

# Fuzzy P-graph for Optimal Synthesis of Polygeneration Systems

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Polygeneration systems have been utilized to simultaneously generate a number of energy and utility products such as heat, power, cooling and treated water. Its implementation has proven to increase fuel efficiency and to reduce the associated carbon footprint in products in comparison to stand-alone production systems. The polygeneration system consists of interdependent process units whose design capacities will depend on the expected product demands. Because of the multiple product streams generated and the associated demands, it is necessary to design a system which aims to simultaneously meet potentially conflicting product demand targets. Fuzzy optimization has initially been used to identify the optimal solution which simultaneously satisfies multiple product demand targets. However, real life decision-making may require an evaluation of alternative solutions. This aspect can be addressed by the P-graph methodology which is able to provide both optimal and sub-optimal network designs. This work thus proposes the development of a fuzzy P-graph model for the design of a polygeneration system. The capabilities of the model are demonstrated in a case study. The model results identify both optimal and sub-optimal design options which generate products within the defined demand targets and which can be further evaluated for other parameters such as robustness for final decision-making.

## 1. Introduction

Enhancement of energy efficiency in industrial systems is an important strategy for achieving sustainability. Systematic approaches for achieving such gains include Process Integration (PI), which in particular has gained a significant role as demonstrated by developments in both methodology and industrial applications (Klemeš, 2013). PI opportunities naturally arise in multi-functional systems such as polygeneration plants due to the utilization of waste heat and material streams (Serra et al., 2009). Adams and Ghouse (2015) give a comprehensive survey of polygeneration systems configurations. Systematic design of polygeneration systems can be done using rigorous Process Systems Engineering (PSE) methodology such as mathematical programming (Liu et al., 2007). Specific formulations range from simple linear programming (LP) models (Lozano et al., 2009) to mixed integer programming (Liu et al., 2009), multi-objective programming (Liu et al., 2010) and fractional programming (Ubando et al., 2013), among others. These methods enable optimal solutions to be determined through specification of polygeneration system configuration and component capacities (Mancarella, 2014). However, it has also been argued that the analysis of near-optimal solutions of models for the synthesis of energy systems is an important step in identifying robust solutions to practical problems (Voll et al., 2015).

An alternative approach to mathematical programming is the use of P-graph methodology (Friedler et al., 1992a) which has the advantage of having an intrinsic ability of generating optimal and near optimal solutions linked to a graphical representation of the system being studied. This framework has been used for the synthesis of fuel cell-based cogeneration systems (Varbanov and Friedler, 2008), optimal dispatch of polygeneration plants under abnormal conditions (Tan et al., 2014) and multi-period optimization of isolated energy systems (Aviso et al., 2016). In this paper, a fuzzy P-graph model is proposed for the synthesis of polygeneration systems. The concept of fuzzy P-graphs was first proposed by Tick (2009) for workflow planning, by integrating principles of fuzzy decision-making (Bellman and Zadeh, 1970) into the graph theoretic framework. However, this paper is the first to apply such an approach to a PI/PSE application. The rest of the paper is organized as follows. Section

2 gives a formal problem statement. Section 3 gives a description of general P-graph methodology, while Section 4 discusses fuzzy P-graphs. The latter methodology is illustrated with a case study in Section 5. Finally, conclusions and prospects for future work are given in Section 6.

## 2. Problem Statement

The formal problem statement can be stated as follows. A polygeneration system is to be designed given  $N$  number of process units which can provide  $M$  number of material or energy streams. The desired product output of each product stream is defined by fuzzy limits. The objective then is to generate the optimal system design which satisfies the demand of all product streams simultaneously.

## 3. Methodology

The Process graph or P-graph model is utilized to generate optimal and near-optimal solutions to the polygeneration system considered. The P-graph framework was initially developed by Friedler et al. (1992a) for Process Network Synthesis (PNS) and it works by identifying all combinatorially feasible pathways from raw material acquisition and processing to product manufacture and distribution. It has been used for a wide range of PNS applications, as described in a recent review by Lam (2013), while a more recent survey notes its application to structurally related problems in more diverse areas (Klemeš and Varbanov, 2015).

The P-graph framework is based on five axioms as proposed in Friedler et al. (1992b):

- Every final product is represented in the graph
- A vertex of the M-type has no input if and only if it represents a raw material
- Every vertex of the O-type represents an operating unit defined in the synthesis problem
- Every vertex of the O-type has at least one path to a vertex of the M-type representing a final product
- If a vertex of the M-type belongs to the graph, it must be an input to or an output from at least one vertex of the O-type in the graph

Furthermore, P-graph makes use of three algorithms in generating the optimal and near-optimal solutions. The algorithms are

- Maximal Structure Generation (MSG) – rigorously identifies a superstructure network (Friedler et al., 1993) based on the five axioms.
- Solution Structure Generation (SSG) – finds all the combinatorially feasible networks as extracted from the MSG.
- Accelerated Branch and Bound (ABB) – a more efficient algorithm for finding solutions in a combinatorial problem in comparison to conventional branch and bound algorithm.

Detailed discussion of the methodology can be found in key textbooks (Klemeš et al., 2011); also, on-line tutorials and software are available from the dedicated website (P-graph, 2016). This framework offers a viable alternative framework to equation-based modelling approaches (Lam et al., 2016). The basic P-graph framework is capable of solving single objective problems which have a similar structure with PNS. However, several optimization problems are multi-objective in nature and it will be advantageous to merge the intrinsic capability of P-graph in finding both optimal and near optimal solutions with the ability of simultaneously satisfying the multiple objectives. A fuzzy P-graph model is thus developed in the next section.

## 4. Fuzzy P-Graph Optimization Model

Fuzzy optimization has been implemented previously using mathematical models particularly for finding the “satisficing” solution when there are multiple objectives to be considered (Zimmermann, 1978). This work, however, presents how fuzzy optimization of polygeneration systems can be modelled within the P-graph framework. However, the fuzzy optimization model will enable the integration of multiple objectives into the P-graph framework. The general fuzzy optimization model can be represented by Eq. (1) to Eq. (6) where Eq. (1) represents the over-all objective of maximizing the degree of satisfaction. Eq. (2) consists of an equality constraints while Eq. (3) represents all inequality constraints. Eq. (4) represents the satisfaction of variables which must be maximized with fuzzy limits  $y^L$  as the lower limit and  $y^U$  as the upper limit. Eq. (5), on the other hand, represents the satisfaction of variables which must be minimized with fuzzy limits  $x^L$  as the lower limit and  $x^U$  as the upper limit. Furthermore, the degree of satisfaction is normalized according to the fuzzy limits and thus must have a value between zero and 1 as seen in Eq. (6).

Fuzzy optimization can be modelled in P-graph through the inclusion of a fictitious operating unit which represents the over-all degree of satisfaction. A simple example which considers two objectives is shown in Fig. 1. Fig. 1 shows two operating units (OU1 and OU2) which process material 1 to generate products M1 and M2. M1 and M2 have defined fuzzy upper ( $M1^U$ ,  $M2^U$ ) and lower ( $M1^L$ ,  $M2^L$ ) limits and the objective is to maximize

the over-all degree of satisfaction represented by the product node LAMBDA. The simultaneous consideration of maximizing M1 and M2 within the fuzzy limits is accomplished through the fictitious operating unit OU\_LAMBDA. The difference between the upper and lower fuzzy limit is utilized as the flow rate of the stream coming from the product node (M1 or M2) going to OU\_LAMBDA. A simple case study on the design of a polygeneration system is considered in the next section to show how the model works.

$$\begin{aligned} \max \lambda & & (1) \\ f(x, y) &= 0 & (2) \\ g(x, y) &\leq 0 & (3) \\ y &\geq y^L + \lambda(y^U - y^L) & (4) \\ x &\leq x^U - \lambda(x^U - x^L) & (5) \\ 0 &\leq \lambda \leq 1 & (6) \end{aligned}$$

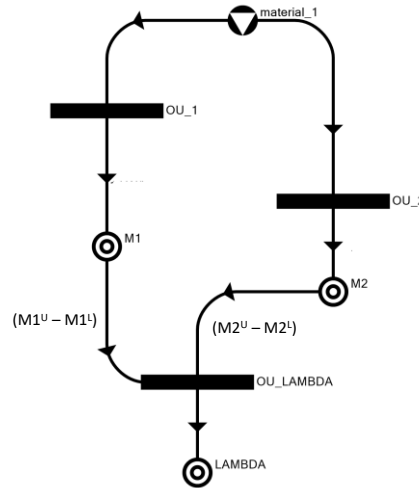


Figure 1: Fuzzy optimization representation in P-graph

## 5. Case Study

The case study considered here involves the design of a polygeneration system consisting of six (6) process units with six (6) material and energy streams. The technology matrix for the different processes is adapted from Kasivisvanathan et al. (2013) for the boiler, CHP, reverse osmosis module and electric chiller while the rest were taken from Carvalho et al. (2012). These are summarized in Table 1 where the rows represent the flow of material or energy streams in the process indicated by the column. It is important to note that a negative entry indicates an input to the process, while a positive entry indicates the generation of the material or energy stream by the process. The four product streams – heat, power, cooling and treated water have identified fuzzy limits with respect to the demand.

Table 1: Technology matrix of polygeneration system (adapted from Kasivisvanathan et al., 2013 and Carvalho et al., 2012)

|               | Units                                   | Boiler | CHP   | Engine | Reverse Osmosis | Absorption Chiller | Electric Chiller |
|---------------|---|--------|-------|--------|-----------------|--------------------|------------------|
| Heat          | kW                                      | 1.00   | 1.50  | 0.00   | 0.00            | -0.83              | 0.00             |
| Power         | kW                                      | -0.01  | 1.00  | 1.00   | -3.00           | -0.01              | -0.20            |
| Cooling       | kW                                      | 0.00   | 0.00  | 0.00   | 0.00            | 1.00               | 1.00             |
| Treated Water | m <sup>3</sup> /h (x 10 <sup>-3</sup> ) | -2.09  | -9.83 | 0.00   | 3,600           | 0.00               | 0.00             |
| Freshwater    | m <sup>3</sup> /h (x 10 <sup>-3</sup> ) | 0.00   | 0.00  | 0.00   | -9,000          | 0.00               | 0.00             |
| Fuel          | m <sup>3</sup> /h (x 10 <sup>-4</sup> ) | -1.19  | -5.40 | -4.32  | 0.00            | 0.00               | 0.00             |

The lower limit  $y_i^L$  represents the minimum amount of product  $i$  that must be produced ( $\lambda = 0$ ) while  $y_i^U$  is the desired value which corresponds to full satisfaction ( $\lambda = 1$ ). The degrees of satisfaction increase linearly as the product demands increase from  $y_i^L$  to  $y_i^U$ . In addition, the resources have an identified limit of availability. The fuzzy demand and resource limits to the system are listed in Table 2.

The case study is then illustrated in Figure 2. Optimizing the polygeneration system such that the over-all degree of satisfaction is maximized results in the network shown in Figure 3 which corresponds to an over-all satisfaction of 0.75. This solution selects the use of the electric chiller instead of the absorption chiller. In addition, there are 5 near-optimal solutions, a comparison between the optimal and the first near optimal solution is given in Tables 3 and 4.

Table 2: Fuzzy demand and resource limits of material and energy streams

|               |                   | Lower Limit<br>$y_i^L$ | Upper Limit<br>$y_i^U$ |
|---------------|-------------------|------------------------|------------------------|
| Heat          | kW                | 20,000                 | 25,000                 |
| Power         | kW                | 8,000                  | 10,000                 |
| Cooling       | kW                | 7,500                  | 9,000                  |
| Treated Water | m <sup>3</sup> /h | 216                    | 360                    |
| Freshwater    | m <sup>3</sup> /h | 1,080                  | 1,260                  |
| Fuel          | m <sup>3</sup> /h | 6.84                   | 8.64                   |

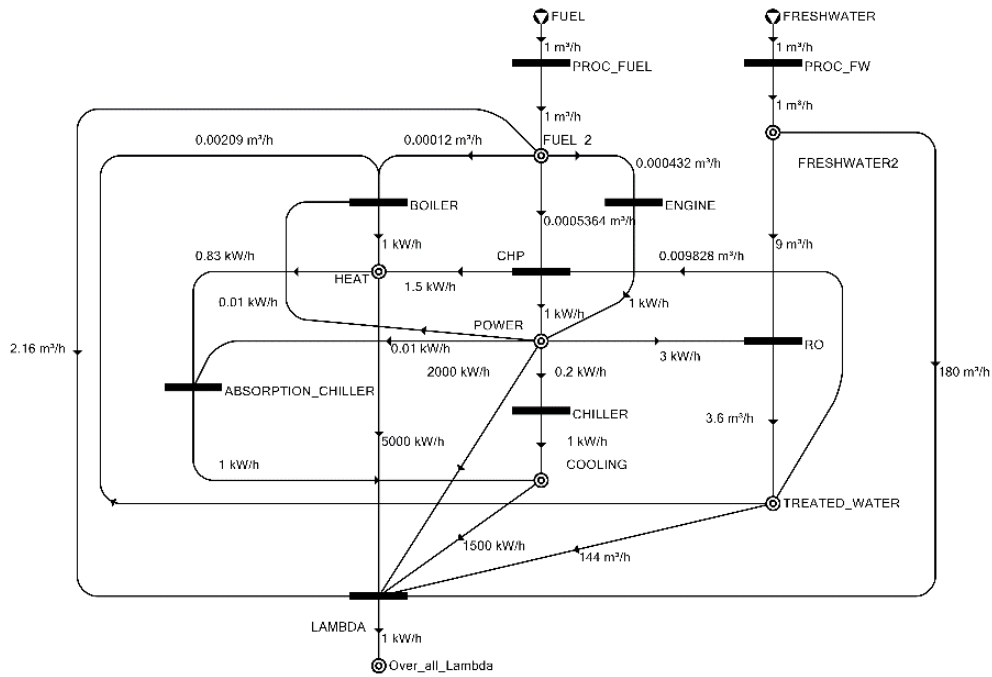


Figure 2: Fuzzy P-graph representation of case study

Table 3: Resulting parameters of the optimal solution ( $\lambda = 0.75$ )

| Process Units      | Capacity  | Streams                             | Flow of Streams |
|--------------------|-----------|-------------------------------------|-----------------|
| Boiler             | 6,573.47  | Heat (kW)                           | 23,744.39       |
| CHP                | 11,447.30 | Power (kW)                          | 9,497.76        |
| Engine             | 215.94    | Cooling (kW)                        | 8,623.332       |
| Reverse Osmosis    | 125.02    | Treated water (m <sup>3</sup> /h)   | 323.84          |
| Absorption Chiller | 0.00      | Freshwater used (m <sup>3</sup> /h) | 1,125.2         |
| Electric Chiller   | 8,623.32  | Fuel used (m <sup>3</sup> /h)       | 8.64            |

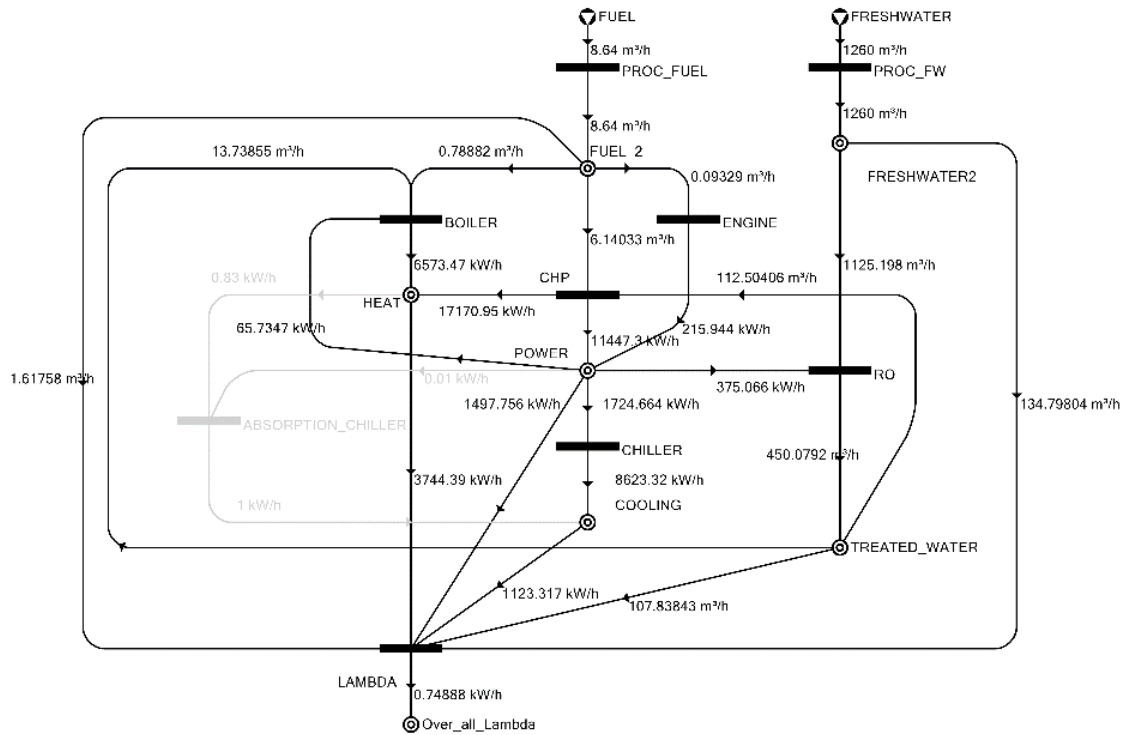


Figure 3: Optimal network for case study

Table 4. Resulting parameters of the near optimal solution ( $\lambda = 0.74$ )

| Process Units      | Capacity  | Streams                             | Flow of Streams |
|--------------------|-----------|-------------------------------------|-----------------|
| Boiler             | 7,755.66  | Heat (kW)                           | 23,699.59       |
| CHP                | 11,392.80 | Power (kW)                          | 9,479.84        |
| Engine             | 0.00      | Cooling (kW)                        | 8,609.88        |
| Reverse Osmosis    | 125.20    | Treated water (m <sup>3</sup> /h)   | 322.55          |
| Absorption Chiller | 1,379.85  | Freshwater used (m <sup>3</sup> /h) | 1,126.81        |
| Electric Chiller   | 7,230.03  | Fuel used (m <sup>3</sup> /h)       | 7.04            |

The first near optimal solution has a degree of satisfaction of  $\lambda = 0.74$ , which is only slightly less than the optimal solution of  $\lambda = 0.75$ . This slight reduction in satisfaction level results in the selection of a different set of technologies. The optimal solution does not choose the absorption chiller to achieve the required product demands but the near optimal solution does not choose the engine and chooses both the absorption and electric chiller. The fifth near optimal solution has a  $\lambda = 0.45$  and selects the boiler, engine, absorption chiller and reverse osmosis process units. The alternative solutions may be useful for decision-makers since it provides them with options to select from. The reduction in satisfaction may be justified by other system characteristics, such as robustness, which can be implemented using Monte Carlo simulation, looking at the probability of network failure when uncertainties in process flow rates and product demands are present (Tan et al., 2017). These designs may be more practical for engineers to implement.

## 6. Conclusions

A fuzzy optimization model was developed within the P-graph framework with the design of a polygeneration system used as a case study to demonstrate the capabilities of the model. The integration of P-graph and fuzzy optimization enables the consideration of multiple objectives and the generation of both optimal and near optimal solutions. Further examination of the optimal and near optimal solutions in terms of the trade-off between the objective function and the network design parameters can provide decision makers with insights on which can be implemented. The model however is limited for process synthesis applications and is not meant for detailed design models. Future work can look into the integration of cost considerations, environmental impact, existing part-load limits of process units and multi-period operations.

## References

- Adams T.A., Ghouse J.H., 2015, Polygeneration of fuels and chemicals. *Current Opinion in Chemical Engineering*, 10, 87-93.
- Aviso K.B., Lee J.-Y., Tan R.R., 2016, A P-graph model for multi-period optimization of isolated energy systems, *Chemical Engineering Transactions*, 52, 865 – 870.
- Bellman R.E., Zadeh L.A., 1970, Decision-Making in a Fuzzy Environment. *Management Science*, 17(4), B141-B164.
- Carvalho M., Lozano M. A., Serra L.M. 2012, Multicriteria synthesis of trigeneration systems considering economic and environmental aspects. *Applied Energy*, 91(1), 245-254.
- Friedler F., Tarjan K., Huang Y.W., Fan L.T., 1992a, Combinatorial algorithms for process synthesis. *Computers and Chemical Engineering*, 16, 313 – 320.
- Friedler F., Tarjan K., Huang Y.W., Fan L.T., 1992b, Graph-theoretic approach to process synthesis: axioms and theorems. *Chemical Engineering Science*, 47, 1973-1988.
- Friedler F., Tarjan K., Huang Y.W., Fan L.T., 1993, Graph-theoretic approach to process synthesis: polynomial algorithm for maximal structure generation. *Computers and Chemical Engineering*, 17, 929-942.
- Kasisvisvanathan H., Barilea I D.U., Ng D.K., Tan R.R., 2013, Optimal operational adjustment in multi-functional energy systems in response to process inoperability. *Applied Energy*, 102, 492-500.
- Klemeš J.J. (Ed), 2013, *Handbook of Process Integration (PI): Minimisation of Energy and Water Use, Waste and Emissions*, Woodhead/Elsevier, Cambridge, UK, 1184 ps. ISBN: 987-0-85709-0.
- Klemeš J.J., Friedler F., Bulatov I., Varbanov P., 2011, *Sustainability in the Process Industry: Integration and Optimization*. McGraw-Hill, New York, USA.
- Klemeš J.J., Varbanov P.S., 2015, Spreading the Message: P-graph Enhancements: Implementations and Applications. *Chemical Engineering Transactions*, 45, 1333-1338.
- Lam H.L., 2013, Extended P-graph applications in supply chain and process network synthesis. *Current Opinion in Chemical Engineering*, 2, 475-486.
- Lam H.L., Aviso K.B., Tan R.R., 2016, Implementation of P-Graph Modules in Undergraduate Chemical Engineering Degree Programs: Experiences in Malaysia and the Philippines. *Journal of Cleaner Production*, 136, 254-265.
- Liu P., Gerogiorgis D.I., Pistikopolous E.N., 2007, Modeling and optimization of polygeneration energy systems. *Catalysis Today*, 127, 347-359.
- Liu P., Pistikopolous E.N., Li Z., 2009, A mixed-integer optimization approach for polygeneration energy systems design. *Computers and Chemical Engineering*, 33, 759-768.
- Liu P., Pistikopolous E.N., Li Z., 2010, A multi-objective optimization approach to polygeneration energy systems design. *AIChE Journal*, 56, 1218-1234.
- Lozano M.A., Carvalho M., Serra L.M., 2009, Operational strategy and marginal costs in simple trigeneration systems. *Energy*, 34, 2001-2008.
- Mancarella P., 2014, MES (multi-energy systems): An overview of concepts and evaluation models. *Energy*, 65, 1-17.
- P-graph, 2016, Available at: [www.p-graph.com](http://www.p-graph.com), (accessed on 04.10.2016).
- Serra L.M., Lozano M.A., Ramos J., Ensinas A.V., Nebra S.A., 2009, Polygeneration and efficient use of natural resources. *Energy*, 34, 575-586.
- Tan R.R., Cayamanda C.D., Aviso K.B., 2014, P-graph approach to optimal operational adjustment in polygeneration plants under conditions of process inoperability. *Applied Energy*, 135, 402-406.
- Tan R.R., Aviso K.B., Foo D.C., 2017, P-Graph and Monte Carlo Simulation Approach to Planning Carbon Management Networks. *Computers and Chemical Engineering*. Article in press [doi.org/10.1016/j.compchemeng.2017.01.047](https://doi.org/10.1016/j.compchemeng.2017.01.047).
- Tick J., 2009, Fuzzy extension to P-graph based workflow models. *IEEE 7th International Conference on Computational Cybernetics 2009*, Pages 109-112, DOI: 10.1109/ICCCYB.2009.5393953.
- Ubando A.T., Culaba A.B., Aviso K.B., Tan R.R., 2013, Simultaneous carbon footprint allocation and design of trigeneration plants using fuzzy fractional programming. *Clean Technologies and Environmental Policy*, 15, 823-832.
- Varbanov P., Friedler F., 2008, P-graph methodology for cost-effective reduction of carbon emissions involving fuel cell combined cycles. *Applied Thermal Engineering*, 28, 2020-2029.
- Voll P., Jennings M., Hennen M., Shah N., Bardow A., 2015, The optimum is not enough: A near-optimal solution paradigm for energy systems synthesis. *Energy*, 82, 446-456.
- Zimmermann H.-J., 1978, Fuzzy Programming and Linear Programming with Several Objective Functions. *Fuzzy Sets and Systems*, 1, 45-55.