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Assessing the Robust Stability and Robust Performance by Classical Statistical Concepts

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The robustness of a control system is concerned with the ability of such a system to keep the characteristics of stability and performance when subjected to disturbances and uncertainties in the parameters, or in plants with unknown dynamics. The analysis of robustness enables an assessment to be made of whether the desired performance of the control system is maintained even when there are changes in the process, thus making such systems insensitive to such changes, and consequently, allowing robust control systems to be designed which may be able to meet the requirements of the project.
Successful analysis of a control system, particularly the analysis of stability, may involve both deterministic and statistical treatments. However, if deterministic techniques are used as if the systems were deterministic, such an approach can have severe implications and lead to misleading results or at least, yield substantially conservative results. While the approach with statistical techniques may not always reduce the uncertainties, this can, however, lead to more precise statements thus enabling better decisions to be made. Since the system under study is considered stochastic by nature, this paper is devoted to assessing the robustness of stability by statistical methods whereby the confidence region for each root of the characteristic equation can be established. Such a technique considers as a metric the statistical distance which is associated with the chi-square distribution, thus permitting the quadratic form to be established, as well the contour of the resulting ellipse, which thus reveals the robustness sought.

1. Robustness region inspired by statistical principles

Despite the high volume of articles in the literature on robustness of stability, the presence of uncertainties in a typical control structure continues to be recognized as a challenging problem for engineers and practitioners, who try to keep the three basic characteristics of a control system, namely: observability, controllability and stability. Relatively little attention has been focused on robust methods based on a classical statistical approach (Calafiore et al., 2011), although it is acknowledged that such an approach can demand a high number of samples or the results obtained are rather complex for practical applications. Most require the determination of measures of probability and, should the Monte Carlo method be used, the accuracy of the results cannot be guaranteed.
It is well-known that in any process even those that are well designed, well conducted or carefully maintained, a certain amount of natural variability is always present, as a result of the cumulative effects of several unavoidable causes. As to the control structure in which the control strategy is based on the model, such variability has pivotal influence for finding the deterministic values of the model parameters or tuning parameters. With a view to overcoming such problems, a procedure was presented by Silva et al.(2012), which is related to establishing the joint confidence region for the parameters involved in the control system. Several proposals have appeared in the literature to deal with the subject such as the classical text book of Himmelblau (1970), the fundamentals of which were applied by Silva et al. (2012) to the conventional PID control resulting in a procedure which expresses the variability of the parameters based on the residual variance of the output signal of the controller.
There is a large spectrum of classical and modern methods for dealing with the uncertainties of models and therefore the robustness of a control system, such as the root-locus method or the singular value analysis, inter alia. However, the use of such methods imply using deterministic metrics, which are weak when the treatment of uncertainties present in the system is considered, therefore, resulting in a conservative analysis. The analysis of robustness by a structured singular value method can to some extent reduce the conservatism of the analysis, although this treatment remains deterministic.

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Furthermore, such methods are neither easy to understand nor to work with and as to their practical implementation, they can be very laborious.

2. Euclidean and statistical distance

Consider any point, P, of a plane or, in particular, of a complex plane with its real (x_1) and imaginary (x_2) coordinates. The Euclidian distance from P to origin O, given by

$$d(O,P)=\sqrt{(x_1)^2+(x_2)^2} \tag{1}$$

is seen to be inadequate for most statistical purposes, because, in most cases, the dispersions of the axial coordinates, representing variability, are not equal, as shown in Figure 1 (Johnson and Wichern, 1992).

Taking such dispersion into account, it is easy to derive, by using standardized coordinates, the so-called statistical distance as follows:

$$d(O,P)=\sqrt{\left(\frac{x_1}{\sigma_{x_1}}\right)^2+\left(\frac{x_2}{\sigma_{x_2}}\right)^2} \tag{2}$$

Clearly, Eq. (2) is a quadratic form, which represents an ellipse centered at the origin. It should also be observed that the Euclidian distance is a particular case of statistical distance when $\sigma_x = \sigma_y$.
In the case of an Ellipse, it is centered at the expected value of the random variable X, a point different from the origin, and also considering Σ as the covariance matrix, Eq.(2) can be generalized to yield:

$$d^2(O,P)=(X-\mu_X)'\sum{}^{-1}(X-\mu_X) \tag{3}$$

For a two-dimensional variable X, Eq. (3) describes the contours of a ellipse centred at $\mu(\mu_{x_1},\mu_{x_2})$, the axes of which are not, necessarily, in the same direction as the coordinate plane.

The graph of Eq. (3) for the plane is illustrated in Figure 1.

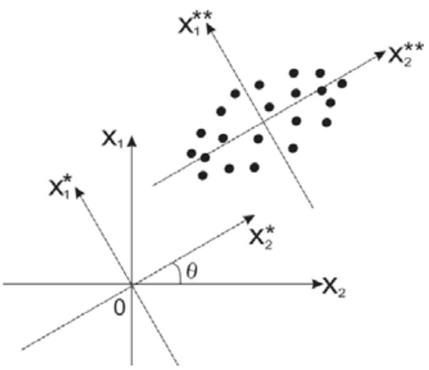

Figure 1: A cluster of points representing an ellipse.

If X_1 and X_2 are correlated which corresponds to a full variance-covariance Σ then by using the combined movements of axial rotations (X_1*, X_2*) and translations (X_1**, X_2**) resulting from a linear transformation, it is possible to suppress the cross-covariance term involved in Eq. (3). Considering also the definition of the random variable χ^2 (chi-square) (Wilson, 1931), then Eq. (3) has a χ^2 distribution with ν degrees of freedom. Hence,

$$d^2(\mu_{X*},P)=\frac{(x_1^*-\mu_{x_1^*})^2}{\sigma_{x_1^*}^2}+\frac{(x_2^*-\mu_{x_2^*})^2}{\sigma_{x_2^*}^2}\cong\chi^2_{2\,(\alpha)} \tag{4}$$

i.e., the indicative of $d^2(\mu_{X*},P)=\chi^2$ enables the contour of an ellipse to be determined that contains $(1-\alpha)100\%$ of the probability, for which α corresponds to the level of significance and $\nu = 2$.

Since X_1* and X_2* can describe the rectangular coordinates of the points representing the roots of the characteristic equation of a second-order system, which give us the first statement about the stability of the system, then the depicted ellipse is related to the robustness of the stability.

3. Robustness of stability

The robustness of stability and of performance are dealt with by a deterministic procedure (Morari and Zafiriou, 1989; Skogestad and Morari, 1987; Doyle and Stein, 1981) whereby what is evaluated, in the sense of stability and performance, is the variability imposed on the parameters without taking into account the most appropriate distribution of probability of each parameter.
The procedure developed in this article is concerned with establishing the region of stability as well as of performance considering the basic concepts of probability applied to the roots of the characteristic equation for a generalized feedback system and in particular for the PID controller. It should be emphasized that the roots are functions of the model and tuning parameters. Since such parameters can be considered random variables, then the roots of the characteristic equation can be also regarded as random variables, as in reality they are. Therefore, all the theoretical formulations set out in the previous section can be used for mapping, which means establishing a region or its contour within which all the possible values for the roots of the characteristic equation are located. Since the classical statement asserts that the system is stable if all the roots of its characteristic equation are to the left of the imaginary axis, then it can be established thereby that if the region containing all the possible values for the roots of characteristic equation is to the left of the imaginary axis, consequently the system is stable. If, at least, a small part of this region is to the right of the imaginary axis, such a condition indicates a potentially unsustainable operating condition, which requires a quick response in order to restore the stability. This region is recognized to be the stability region.
It is worth observing that in spite of the number of parameters considered as random variables which are involved in the second-order system, which represents the greatest part of the processes, the procedure always deals with two parameters that are not correlated (real and imaginary parts).

4. Robustness of performance

The evaluation of the dynamic performance of a control system can be established by considering its ability to keep the variable to be controlled at a desired value in spite of there being disturbances in the system. Hence, since the contour of the region of stability has been obtained, it is possible generate the curve of responses with respect to the points that are elements of this contour, thus recovering the tuning parameters, which can yield the curve or region of robustness of the performance. Additionally, the distribution of probability for the output of the system can also be obtained.
It should be highlighted that the recovery of the tuning parameters from the points of the contours is not an easy task due to the coupled nonlinear system resulting.
Since the response for the output variable of the system has been generated and by choosing an appropriate performance criterion based on some characteristic features of the closed-loop response, the assessment and the mapping of the robustness of the performance can be effectively carried out.

5. The case study

To illustrate the methodology formulated, a jacketed vessel heater connected to a control structure including a supervisory system was considered for analysis, as per Figure 2. In order to obtain a representation similar to an industrial plant, a virtual plant was developed by means of Object Linking and Embedding for Process Control (OPC) Elipse supervisory SCADA software, together with MATLAB, as the computational tool.

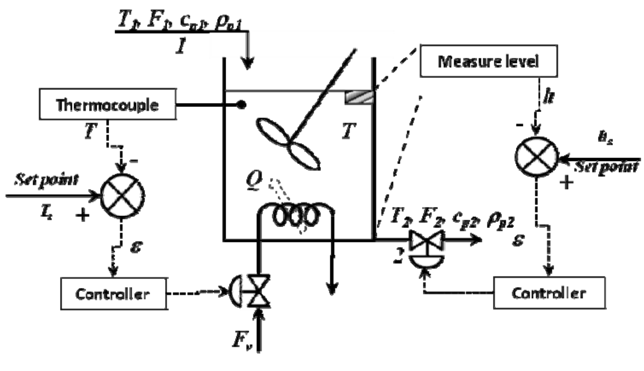

Figure 2: Representation of the diagram level and temperature control networks

The following equations (5) and (6) represent the mass and energy balances for the system where h and T are state variables denoting the level and temperature of the system, respectively. F_1 and F_v correspond to the flow rates of the input stream and the feed stream of steam.

$$\frac{dh}{dt} = \frac{\rho_1}{\rho A} F_1 - \frac{\rho_2}{\rho A} \frac{\pi D_2^2}{4} \sqrt{2g(h_L + P_m / g\rho)} \tag{5}$$

$$\frac{dT}{dt} = \frac{[F_1(T_1 - T)]}{V} + \frac{\dot{Q}}{\rho V \hat{c}_p} \tag{6}$$

Due to the need to consider a model coupled to a realistic system, Eqs.(5) and (6) will be considered for the purposes of simulating the description of the real process while the so-called model is represented by a combination of the convolution model and the autoregressive model with exogenous input expressed by:

$$y_{(t)} = \theta^T \varphi_{(t)} + v_{(t)}^* \tag{7}$$

where y(t), θ, φ, are the output of the model, the matrices of the coefficient and of the variables respectively and ν*(t) denotes the unmeasured disturbance (Silva et al., 2012).
In order to fit the model described by Eq. (7) to data provided by the process, the classical recursive least square method was required.
Figure 3 illustrates how the information flows, besides clarifying the meaning of each block.

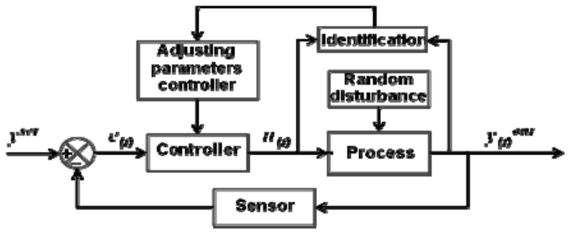

Figure 3: Diagram of the generalized structure in a closed network with blocks for the identification and for the tuning parameters

6. Results and discussion

In spite of the identification method providing a good fit, the model given by Eq.(7) depends on the order of the regressive model, given by the values of n and m, in order to choose the appropriate functional form, taking into account the one that best describes the process. After conducting a few simulations in which the process was submitted to a disturbance of 5% in the value of the set point, it can be found that if n=m=100, the functional form representing the model becomes sufficiently closed for the process, with minimum computational efforts. For convenience, it is assumed n and m have the same value
With the aim of adjusting the parameters of the PID controller on-line, an auto-tuning configuration for the control system was established, consisting of a relay, a convolution model and an algorithm for tuning parameters, in accordance with the strategy adopted by Aström and Hagglund (1995) and based on the Ziegler-Nichols method. Hence, a set for the tuning parameters was found for each sampling interval, yielding a response in close-loop with an auto-tuning structure as shown in Figure 4b while Figure4a shows the results without the procedure of auto-tuning. Despite the presence of disturbances, the results indicate a good performance of the control system while that of the response with the auto-tuning is shown to be superior.

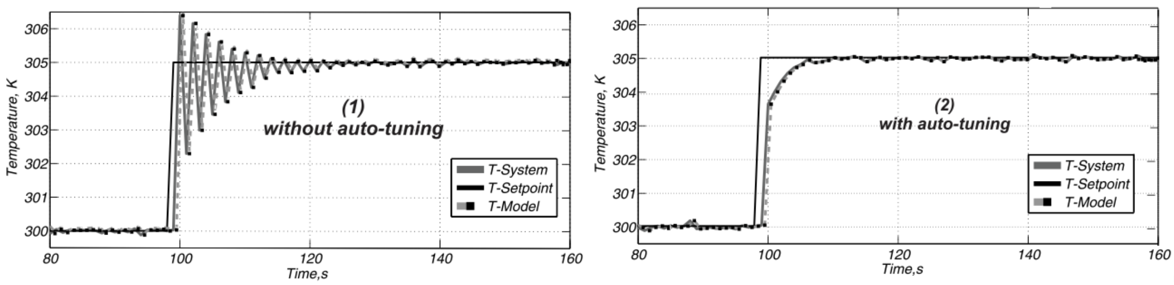

Figure 4: The behavior of the output temperature for the model and process: without auto-tuning –(1), auto-tuning – (2)

6.1 Analysis of the robust region of stability

With the auto-tuning device implemented on-line in a system in which the PID controller is used, the values of the tuning parameters were recorded and introduced in the characteristic equation resulting in a set of values for the real and imaginary parts of the complex plan that can expressed as a distribution of probability, notedly normal (Gaussian). Thus, since all the pairs of points representing the roots can be plotted on a graph, the results follow below:

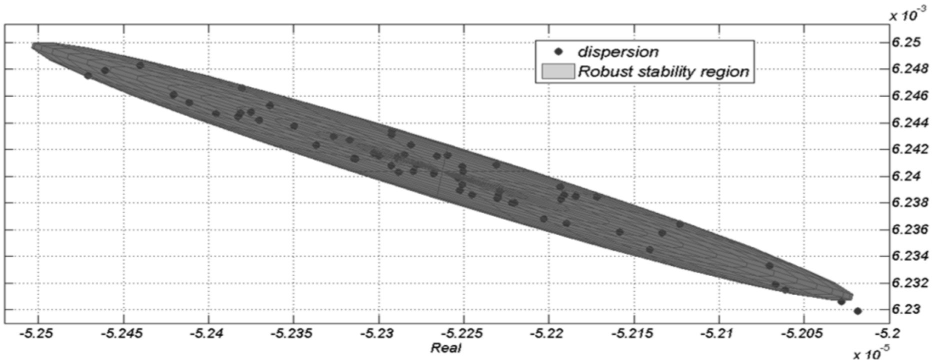

Figure 5: The robustness region for the other root of s also based on α= 5% while the system is submitted to disturbances present in the inlet variables and set point.

It can be observed in Figure 5 that the variability in each axis is not the same, indicating that the statistical distance $d(\mu_{X^*},P)$ or $d(\mu_{X^*},P)^2$ should be used for an appropriate treatment of the data, the result being an ellipse that can be easily established for a specified level of significance (α) by using Eq. (3).

It should be emphasized that due to the dynamic nature of the system, the ellipse can move in the complex plane when new pairs of points are captured by the data acquisition system, thus revealing the new contours for the stability. Furthermore, whenever for some motive, the control system for the process considered is inoperative, then, the stability of the process represented by the ellipse can be directed toward the unstable region, and even if only a small part of the robust confidence region is partially inside the unstable region, indicating a potentially unsustainable operating condition, immediate action must be taken with a view to restoring the stability conditions of the system.

6.2 Analysis of the robust range of performance

Having established the robust region of stability from the dispersion of the roots of the characteristic equation plotted on the complex plane, the strategy for obtaining the robust region of performance, or more appropriately the robust range of performance, is to recover the tuning parameters, k_c, τ_i and τ_d (Figure 6b), related to the points exclusively over the edge of the robust region (Figure 6a) and then using these parameters, to generate the output signal of the process.

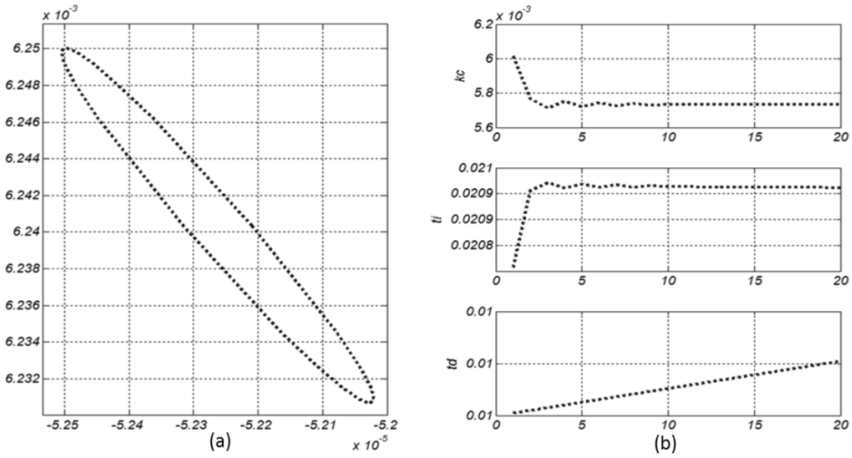

Figure 6: The contour of the robust region of stability (6a) and the tuning parameters associated with the edge of the robust region (6b).

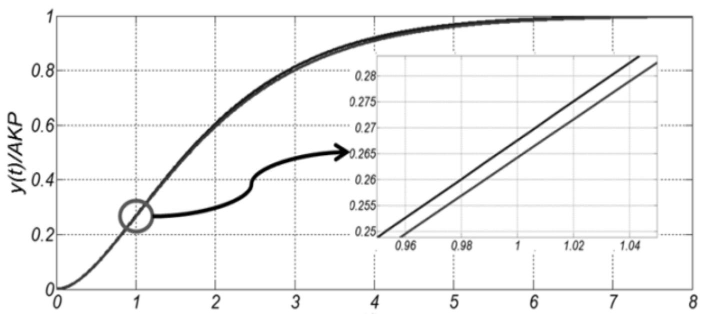

Figure 7: The closed-loop response for the process by using the tuning parameters obtained from the edge of the robust region, as well as, the robustness range of performance

Since each triad of tuning parameters associated with the edge of the robust region generates a typical process reaction curve, as well as the robust range of performance, as shown in Figure7, then in order to assess the complete closed-loop response it is possible to select some appropriate criterion of performance that satisfies the designer's objectives. Assuming, for instance, that the criterion for performance is given by the rise time, measured at the point $t/\tau_p=1$ where τ_p is the period of oscillation of the process, thus the robust range of performance is readily established as per Figure7. Evidently several other performance criteria can be simultaneously adopted and assessed, the need for which will depend on the judgment of the process engineering team.

7. Conclusion

Due to the need to capture data from simulations, a structure consisting of a block for estimating recursively the parameters and automatic tuning connected with the process was developed, in which the parameters of the process model are updated on-line. This enabled the variables to be considered as random, besides generating information on-line that will be used in determining the robust region of stability and the robust range of performance.
Hence, a structure developed from the statistical distance as a metric, in connection with the chi-square distribution resulted in a methodology that has the ability to map the contour of a region and also of a range, which can be considered as robust for a given level of significance (α).
By using the auto-tuning strategy, the values of the roots of the characteristic equation were determined and plotted on the complex plane, revealing the robust region of stability, this being easily established by the methodology developed. Furthermore, it was shown how it is possible to determine the range of robust performance associated with robust stability, taking a criterion of performance into consideration.
Finally, the methodology is innovative in purpose, easy to understand, does not require hard mathematical treatment, and can be done at low computational cost.

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