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Robust Model Predictive Control of Heat Exchangers

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The paper attempts to show that using the robust model-based predictive control (RMPC) strategy for control of thermal processes can lead to energy savings in comparison with classical control approaches. RMPC is applied for control of a tubular heat exchanger that is used for pre-heating petroleum by hot water. The heat exchanger is a nonlinear system with time delay and uncertainty. The control objective is to keep the output temperature of the heated stream at a reference value and minimize the energy consumption needed for petroleum heating. The advantage of RMPC is that it is the optimisation based strategy, the control input and controlled outputs constraints are directly included into the synthesis and uncertainty of the process model is taken into account. RMPC of the heat exchanger is compared with the classical optimal linear quadratic (LQ) control by simulations experiments. In the presence of uncertainty and boundaries on control inputs, using the RMPC approach increases the quality of the control performance and decreases energy supplied to the heating medium.

1. Introduction

Heat exchangers belong to key process units in industry and they are characterised by high energy demands (Chen et al., 2010). Optimal processing of heat exchangers can lead to significant energy savings, especially in heat exchanger networks (Varbanov et al., 2011). Modelling heat exchangers is a difficult task because of their complex dynamics characterized by distributed parameters, non-linearity, asymmetric dynamics, transport delays and varying parameters. Most of control strategies that are able to assure optimal regime are model-based strategies and the control system does not work optimally if the dynamics of the process model differs from the dynamics of the real plant. Application of advanced control approaches is a way to solve them.

One of recently intensively developed robust control strategies is robust model-based predictive control (RMPC). Model-based predictive control (MPC) (Darby and Nikolaou, 2012) refers to a class of algorithms that optimize future behaviour of a plant and the process model is used for prediction of future process outputs. MPC technology can now be found in a wide variety of application areas. The main reasons for such popularity of the predictive control strategies are intuitiveness and explicit constraints handling (Keshavarz et al., 2010, Pannocchia et al., 2011).

Robust model-based predictive control (RMPC) represents adaptation of MPC focused on the model-plant mismatch problem (Ramirez et al., 2004), (Wang and Rawlings, 2004) and is studied in this paper. The controlled process is a tubular heat exchanger controlled by the mass flow rate of heating medium and not by the inlet temperatures (Arbaoui et al., 2007). The parameters of the state-feedback controller are generated in each sampling period and these parameters are obtained as a solution of a constrained optimization problem that is solved on infinite prediction horizon. The symmetric boundaries on control inputs are formulated in the form of LMIs and so the problem of robust stabilizing

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controller design is transformed to the solution of a convex optimization problem (Kothare et al. 1996). The RMPC of the heat exchanger is compared with the discrete-time optimal LQ controller (Diaz, 2007). The possibility to use above mentioned strategies for control of the described heat exchanger has been studied by simulation experiments.

2. Controlled heat exchanger

Controlled process is a copper co-current tubular heat exchanger, in which petroleum is heated by hot water. The heat exchanger is depicted in Figure 1. Petroleum flows in the inner tube and water flows in the outer tube. The controlled variable is the temperature of the outlet stream and the control input is the mass flow rate of hot water. The heat exchanger represents a non-linear system with transport delay. The disturbance is represented by changes of the temperature of the inlet stream of petroleum. There are also several uncertain parameters in the heat exchanger. The heat-transfer coefficient changes as the flow rate of heating media changes and there are temperature depended parameters in the heat exchanger as densities and specific heat capacities. The objective is to heat the outlet temperature of the petroleum to the reference value 46 ^{o}C and to minimise energy consumption that is necessary for heating water to the 85 ^{o}C.

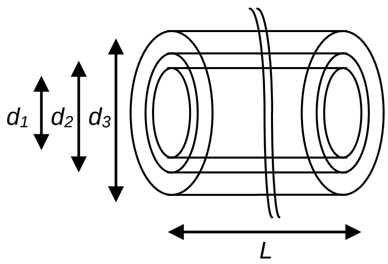

Figure 1: Tubular heat exchanger

Table 1: Parameters and steady-state inputs of the heat exchanger

Variable	Value	Unit	Variable	Value	Unit
L	2.000	m	ρ_1	kg m^{-3}	810
d_3	0.050	m	ρ_2	kg m^{-3}	8930
d_2	0.028	m	ρ_3	kg m^{-3}	1000
d_1	0.025	m	c_{P1}	kJ kg^{-1} K^{-1}	2.1
α_{32}	750	W m^{-2} K^{-1}	c_{P2}	kJ kg^{-1} K^{-1}	0.385
α_{21}	1480	W m^{-2} K^{-1}	c_{P3}	kJ kg^{-1} K^{-1}	4.186
m_1^s	0.0556	kg s^{-1}	$T_{1,0}^s$	°C	20
m_3^s	0.0417	kg s^{-1}	$T_{3,0}^s$	°C	85

Parameters and steady-state inputs for the tubular heat exchanger are enumerated in Table 1, where the L is the length of the tube, d is the diameter. Parameter α is the heat transfer coefficient, m^s is the steady state value of the mass flow, parameter ρ is the density, c_P is the specific heat capacity, T_0^s is the inlet steady-state temperature of both streams, 1 is petroleum, 2 is copper, 3 is water.
The mathematical model of the heat exchanger is represented by three nonlinear partial differential equations with delayed inputs and varying coefficients. For control system design the mathematical model of the heat exchanger was identified from input-output data in the form of a discrete-time linear state-space system in the form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), x(0) = x_0 \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where the $x(k) \in \Re^{N_x}$ represents the real vector of states, $u(k) \in \Re^{N_u}$ is the real vector of system inputs, $y(k) \in \Re^{N_y}$ is the real vector of system outputs, and matrices *A, B, C* have appropriate dimensions. The sampling period 10 s was used. As the input-output data used for identification were obtained in various initial conditions and operating points, the system (1) is an uncertain system with parametric polytopic uncertainty. Then matrices *A*, *B* represent the convex hulls of matrices A_v, B_v, v = 1,…,8, which describe the vertex systems of the uncertain system (1). The matrices of the discrete-time nominal system have following form

$$A_0 = \begin{bmatrix} 0.6068 & -0.0043 \\ 7.9169 & 0.9767 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0.0003 \\ 0.0015 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{2}$$

3. Robust model-based predictive control

The robust static state-feedback control problem in the discrete-time form can be formulated as follows: find the state-feedback control law

$$u(k) = F_k x(k) \tag{3}$$

for the system (1), where the matrix F_k represents the static state-feedback robust controller for the *k*-th control step.

The quality of the control performance can be described using the quadratic cost function *J*

$$J = \sum_{k=0}^{N} \left(x(k)^T \boldsymbol{Q} x(k) + u(k)^T \boldsymbol{R} u(k) \right) \tag{4}$$

where *N* is the number of control steps and $Q \in S_{++}^{N_x \times N_x}$, $R \in S_{++}^{N_u \times N_u}$ are the real symmetric positive-definite weight matrices of states *x*(*k*) and system inputs *u*(*k*), respectively. The matrix *Q* is 2×2 matrix in the form *Q* = diag(*q*), where diag(*q*) means the diagonal matrix with the same elements *q* on the main diagonal. The matrix *R* is in fact 1×1 matrix in the form *R = r.* The aim is to design such a controller F_k that ensures robust stability of all considered vertex systems and minimizes the quadratic cost function *J* (4).

The robust quadratic stability condition has the form

$$A_{CL,v}^T P_k + P_k A_{CL,v} \prec 0 \quad v = 1, \ldots, N_v \tag{5}$$

where $P_k \in S_{++}^{N_x \times N_x}$ is the Lyapunov matrix and N_v is the number of vertices of uncertain system. The condition (5) can be transformed using the model of the uncertain system (1) and considering the cost function (4) into following form (Kothare et al., 1996)

$$[A_v + B_v F_k]^T P_k + P_k [A_v + B_v F_k] - P_k + F_k^T R F_k + Q \prec 0 \quad v = 1, \ldots, N_v \tag{6}$$

The other demand on control performance can be to take into account the symmetric constraints on the system outputs *y*(*k*) and inputs *u*(*k*) in the form

$$\|y(k)\|_2 \le y_{max}, \quad \|u(k)\|_2 \le u_{max}, \quad |u_j(k)| \le u_{j,max}, \quad j = 1, 2, \ldots, N_u \tag{7}$$

For the Lyapunov matrix P_k and the feedback controller F_k following conditions hold

$$P_k = \lambda_k X_k^{-1}, \quad Y_k = F_k X_k \tag{8}$$

where λ_k is the auxiliary parameter, $X_k \in S^{++}$ and Y_k represent auxiliary matrices enabling the evaluation of the robust feedback controller matrix F_k in the form (Kothare et al., 1996)

$$F_k = Y_k X_k^{-1} \tag{9}$$

Using substitutions and Schur complement formula the robust stabilization problem can be transformed as the RMPC convex optimization problem based on the LMIs as follows (Kothare et al., 1996)

$$min_{\lambda_k, X_k, Y_k} \lambda_k \tag{10}$$

$$\begin{bmatrix} 1 & x_k^T \\ * & X_k \end{bmatrix} \geq 0 \tag{11}$$

$$\begin{bmatrix} X_k & X_k A_v^T + Y_k^T B_v^T & X_k \sqrt{Q} & Y_k^T \sqrt{R} \\ * & X_k & 0 & 0 \\ * & * & \lambda_k I & 0 \\ * & * & * & \lambda_k I \end{bmatrix} \geq 0 , \; v = 1, \ldots, N_v \tag{12}$$

The symmetric Euclidean norm and symmetric peak constraints on control inputs in the form (11) can be added to the optimization problem (10)-(12) in the following LMI form

$$\begin{bmatrix} u_{max}^2 I & Y_k \\ * & X_k \end{bmatrix} \geq 0 , \quad \begin{bmatrix} U_k & Y_k \\ * & X_k \end{bmatrix} \geq 0 , \quad U_{j,j}(k) \leq u_{j,\max}^2 , \quad j = 1, \ldots, N_u \tag{13}$$

Similarly, the symmetric Euclidean norm constraints on controlled outputs in the form (7), can be added into optimization problem in the following LMI form

$$\begin{bmatrix} X_k & (A_v X_k + B_v Y_k)^T C^T \\ * & y_{max}^2 I \end{bmatrix} \geq 0 \quad v = 1, \ldots, N_v \tag{14}$$

The speed of closed-loop control can be modified via setting the parameter ω used in the following LMI (Kothare et al., 1996)

$$\begin{bmatrix} \omega X_k & (A_v X_k + B_v Y_k)^T \\ * & X_k \end{bmatrix} \geq 0 \quad v = 1, \ldots, N_v \tag{15}$$

The algorithm for robust MPC can be formulated as follows (Kothare et al., 1996).
1. Set parameter $k = 0$.
2. Set number of control steps N, initial conditions of states $x(0)$, values of the symmetric constraints on control input $u_{\max}$ and output $y_{\max}$ and the value of the closed-loop speed parameter ω.
3. Set parameter $k = k + 1$.
4. Set the values of states $x(k)$.
5. Solve optimization problem described by (10) – (15) to evaluate the matrices X_k and Y_k.
6. Using (9) design the matrix F_k of the feedback controller.
7. Calculate the control input $u(k)$ using the control law (3).
8. If the parameter $k < N$ then go to the *Step 3* else *Stop*.

4. Results and discussion

The robust model predictive control of the heat exchanger was studied using simulation of control of the heat exchanger in MATLAB-Simulink. The obtained results were compared with the control performance ensured by the designed discrete-time LQ optimal controller F_{LQ} (Diaz, 2007)

$$F_{LQ} = \begin{bmatrix} -59.1633 & -2.6717 \end{bmatrix} \tag{16}$$

where the gain matrix of the controller F_{LQ} has been obtained using the weight matrices Q, R of the cost function (4). Both strategies were compared by evaluating of the energy E that was supplied for heating water from the source temperature 15 C to the temperature 85 . The energy E was calculated from the total mass of water consumed during control running 150 s. The simulation results are summarised in the Table 2, where $T_{1,0}$ is the temperature of the inlet stream of the petroleum, q, r are the coefficients in the matrices Q, R, E_{RMPC} is the energy consumed using RMPC control strategy, E_{LQ} is the energy consumed using optimal LQ control strategy, J_{RMPC} is the value of the const function (4) assured using RMPC control strategy and J_{LQ} is the value of the const function (4) assured using optimal LQ control strategy. The control performance of the *Case* 3 (Table 2) is presented in Figure 2 and associated control inputs are shown in Figure 3 as an illustrative example.

According to the presented results it is possible to state that in all tested situations the RMPC strategy leads to lower energy consumption and higher quality of control.

Table 2: Results of robust MPC and discrete-time LQ optimal control

Case	$T_{1,0}$ [°C]	q	r	E_{RMPC} [kJ]	E_{LQ} [kJ]	J_{RMPC}	J_{LQ}
1	20	100	1	1831	1857	0.145×10^6	3.517×10^6
2	20	1000	1	1834	2026	1.449×10^6	4.817×10^6
3	18	100	1	1856	1889	0.174×10^6	3.546×10^6
4	18	1000	1	1860	2216	1.740×10^6	5.094×10^6
5	16	100	1	1862	1924	0.233×10^6	3.604×10^6
6	16	1000	1	1862	2444	2.322×10^6	5.684×10^6

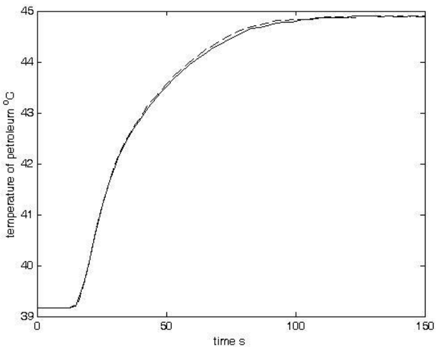

Figure 2: Control performance of the output temperature of petroleum assured using RMPC (solid) and LQ optimal (dashed) controllers (Case 3)

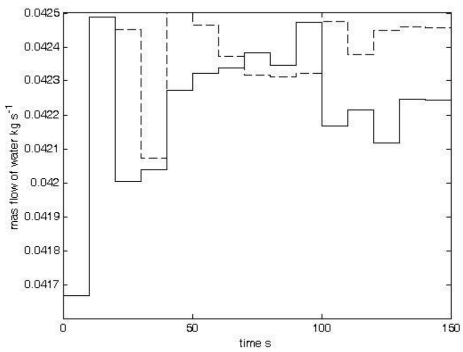

Figure 3: Control inputs generated by RMPC (solid) and LQ optimal (dashed) controllers (Case 3)

5. Conclusion

Simulation results demonstrate the effectiveness of the proposed RMPC approach because of the smaller energy consumption that is needed for production of heating medium. The example was chosen so that the investigated tubular heat exchanger can represent one tube in the shell-and-tube heat exchanger. The number of tubes in the shell-and-tube exchanger is high and in such case the energy savings become interesting. The energy savings are many times higher if such shell-and–tube heat exchangers create heat-exchanger network. Therefore it can be stated, that robust MPC strategy used in practical implementations can lead to significant energy savings. The results obtained using optimal LQ control would be better when the model of the process was perfect. In the presence of uncertainty and boundaries on control inputs and controlled outputs, the robust feedback control approach increased the quality of the control performance and the energy consumption has been reduced.

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