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Second-Order Perturbation Solutions to a Liquid Pool Spreading Model with Instantaneous Spill

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Second-order perturbation solutions have been obtained for the simple physical model describing the liquid pool spreading with an instantaneous spill, and are shown to improve over the first-order perturbation solutions. The second-order solutions to the non-dimensional governing equations for the model are derived to obtain more general solutions. Non-dimensional parameters are sought as the governing parameters for the non-dimensional equations, and the non-dimensional evaporation rate per unit area is used as the perturbation parameter. It is revealed that the number of governing parameters is reduced to two from three by non-dimensionalization. The second-order solutions exhibit a definite improvement over the first-order solutions with respect to the pool volume and the pool radius. The second-order solutions for the pool volume agree remarkably with the numerical solutions; however, there is still a little difference between the second-order solutions and the numerical ones for the pool radius at the final stage of spread.

1. Introduction

The study of liquid pool spreading plays an essential role in the quantitative risk assessment of accidentally released cryogenic liquids, such as LNG and liquefied hydrogen because the spreading of such liquids is the first step in the development of multi-staged accident sequences leading to a major disaster. Generally, leak is placed on the top of the Fault Tree Analysis, which means the final step of hazard identification or qualitative risk analysis. Simultaneously, the leak is an initiating event of the Event Tree Analysis, which is the beginning of accident because spilled flammable liquid can cause fires and explosions. It is, therefore, essential to study liquid pool spreading for the estimation of fire and explosion risks.

A number of numerical simulations have been performed for certain model equations governing the pool spread. Various types of governing equations, ranging from a simple physical model to the full Navier-Stokes equation (Venetsanos and Bartzis, 2005), are available for numerical simulations. The model based on shallow layer equations (Stein and Ermak,1980; Verfondern and Dienhart, 1997, 2007; Brandeis and Kansa, 1983; Brandeis and Ermak, 1983; Briscoe and Shaw, 1980) under the assumption of axisymmetry, solves for the velocity and pool height with respect to radius and time. The simplest mathematical model, which can be called the simple physical model (Briscoe and Shaw, 1980) describes the pool spread in terms of how the pool radius and height evolve in time. The corresponding equations consist of two ordinary differential equations with respect to time and one algebraic equation. For the purpose of engineering design and analysis, however, the shallow layer model presents a problem in determining the size of the pool fire because the model allows the leading-edge wave to separate from the spreading pool to form an annulus.

In this study, a set of second-order perturbation solutions are derived, as an improvement over the previous first-order perturbation solutions (Kim et al., 2011), for the simple physical model describing the liquid pool spreading with an instantaneous spill. The second-order solutions are derived for the governing equations after introducing dimensionless governing parameters in order to obtain more generic solutions.

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The normalized, dimensionless evaporation rate per unit area is used as the perturbation parameter. The results demonstrate that the second-order solutions exhibit a definite improvement over the first-order solutions with respect to the pool volume; however, there is still a little difference between the second-order solutions and the numerical ones with respect to the pool radius at the final stage of spread.

2. Governing equations

Neglecting the surface tension and viscous drag, the driving force for the pool spread is gravity. Although this force acts downwards, it creates an unbalanced pressure distribution in the pool, causing the pool to spread laterally. Pool spread is governed by the following set of equations (Briscoe and Shaw, 1980):

$$\frac{dR}{dT} = \sqrt{\alpha H} \tag{1}$$

where *R* is pool radius (m), *T* is time (s), α is 2*g* (m/s^2), *g* is gravity (m/s^2) and *H* is pool height (m), respectively.

$$\frac{dV}{dT} = -E\pi R^2 + \beta \tag{2}$$

where *V* is pool volume (m^3), *E* is evaporation rate per unit area (m/s) and β is the spill source rate (m^3/s) for a continuous spill, respectively. Therefore, for an instantaneous spill, β becomes zero. To complete the model, the following algebraic equation is required:

$$H = \frac{V}{\pi R^2} \tag{3}$$

3. Initial conditions and solutions

If the liquid is instantaneously released from storage, the following initial conditions can be used:

$$V(0) = V_i, \quad R(0) = R_{i,} \quad H(0) = H_i \tag{4}$$

From Eq. 1 through 4 it is understood that two initial conditions and the evaporation rate per unit area, E, govern the model equations. To make the governing equations dimensionless, the following variables are introduced:

$$v = V / V_i, \quad r = R / R_i, \quad h = H / H_i, \quad t = T / \tau_i \tag{5}$$

where *v* is dimensionless pool volume, *r* is dimensionless pool radius, *h* is dimensionless pool height and *t* is dimensionless time and τ_i is characteristic time $\left(= \sqrt{R_i / \alpha}\right)$, respectively. Using the dimensionless variables in Eq. 5, the following non-dimensional governing equations are derived:

$$\frac{dv}{dt} = -\varepsilon r^2 \tag{6}$$

$$\frac{dr}{dt} = \delta \sqrt{h} \tag{7}$$

$$h = \frac{v}{r^2} \tag{8}$$

where ε is dimensionless evaporation rate $\left(= E\tau_i / H_i\right)$ and $\delta = \sqrt{H_i / R_i}$. From Eq. 6 through 8 it can be seen that the two dimensionless parameters, ε and δ, corresponding to the dimensionless evaporation rate per unit area and the aspect ratio of the initial pool, respectively, control the non-dimensional governing equations. The initial conditions are

$$v = 1, \quad r = 1, \quad h = 1 \text{ at } t = 0 \tag{9}$$

The evaporation rate per unit area of the pool *E*, which is referred to as the constant regression velocity, generally exhibits very small values (about 4.2×10^{-4} m/s for LNG). Therefore, the dimensionless evaporation rate, ε, is chosen as the perturbation parameter. The perturbation solutions can then be expressed in the following forms:

$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 \tag{10}$$

$$r = r_0 + \varepsilon r_1 + \varepsilon^2 r_2 \tag{11}$$

$$h = h_0 + \varepsilon h_1 + \varepsilon^2 h_2 \tag{12}$$

where terms higher than O(ε^2) are omitted. In this study, a second-order expansion is used, and terms up to O(ε^2) are retained. Substituting Eq. 10 through 12 into Eq. 6 through 8 and equating the coefficients of ε^0, ε and ε^2 on both-hand sides, we obtain

$$\frac{dr_0}{dt} = \delta\sqrt{h_0} \tag{13}$$

$$\frac{dr_1}{dt} = \frac{\delta}{2}\frac{h_1}{\sqrt{h_0}} \tag{14}$$

$$\frac{dr_2}{dt} = \frac{\delta}{2}\frac{h_2}{\sqrt{h_0}} - \frac{\delta h_1^2}{8h_0^{3/2}} \tag{15}$$

$$\frac{dv_0}{dt} = 0 \tag{16}$$

$$\frac{dv_1}{dt} = -r_0^2 \tag{17}$$

$$\frac{dv_2}{dt} = -2r_0 r_1 \tag{18}$$

$$h_0 = \frac{v_0}{r_0^2} \tag{19}$$

$$h_1 = \frac{v_1}{r_0^2} - \frac{2h_0 r_1}{r_0} \tag{20}$$

$$h_2 = \frac{v_2}{r_0^2} - h_0\left[\frac{2r_2}{r_0} + \left(\frac{r_1}{r_0}\right)^2\right] - \frac{2h_1 r_1}{r_0} \tag{21}$$

Nine new equations (Eq. 13 through 21) have been obtained; therefore, the number of initial conditions must increase to nine. Applying the conditions in Eq. 9 to Eq. 10 through 12 and equating the coefficients of ε^0, ε and ε^2 on both sides, we get

$$v_0 = 1, \quad r_0 = 1, \quad h_0 = 1 \text{ at } t = 0 \tag{22}$$

$$v_1 = 0, \quad r_1 = 0, \quad h_1 = 0 \text{ at } t = 0 \tag{23}$$

$$v_2 = 0, \quad r_2 = 0, \quad h_2 = 0 \text{ at } t = 0 \tag{24}$$

Solving Eq. 13 through 21 with the initial conditions in Eq. 22 through 24 yields

$$r_0 = \sqrt{1 + 2x} \tag{25}$$

$$r_1 = -\frac{\delta}{2}\frac{\frac{1}{2}t^2 + \frac{\delta}{3}t^3}{\sqrt{1+2x}} \tag{26}$$

$$r_2 = -\frac{1}{1440}\frac{\delta t^3}{\left(1+2x\right)^{3/2}}\left(68x^3 + 204x^2 + 225x + 60\right) \tag{27}$$

where $x = \delta t.$ And

$$v_0 = 1 \tag{28}$$

$$v_1 = -\left(t + \delta t^2\right) \tag{29}$$

$$v_2 = \frac{\delta^2 t^4}{12} + \frac{\delta t^3}{6} \tag{30}$$

$$h_0 = \frac{1}{1+2x} \tag{31}$$

$$h_1 = -\frac{\left(1+x\right)t}{1+2x} + \frac{\delta\left(\delta t^3/3 + t^2/2\right)}{\left(1+2x\right)^2} \tag{32}$$

$$h_2 = -\frac{1}{180}\frac{\delta t^3}{\left(1+2x\right)^3}\left(28x^3 + 84x^2 + 105x + 45\right) \tag{33}$$

4. Results and discussion

For the purpose of numerical evaluation, a spreading of LNG on the ground with the values of E = 4.2×10^{-4} m/s and density (ρ = 420 kg/m^3) has been considered. The result (Kim et al., 2011) for the pool volume is shown in Figure. 1, in which the difference between the numerical solution and the first-order solution becomes evident in the late stage of spread when the time is large. This discrepancy comes from the secular terms (Nayfeh, 1981) that lead to the non-uniform expansion when time is large.
From Eqs. 28 and 29, the first-order expansion for the pool volume can be expressed as

$$V = V_i - \pi E\left(aT + \frac{b}{2}T^2\right) \tag{34}$$

where $a = R_i^2$ and $b = 2\left(\alpha V_i/\pi\right)^{1/2}$.

In Eq. 34, the second term within the bracket should be much smaller than the first term because the second term has been introduced as the first correction term for the first term. As time increases, however, the order of the correction term, *O(ET)*, approaches *O(1)*. That is, the first correction term becomes of the order of the main term for a large *T*. To address this anomaly, a uniform expansion would be adequate; however, it is beyond the scope of the present study. Instead, the second-order expansion has been pursued.
The second-order solutions have been calculated for the initial radius of the pool of 0.1 m, 1 m, and 10 m with the constant initial volume in order to explore the effect of the major parameters on the solutions. As can be seen in Figures. 2-4, the second-order solutions for the pool volume agree remarkably with the numerical solution. Moreover, the second-order solutions display a noticeable improvement over the first-order solutions even in the late stage of spread, and the two perturbation solutions are nearly indistinguishable except the final stage. The results also demonstrate that the second-order perturbation solutions readily accommodate the change in the aspect ratio of the initial pool.

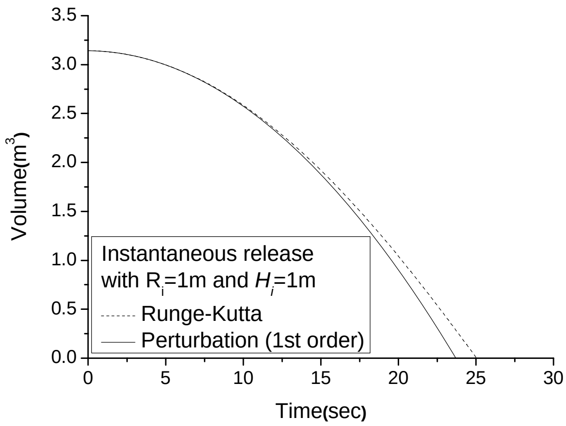

Figure 1: The pool volume vs. time

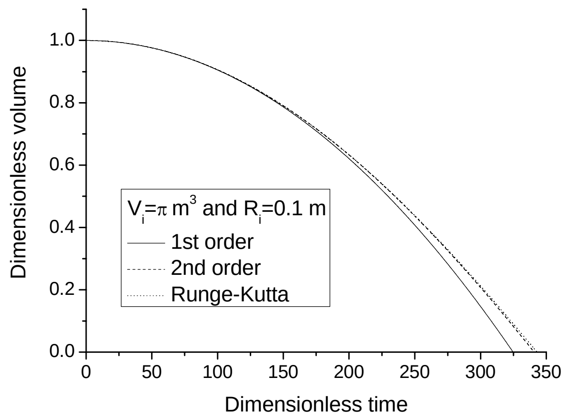

Figure 2: v vs. t with $V_i = \pi\, m^3$ and $R_i = 0.1$ m

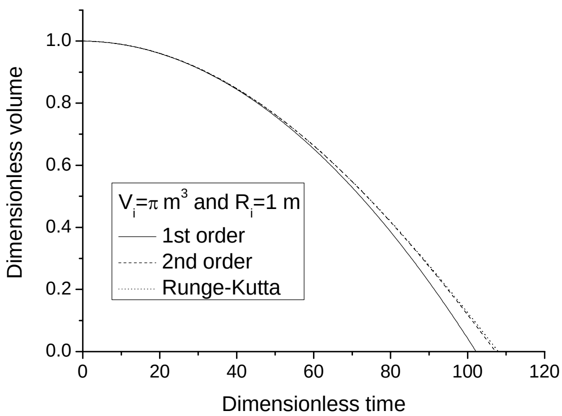

Figure 3: v vs. t with $V_i = \pi\, m^3$ and $R_i = 1.0$ m

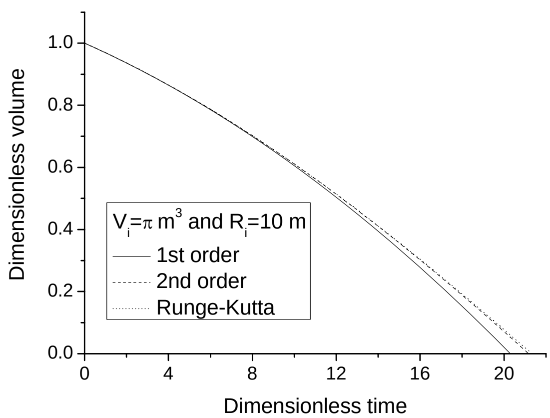

Figure 4: v vs. t with $V_i = \pi\, m^3$ and $R_i = 10$ m

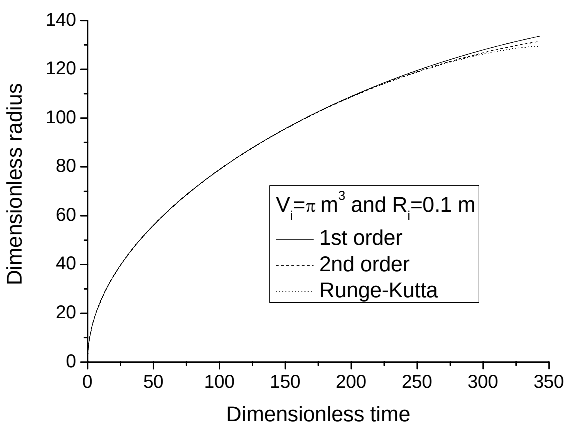

Figure 5: r vs. t with $V_i = \pi\, m^3$ and $R_i = 0.1$ m

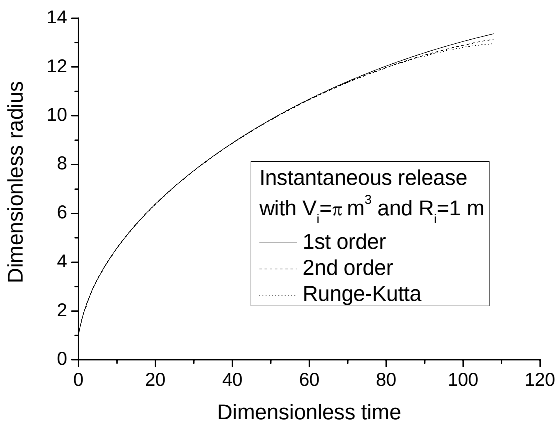

Figure 6: r vs. t with $V_i = \pi\, m^3$ and $R_i = 1.0$ m

In the case of the pool radius, however, there is virtually no difference between the two types of perturbation solutions when time is large, as seen in Figures. 5-7. It is shown that the perturbation solutions generally allow for a change in the aspect ratio of the initial pool. As in the case of the pool volume, the second-order solutions for the pool radius demonstrate the improvement over the first-order ones. However, there is still a little difference between the second-order solutions and the numerical ones at the final stage of spread. Therefore it can be said that the second-order solutions for volume are more accurate than them for radius.

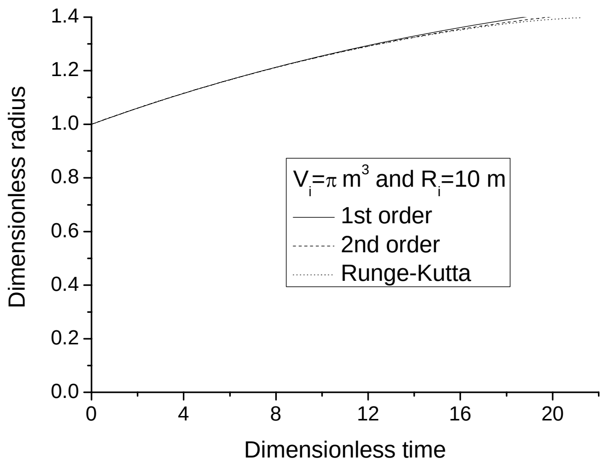

Figure 7: r vs. t with $V_i = \pi m^3$ and $R_i = 10$ m

5. Conclusion

The model equations for the spread of a liquid pool with instantaneous spill have been made dimensionless to reveal that these equations are governed by two parameters, namely, the dimensionless evaporation rate and the aspect ratio of the initial pool. It is noteworthy that the original governing equations, before being made dimensionless, contain three governing parameters instead of two.

For the dimensionless governing equations, the second-order perturbation solutions are obtained. It is found that the second-order solutions exhibit a definite improvement over the first-order solutions with respect to the pool volume and the pool radius. The second-order solutions for the pool volume agree remarkably with the numerical solutions; however, there is still a little difference between the second-order solutions and the numerical ones for the pool radius at the final stage of spread. The second-order solutions readily accommodate changes in the major parameters.

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