CHEMICAL ENGINEERING TRANSACTIONS
VOL. 29, 2012
A publication of AIDIC
The Italian Association of Chemical Engineering
Online at: www.aidic.it/cet
Guest Editors: Petar Sabev Varbanov, Hon Loong Lam, Jiří Jaromír Klemeš

ISBN 978-88-95608-20-4; **ISSN** 1974-9791
DOI: 10.3303/CET1229061

MIMO LQ Control of the Energy Production of a Synchronous Generator in a Nuclear Power Plant

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A multiple-input multiple-output (MIMO) LQ servo controller was proposed for an industrial size synchronous generator that keeps the active power at the desired level and performs reactive power reference tracking using the reactive power demand from the central dispatch center. The controller design was based on the locally linearized version of a previous dynamical model of the synchronous electrical generator (Anderson and Fouad, 1977, Fodor et al., 2012) the parameters of which have been identified using measured data from Paks Nuclear Power Plant, Hungary.

1. Introduction

A major portion of industrial energy distribution and transportation is performed in the form of electrical energy using large-scale electrical power grids. Both of the energy consumers and energy producers are connected to this grid that should be operated in a balanced way taking the time varying power demand of the consumers into account that is difficult to predict (Lopes et al., 2007).
Besides of the active power produced by a power plant, other characteristic variables of the produced electrical energy are of importance, such as the reactive power, the frequency and the distortion characteristics from the ideal sinusoidal waveform. The reactive power is tightly related to bus voltages throughout a power network, and hence it has a significant effect on system security. Its importance is indicated by the fact that insufficient reactive power of the system may result in the voltage collapse. Therefore, it is widely accepted that the consumer of reactive power should pay for the reactive power support service and the producers of reactive power are remunerated (Dittmar, 2012).
From the viewpoint of the power grid the electric power generation of these plants can be characterized by the operation of the electrical generators, the subject of our study. These power plants should be able not only to follow the time-varying active and reactive power demand of the consumers and the central dispatch center, but also keep the quality indicators (frequency, waveform, total harmonic distortion) of the grid on the expected level. This can be achieved by applying proper control methods based on dynamic models of the involved generators.

2. The model of the synchronous generator

In this section the bilinear state-space model for a synchronous generator is presented based largely on (Anderson and Fouad, 1977) and (Fodor et al., 2012) that will be used for controller design.
The synchronous generator (SG) model is based on the following simplification assumptions:

Please cite this article as: Fodor A., Magyar A. and Hangos K. M., (2012), MIMO LQ control of the energy production of a synchronous generator in a nuclear power plant, Chemical Engineering Transactions, 29, 361-366

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- there are two amortisseur or damper windings in the machine,
- all of the windings are magnetically coupled,
- the flux linkage of the windings is a function of the rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal,
- the stator and rotor permeability are assumed to be infinite,
- the power of the generator is negligible compared to the network.

It is also assumed that all the losses due to wiring, saturation and slots can be neglected.

The six windings (three stators, one rotor and two dampers) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linking of the windings is also a function of the rotor position. The actual terminal voltage *v* of the windings can be written in the form

$$v = \pm\sum_{j=1}^{J}(r_j i_j) \pm \sum_{j=1}^{J}(\dot{\lambda}_j), \tag{1}$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both travelling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and another phase shifted by 90˚. With the above, one can construct an air-gap field with its maximal aligned to the rotor poles (*d* axis), while the other is aligned to the *q* axis (between poles). This method is called the Park's transformation.

As a result of the derivation in (Fodor *et al.*, 2012) the vector voltage equation is as follows:

$$v_{dFDqQ} = -Ri_{dFDqQ} - L\dot{i}_{dFDqQ}\,, \tag{2}$$

with $i_{dFDqQ} = \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q \end{bmatrix}^T$ and $v_{dFDqQ} = \begin{bmatrix} v_d & -v_F & v_D = 0 & v_q & v_Q = 0 \end{bmatrix}^T$, where v_d and v_q are the direct and the quadratic components of the stator voltage of the SG, v_D and v_Q are the direct and the quadratic components of the rotor voltage of the SG, i_d and i_q are the direct and the quadratic components of the stator current, i_D and i_Q are the direct and the quadratic components of the rotor current, while v_F and i_F are the exciter voltage and current. Furthermore, R and L are the following matrices

$$R = \begin{bmatrix} r+R_e & 0 & 0 & \omega L_Q & \omega kM_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega kM_F & -\omega kM_D & r+R_e & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix}, L = \begin{bmatrix} L_d+L_e & kM_F & kM_D & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q+L_e & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix}, \tag{3}$$

where *r* is the stator resistance of the SG, r_F is the exciter resistance, r_D and r_Q are the direct and the quadratic part of the rotor resistance of the SG, L_d, L_q, L_D and L_Q are the direct and the quadratic part of the stator and rotor inductance, ω is the angular velocity, and M_F, M_D and M_R are linkage inductances. The resistance R_e and inductance L_e represent the output transformer of the synchronous generator and the transmission-line.

The state-space model for the currents is obtained by expressing $\dot{i}_{dFDqQ}$ from (2), i.e.

$$\dot{i}_{dFDqQ} = -L^{-1}Ri_{dFDqQ} - L^{-1}v_{dFDdQ} \tag{4}$$

The purely electrical model (4) has to be extended with the equation (5) of rotational motion that gives the mechanical sub-dynamics, that is

$$\dot{\omega} = \begin{bmatrix} -\frac{L_d i_q}{3\tau_j} & -\frac{kM_F i_q}{3\tau_j} & -\frac{kM_D i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{kM_Q i_d}{3\tau_j} & -\frac{D}{\tau_j} \end{bmatrix} \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q & \omega \end{bmatrix} + \frac{T_{Mech}}{\tau_j} \tag{5}$$

Altogether, there are six state variables: i_d , i_F , i_D , i_q , i_Q and ω. The manipulated input vector of the generator is $u = \begin{bmatrix} v_F & T_{mech} \end{bmatrix}$, the disturbance input vector is $d = \begin{bmatrix} v_d & v_q \end{bmatrix}$. Observe that the state equations (4-5) *are bilinear in the state variables* because matrix R depends linearly on ω.
On the other hand, the generator is assumed to be negligible compared to the network. Then it follows that ω is constant, i.e. (5) can be replaced with its steady state version. The algebraic constraint can be embedded to the remaining dynamics by solving it for ω and substituting it into (4). The obtained model is still nonlinear and it has five state variables: i_d , i_F , i_D , i_q and i_Q. (Fodor *et al.*, 2010)
The *performance outputs* of the generator are the active and reactive power:

$$y_{perf} = \begin{bmatrix} p_{out} & q_{out} \end{bmatrix}^T, \quad p_{out} = v_d i_d + v_q i_q, \quad q_{out} = v_d i_q - v_q i_d, \tag{6}$$

Note, that *performance output equations are bi-linear in the state and input variables*.
It is convenient to define the measurable/computable state variables of the system are i_d, i_F, and i_q to be the *measured outputs* of the system, i.e. $y_{meas} = \begin{bmatrix} i_d & i_F & i_q \end{bmatrix}^T$.
The model parameters have been estimated by using measured data of a sufficiently exciting load changing transient from an industrial generator operating in the Paks NPP. The estimated dimensionless (in p.u.) parameters of the generator (from Fodor *et al.*, 2012) are collected in Table 1.

Table 1: Parameters of the synchronous generator model (4-5)

Parameter	Value	Parameter	Value	Parameter	Value
r	0.0211	M_F	1.2656	L_F	1.6510
R_e	0.0000	M_D	1.2656	L_D	1.6050
r_F	0.0006	M_Q	1.2468	L_q	1.5260
r_D	0.0131	M_R	1.5500	L_Q	1.5260
r_Q	0.0540	L_d	2.1000	τ_j	1786.0
k	1.2247	L_e	0.0000	D	2.0040

The quality of the model has also been evaluated by the fit in the active and reactive power in (Fodor *et al.*, 2012), and a good fit could be achieved.

2.1 The locally linearized model of the synchronous generator
The steady-state values of the state variables can be obtained from the steady-state version of state equations (4, 5) using the above parameters. The equilibrium point of the system is found to be

$$i_d = -1.751857 \quad i_q = 0.78545 \quad i_F = 2.9790 \quad i_D = 1.24 \cdot 10^{-7} \quad i_Q = 7.18 \cdot 10^{-8} \tag{7}$$

The locally linearized state-space model has the form of (8), where the two distinct output equations stand for the performance output and the measured output, respectively.

$$\dot{x} = Ax + B_u u + B_d d, \qquad y_{meas} = C_{meas} x, \qquad y_{perf} = C_{perf} x + D_u u + D_d d \tag{8}$$

3. Observer based LQ-servo control of the synchronous generator

The advanced control design of synchronous generators of power plants is far from being trivial because of the nonlinearity of the generator and the widely varied power demand of the consumers (see e.g. (Fernandez et al., 2008) or (Leon et al., 2011) for recent studies).
The control aims and expectations against the controller can be summarized as follows.

- The performance output (y_{perf}) of the system should follow the prescribed time varying piecewise constant reference signal.
- The closed loop system should be locally asymptotically stable.
- The performance output (y_{perf}) should be insensitive with respect to disturbance d.

The above aims suggest the use of LQ-servo technique for the controller design. The choice is justified by the following properties of LQ-servo method. It is a robust method, i.e. relatively insensitive for the difference between the linear and nonlinear models, moreover it performs trajectory tracking that was the primary control aim; also it has good disturbance rejection properties. As an LQ-servo controller applies full state feedback and not all of the state variables are measurable, the need of designing a state observer (Neimeier *et al.*, 1997, Wozny *et al.*, 1989) arises

3.1 State observer design

The primary task of the observer is to determine the estimated value of the state vector x_{obs} from the measured output y_{meas}. The observer is designed by pole placement technique using the system parameters A, B_u and C_{meas}. The poles λ_{obs} of the observer error dynamics are designed to be faster than the locally linearised system's poles in order that the error transients decay faster than the system transients.

$$\lambda_{obs} = \begin{bmatrix} -5 & -6 & -7 & -8 & -10 \end{bmatrix}^T \qquad (9)$$

The observer structure can be seen on Figure 1, the observer gain matrix L_{obs} has the following value

$$L_{obs} = \begin{bmatrix} 12.1913 & -1.4484 & -10.3415 & 8.5417 & -39.2208 \\ 0.3850 & 7.6033 & 1.1784 & 7.9901 & -5.1052 \\ 0.1961 & 0.0263 & 8.1142 & 14.9986 & -13.4230 \end{bmatrix}^T \qquad (10)$$

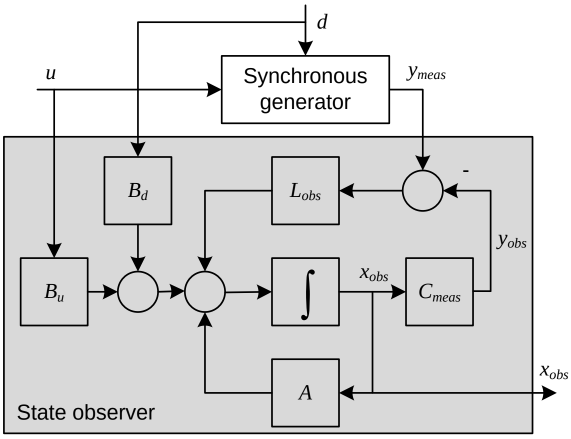

Figure 1: State observer structure for the synchronous generator.

3.2 LQ servo controller design

The LQ-servo control itself is based on the extension of the linear model (8) with a tracking error variable z that represents the difference between the reference input r and the performance output y_{perf}. The extended state equation has the form (11).

$$\dot{x} = Ax + B_u u + B_d d$$
$$\dot{z} = r - y_{perf} = r - C_{perf} x - D_u u - D_d d \tag{11}$$

The extended state equation in block matrix form detailed by equation (12) below

$$\dot{\tilde{x}} = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C_{perf} & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B_u \\ -D_u \end{bmatrix} u + \begin{bmatrix} B_d \\ -D_d \end{bmatrix} d + \begin{bmatrix} 0 \\ I \end{bmatrix} r, \quad \text{where} \quad \tilde{x} = \begin{bmatrix} x \\ z \end{bmatrix} \tag{12}$$

The structure of the LQ-servo control loop can be seen in Figure 2, where the inner loop is the stabilizing state feedback controller, and the outer integrating loop implements the servo control.

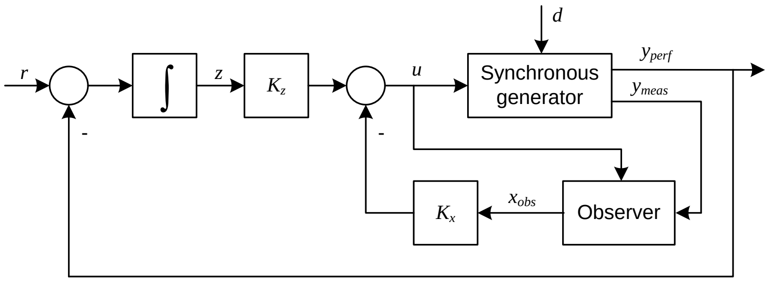

Figure 2: Observer based LQ-servo controller.

As in the case of LQR design, the functional to be minimized with respect to (11) is the object function

$$J(\tilde{x}, u) = \int_0^\infty \tilde{x}^T Q \tilde{x} + u^T R u \, dt \,, \tag{13}$$

where the state and input penalty factors has chosen to be

$$Q = 100\,\text{diag}\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 100 & 10\end{bmatrix}, \qquad R = \text{diag}\begin{bmatrix}1 & 1\end{bmatrix}. \tag{14}$$

The last two diagonal elements of Q in (14) stand for the tracking error variables.
The feedback gain that minimizes (13) with respect to (11) and governs the extended system to its equilibrium is in block matrix form $K = \begin{bmatrix} K_x & K_z \end{bmatrix}$ with numerical values (15).

$$K_x = \begin{bmatrix} \text{-}401.3310 & \text{-}378.4688 & \text{-}375.2029 & 119.7380 & 138.7553 \\ \text{-}252.8653 & \text{-}174.3798 & \text{-}174.6745 & 38.8057 & 68.5166 \end{bmatrix}, \quad K_z = \begin{bmatrix} \text{-}99.6438 & \text{-}2.6667 \\ \text{-}8.4329 & 31.5101 \end{bmatrix} \tag{15}$$

3.3 LQ servo controller design

In order to verify the designed controller the transient response and the tracking properties has been checked. The result of the time domain analysis can be seen in Figure 3, where the transient response and reference tracking of $p_{out,}$ and q_{out} is apparent. The controller's behavior in presence of load change like disturbances has also been examined; the LQ-servo compensates the step-type disturbances at 250 s and 450 s satisfactorily. The closed loop system's local asymptotic stability is guaranteed by the LQ method.

4. Conclusion and further work

In this work an LQ-type multiple-input multiple-output controller has been presented that does not only control active power of a synchronous generator, but can follow also the reactive power demand, as well. This fits well into the recent trends in electrical energy market where it is more widely accepted that consumers pay for the reactive power support service.
Further work includes the investigation of the interplay between the power controller of the plant and the designed LQ-servo controller of the generator to further increase their adaptivity and efficiency.

5. Acknowledgement

This work has been supported by the Hungarian Science Fund through grant 83440.

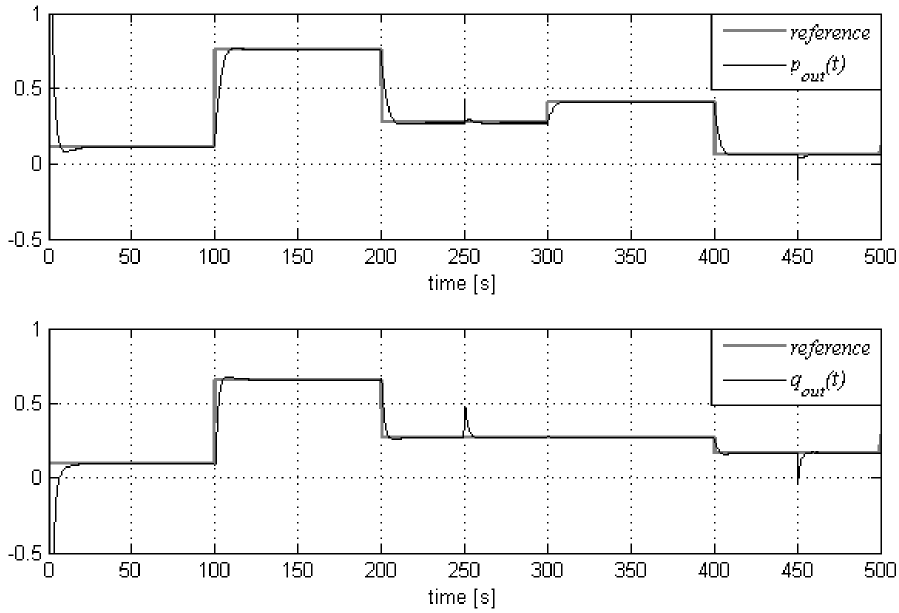

Figure 3: Transient response and reference tracking of active and reactive power.

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