that the set of installed plants to be fixed in the reduced (FP) will be different at each iteration.

Table 1: Model characteristics of different approaches in each example

		(FP)	Bi-level		Lagrangean			Hybrid		
			(LBP)	(UBP)	(P1)	(P2)	Reduced FP	(P1)	(P2)	Reduced FP
Example 1	bin. var.	223	68	115	220	3	218	65	3	218
	cont. var.	2,082	2,082	2,081	2,039	104	2,142	2,039	104	2,142
	constr.	2,144	2,484	2,144	2,114	35	2,204	2,114	35	2,204
Example 2	bin. var.	450	140	430	440	10	440	130	10	440
	cont. var.	4,488	4,488	4,488	4,354	535	4,888	4,354	535	4,888
	constr.	4,286	4,287	4,286	4,214	77	4,686	4,214	77	4,686
Example 3	bin. var.	470	140	450	460	10	460	130	10	460
	cont. var.	6,608	6,608	6,608	5,994	1,215	7,208	5,994	1,215	7,208
	constr.	6,054	6,054	6,054	5,916	145	6,654	5,916	145	6,654
Example 4	bin. var.	700	205	675	690	10	685	195	10	685
	cont. var.	9,603	9,603	9,603	8,989	1,515	10,503	8,989	1,515	10,503
	constr.	9,004	9,005	9,004	8,866	145	9,904	8,866	145	9,904
Example 5	bin. var.	450	140	430	440	10	440	130	10	440
	cont. var.	4,768	4,768	4,768	4,354	815	5,168	4,354	815	5,168
	const.	4,314	4,315	4,314	4,214	105	4,714	4,214	105	4,714

5. Numerical results

We solved five numerical examples to evaluate the performance of the proposed decomposition algorithms. Table 1 presents the model characteristics while Table 2 shows the computational performances for the different approaches in each example. All the examples were implemented and solved in GAMS (Rosenthal, 2008) on an Intel Core i5, 2.3 GHz. The CPLEX 12.1.0 solver was employed for solving the MILP problems. The number of continuous and binary variables and constraints strongly depends on the number of plants to be installed, the number of products to be produced, and the number of discrete options considered for the batch units and storage tanks sizes. Note that the computational complexity of the problem, and consequently the computational burden, grows with the number of binary variables. Moreover, due to some trade-offs involved in the decision-making problem, the model performance varies according to the problem data, as shown in Example 5. In all of the examples, the tolerance error (difference between bounds) for the bi-level, Lagrangean and hybrid algorithms was set to 1 %. An optimality gap of 1% was also defined for CPLEX when solving (FP). The maximum number of iterations for decomposition algorithms was set to 20 and the resolution time limit equal to 14,400 CPU s (4 h).

The first example is taken from Corsano and Montagna (2011). It considers 2 raw material sites that provide 3 different raw materials to 5 potential production plants. These plants produce 4 products through 3 batch stages. For each batch stage, we consider a set of 5 discrete sizes, and the option of duplication of up to 2 units in phase or out of phase is allowed. Also, 3 different tank sizes are available. There are 3 types of warehouses and 3 customer zones.

Table 2: Computational performances for the different approaches in each example

	FP			Bi-level decomposition				Lagrangean spatial decomposition				Hybrid decomposition strategy			
	solution	gap	time (s.)	solution	gap	time (s.)	iter.	solution	gap	time (s.)	iter.	solution	gap	time (s.)	iter.
Example 1	UB=1,019,543.2	0%	79.8	UB=1,019,543.2	0.7%	54.4	5	UB=1,019,543.2	0%	517.1	4	UB=1,019,543.2	1%	31.9	3
	LB=1,019,543.2			LB=1,012,543.6				LB=1,019,543.2				LB=1,030,440.1			
Example 2	UB=1,014,643.2	0.99%	14,244.4	UB=1,014,643.2	0.9%	185.7	2	UB=1,014,643.2	4.3%	14,400*	6	UB=1,014,643.2	0.8%	775.71	8
	LB=1,004,497.3			LB=1,005,993.6				LB=970,718.7				LB=1,006,735.8			
Example 3	UB=1,592,852.4	9.68%	14,400*	UB=1,569,624.6	0.9%	1839	9	UB=1,599,279.2	13.1%	14,400*	4	UB=1,628,520.3	12.1%	14,400*	15
	LB=1,438,601.5			LB=1,583,033.7				LB=1,389,259.6				LB=1,430,749.3			
Example 4	UB=1,580,595	20.64%	14,400*	UB=1,569,624.6	0.4%	9,022	7	UB=1,589,898.6	12%	14,400*	4	UB=1,590,001.1	16.3%	14,400*	9
	LB=1,254,313.6			LB=1,576,071.8				LB=1,398,354.2				LB=1,331,471.6			
Example 5	UB=3,299,862.7	24.22%	14,400*	UB=3,123,332.7	0.77%	11,588	3	UB=3,380,358.6	31%	14,400*	2	UB=3,324,447.2	21.8%	14,400*	6
	LB=2,500,626.6			LB=3,147,634.7				LB=2,332,012.9				LB=2,599,881.3			

* time limit

The second example increases to 10 the number of production plants and 10 warehouses. The number of customer zones and associated demand remain the same as the previous case. The aim here is to test the model performance for a large-size problem.

The third and the fourth examples are also large-size problems and they are presented in order to evaluate the strategies performance. Instance 3 considers the possibility of producing 6 products in each plant and ten customer zones, while instance 4 increases the number of production plants to 15.

Finally, to construct the fifth example we have modified some model parameters values from the second example in order to show how the problem data affect the computational burden due to the existence of inherent tradeoffs.

It is worth to mention that in some cases the lower bound exceeds the upper bound (bi-level decomposition algorithm in Example 3, 4 and 5 hybrid strategy in Example 1). This is due to the use of integer cuts. That means that the best solution identified so far is in turn the global optimum, since it is impossible to obtain any other solution with a better objective function value.

6. Concluding remarks

Taking as a basis an MILP that calculates decisions pertaining to different hierarchical levels in SCM (i.e., single site and multi-site design tasks), we have developed three decomposition algorithms that exploit the problem structure. Several case studies have been solved to test the capabilities of these numerical methods. From these examples, it has been shown that the bi-level decomposition scheme performs better in terms of quality of the final solution produced and time spent in its generation. This finding is consistent with other results published in the recent literature (You et al., 2011).

Lagrangean decomposition is not so good in this type of problem where after applying the spatial decomposition, the subproblems still have many binary variables and several tradeoffs among decisions (product selection, plant configuration, unit sizing, etc.). On the other hand, the hybrid method reduces the number of binary variables in the subproblems, but the lower bounds might be less than the lower bounds provided by Lagrangean method, and consequently the length of the optimality gap is worsened. Future work will extend our approach to tackle other similar problems that integrate several decision-making levels.

References

Corsano G., Montagna J.M., 2011, Mathematical modeling for simultaneous design of plant and supply chain in batch process industry, Computers and Chemical Engineering, 35 (1), 149-164. DOI: 10.1016/j.compchemeng.2010.06.008.

Guillén-Gosálbez G., Mele F., Grossmann I. E., 2010, A bi-criterion optimization approach for the design and planning of hydrogen supply chains for vehicle use, AIChE Journal, 56 (3), 650-667. DOI: 10.1002/aic.12024

Iyer R. R., Grossmann I. E., 1998, A bilevel decomposition algorithm for long-range planning of process networks, Ind. Eng. Chem. Res., 37, 474–481. DOI: 10.1021/ie970383i.

Jackson J. R., Grossmann I. E., 2003, Temporal decomposition scheme for nonlinear multisite production planning and distribution models, Ind. Eng. Chem. Res., 42(13), 3045–3055.

Rosenthal R. E., 2008, GAMS: A User’s Manual; GAMS Development Corporation: Washington, DC.

van den Heever S. A., Grossmann I. E., Vasantharajan S. A., 2001, Lagrangean decomposition heuristic for the design and planning of offshore hydrocarbon field infrastructures with complex economic objectives, Ind. Eng. Chem. Res., 40 (13), 2857–2875. DOI: 10.1021/ie000755e

You F., Grossmann I.E., 2008, Mixed-Integer Nonlinear Programming Models and Algorithms for Large-Scale Supply Chain Design with Stochastic Inventory Management, Ind. Eng. Chem. Res., 47, 7802-7817. DOI: 10.1021/ie800257x.

You F., Grossmann I.E., Wassick J.M., 2011, Multisite Capacity, Production, and Distribution Planning with Reactor Modifications: MILP Model, Bilevel Decomposition Algorithm versus Lagrangean Decomposition Scheme, Ind. Eng. Chem. Res., 50, 4831–4849. DOI: 10.1021/ie100559y.