Minimization of Resource Requirement and Inter-Plant Cross Flow Across Resource Allocation Networks

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Overall resource requirement can be minimized by integrating many resource allocation networks (RANs). The objective of this work is to develop an algorithmic procedure to optimize the overall resource requirement by integrating multiple RANs and also to minimize the inter-plant cross flow, satisfying the targeted overall minimum resource requirement. The applicability of the proposed methodology is limited to two RANs.

1. Introduction

Process integration techniques such as pinch analysis can be applied for resource conservation. Pinch analysis techniques have been applied to single resource allocation networks (RANs). Several networks or plants may be integrated to maximize resource recovery through inter-plant integration. Various process integration methodologies such as mathematical optimization based (Keckler and Allen, 1998; Chew et al., 2008; Chen et al., 2010; Aviso et al., 2010a and 2010b; Tan, 2010) as well as pinch analysis based (Foo, 2008; Chew et al., 2010a and 2010b) approaches have been proposed. The insight-based methods do not guarantee the optimality. In this paper, an insight-based non-iterative algorithm is proposed, with rigorous mathematical results. The proposed algorithm guarantees the optimum requirement of resource and also the minimum inter-plant cross flow.

2. Problem Statement

Let's consider each plant k (k = A and B) has N_{sk} number of sources and N_{dk} number of demands. F_{sik} denotes the flow produced by the ith source of kth plant with a specified quality q_{sik} and F_{djk} indicates the flow required by the jth demand of kth plant with a predetermined maximum quality limit q_{djk}. Let f_{ikjl} represents the flow transferred from source i of plant k to demand j of plant l. Let f_{rjk} and f_{ikw} represent the flow transferred from external resource r to demand j of plant k and flow transferred from source i of plant k to waste, respectively. Due to flow and quality load conservations (Bandyopadhyay et al., 2006), the flow and quality balances for the overall plant may be written as follows.

$$\sum_{j=1}^{N_{dk}} f_{ikjl} + f_{ikw} = F_{sik} \qquad \forall \text{ source } i \in \{1, 2 \ldots N_{sk}\}, k \text{ and } l \in \{A, B\} \qquad (1)$$

$$f_{rjk} + \sum_{i=1}^{N_{sk}} f_{iljk} = F_{djk} \qquad \forall \text{ demand } j \in \{1, 2...N_{dk}\}, k \text{ and } l \in \{A, B\} \qquad (2)$$

$$f_{rjk}q_{rs} + \sum_{i=1}^{N_{sk}} f_{iljk}q_{sil} \leq F_{djk}q_{djk} \; \forall \text{ demand } j \in \{1, 2...N_{dk}\}, k \text{ and } l \in \{A, B\} \qquad (3)$$

where, q_{rs} is the resource quality. As the quality levels for every source and the resource are known, Equation (3) is linear in terms of flow variables. In order to synthesize an optimum inter-plant RAN, a sequential optimization approach is adopted here. First, objective function is to minimize the overall resource requirement (R) subject to the constraints given by Equations (1)–(3).

$$R = \sum_{k=A}^{B} \sum_{j=1}^{N_{dk}} f_{rjk} \qquad (4)$$

Subsequently, the inter-plant cross flow (F_C) is minimized subject to the constraints (1)–(4).

$$F_C = \sum_{i=1}^{N_{sk}} \sum_{\substack{j=1 \\ k \neq l}}^{N_{dk}} f_{ikjl} \qquad \forall \; k \text{ and } l \in \{A, B\} \qquad (5)$$

As all the constraints as well as the objective function are linear, both the optimization problems are linear programming problem. Algebraic algorithms are illustrated to solve these problems in a sequential manner. Before illustrating the algorithm the supported mathematical results are described in the following section.

3. Mathematical results

From the Equations (1) to (4), it can be easily realized that all the sources along with the resource are accessible to all demands. Therefore, the following mathematical result may be concluded.

Theorem 1: *Minimization of resource requirement* (4) *subject to the constraints* (1)–(3) *is equivalent to minimization of resource requirement for the overall plant having all the sources and demands of all the individual plants together.*

The minimum resource requirement for the overall plant can be determined using any of the proposed methodology. According to the principle of pinch, flow and quality load of sources, incorporating the resource and demands are balanced below the overall pinch (P_O) of the system. Hence, pinch point of A (P_A) and pinch point of B (P_B) has to be equal to overall pinch (P_O) for achieving the optimum resource requirement. As there cannot be any cross-pinch flow transfer, every RAN should be separated as two different problems; above and below the overall pinch of the system.

3.1 Determination of minimum cross flow in the below overall pinch region

Source composite curve (SSC) is the plot of cumulative quality load (M_k) versus quality. The cumulative quality load used for SCC of a resource conservation network at a

quality (q_k) for the below overall pinch region can be expressed in the following equation (Bandyopadhyay et al., 2006).

$$M_k = \begin{cases} 0 & \text{for } q_k = P_O \\ \sum_{l=1}^{k-1} F_l(q_l - q_k) & \forall q_l \in (q_k, P_O] \text{ and } q_k \in (q_{rs}, P_O) \end{cases} \qquad (6)$$

The quality load at q_{rs} is zero because of the overall quality load balance in the below overall pinch region and there is no waste generation. This leads to the following result.

Theorem 2: *At optimum resource requirement the SCC, generated by below pinch sources and demands, touches the quality axis (vertical axis) at overall pinch and resource quality.*

Let all the sources and demands from the overall plant are separated as two individual RANs (like plant A and plant B from the overall plant AB). As quality load is conserved at each quality, sum of quality load of the individual source composite curves at a quality q_k (M_{kA} and M_{kB}) are always identical to the overall quality load (M_k) at the quality q_k. This can be represented as:

$$M_{kA} + M_{kB} = M_k \quad \forall k \qquad (7)$$

The possible waste generation corresponds to the maximum reciprocal of the inverse slope of a line originating from (M_{rs}, q_{rs}) to each point on the source composite curve (waste line). According to Theorem 2, at optimum resource requirement, the value of M_{rs} is zero in the below pinch region. Hence, for resulting zero waste generation the value of M_k at each quality level has to be greater than or equal to zero.

Theorem 3: *At minimum resource requirement, SCCs of individual feasible RANs are always bounded by quality (vertical) axis and overall SSC below overall pinch.*

M_{kB} may be negative for some q_k. In this case, SSC crosses the quality axis. Based on Equation (7), the value of M_{kA} is greater than M_k, i.e., the SSC of A crosses the overall source composite curve. The negative value of M_{kB} suggests that there is a deficit of quality load in plant B and at the same time there is a surplus of mass load in plant A. Hence, for making feasible network for A and B the surplus quality load at A has to be passed to B to fulfil the deficit quality load at B. Due to flow or quality load transfer from one RAN to another, SSCs of every RAN are going to be changed. For observing simultaneous availability and requirement of quality load for individual plants, the SSC of one RAN is reflected quality axis. Therefore, M_{kA} are multiplied by –1 and denoted by RM_{kA}. Equation (7) can be rewritten as:

$$M_{kB} - RM_{kA} = M_k \quad \forall k \qquad (8)$$

From Equation (8), it can be realised that at each q_k, the horizontal distance between reflected SCC of A and SCC of B is the cumulative mass load (M_k) of overall SCC. As M_k is always greater than or equal to zero, the horizontal distance between reflected SCC of A and SCC of B at each q_k is always greater than or equal to zero. From Equations (6) - (7), we conclude that $RM_{kA} = M_{kB} = 0$ at P_O and $RM_{kA} = M_{kB}$ at q_{rs}.

Theorem 4: *The reflected SCC of a RAN below overall pinch and the SSC of other RAN never intersect each other; however, they always touch at P_O and q_{rs}.*

At optimum with feasible inter-plant cross flow, SCCs of A and B lie completely in the first quadrant (Theorem 3). This leads to the following result.

Theorem 5: *If plant A and B satisfies the minimum resource requirement of overall plant then reflected of one and SCC of other are separated by the quality axis.*

As all the M_k is changed by their sign for reflected SCC, the inverse slope of a flow line corresponds to the available flow. From Equation (6) it can be easily proved that any flow line that supplies flow from one RAN to another becomes the new quality axis. According to Theorems 4 and 5, to achieve the minimum resource requirement, the flow line should start from the P_O and ends at q_{rs} without intersecting any of the SSCs. Using the polygon law of vector addition, it can be easily proved that the feasible cross flow line with the least length corresponds to the minimum cross flow for achieving the overall minimum resource requirement (for brevity the detailed proof is not shown in this paper.).

Theorem 6: *The feasible cross flow line with the least length is the optimized cross flow line below the overall pinch.*

3.2 Determination of minimum cross flow in the above overall pinch region

In the above pinch region there is no resource requirement, hence, a resource targeting tool known as limiting composite curve (LCC) is used. Due to similarity between SCC and LCC, similar results can be obtained. For brevity detail proofs are not given. Following these results an algorithm is proposed.

4. Algorithm for minimum cross flow across two RANs.

The steps of the proposed algorithm are as follows:

Step 1: Determine the overall resource requirement, overall pinch and the contribution of pinch sources towards below (F_{ps_Bp}) and above overall pinch (F_{ps_Ap}) region using the overall flow and quality load balances.

Step 2: Separate all sources and demands of individual plants as below and above overall pinch region.

Step 3: Draw the SSC of one RAN and reflected SSC for other for below pinch region.

Step 4: Draw the feasible cross flow line with the minimum length. Let i^{th} segment, denoted by L_i, with an inverse slope of R_{Li}. The effective flow transfer by each segment L_i (F_{Li}) can be determined using the following equation.

$$F_{Li} = \begin{cases} R_{Li} & \text{when } i = 1 \\ R_{Li} - F_{Li-1} & \text{when } L_i \text{ deviated from } L_{i-1} \text{ towards RSCC} \\ F_{Li-1} - R_{Li} & \text{when } L_i \text{ deviated from } L_{i-1} \text{ away from RSCC} \end{cases} \quad (9)$$

Step 5: Draw the LCC of one RAN and reflected LCC for other for above pinch region.

Step 6: Draw the feasible cross flow line with the minimum length. Let i^{th} segment, denoted by C_i, with an inverse slope of R_{Ci}. The effective flow transfer by each segment C_i (F_{Ci}) can be determined using the following equation.

$$
F_{Ci} = \begin{cases} R_{Ci} & \text{when } i = 1 \\ R_{Ci} - F_{Ci-1} & \text{when } C_i \text{ deviated from } C_{i-1} \text{ towards RLCC} \\ F_{Ci-1} - R_{Ci} & \text{when } C_i \text{ deviated from } C_{i-1} \text{ away from RLCC} \end{cases} \quad (10)
$$

Step 7: Sum all the flow to target the minimum inter-plant cross flow.

5. Illustrative Example

The applicability of the proposed algorithm is illustrated with this example (Table 1). According to Theorem 1, the minimum resource requirement for the overall plant is targeted to be 141.46 t/h with overall pinch point at 100 ppm. This is lower than the total resource requirement of the non-integrated RANs of 153 t/h (98.33 t/h of A + 54.65 t/h of B). Based on flow balance in the below overall pinch region, F_{ps_Bp} is determined to be 98.698 t/h. Figure 1 represents the reflected SCC of A and SCC of B as well as LCC of B and reflected LCC of A. The optimized cross flow line for below overall pinch region is also shown. The line joining (0, 100) and (-457.2, 80) has positive slope, hence, the 22.86 t/h amount of flow has to be transferred from A to B at 100 ppm. The second line (-457.2, 80) and (-1536.3, 0) deviated away from the RSCC of A. Therefore, according to Equation (9) 9.37125 t/h (22.86-13.48875) is the transferred flow from B to A at 80 ppm. The LCCs, for the above overall pinch region, are already separated by the concentration axis and hence, inter-plant cross flow is not required. Therefore, 32.23 t/h of cross flow is required for achieving the minimum resource requirement. It may be noted that Foo (2008) and Chew et al., (2008) transferred 12.86 t/h of flow from plant A to B at 80 ppm and this lead to a sub-optimal resource requirement of 143.33 t/h (instead of 141.46 t/h).

Table 1: Limiting process data for the example

Plant	S	F (t/h)	C (ppm)	D	F (t/h)	C (ppm)
A	1	20.00	100	1	20.00	0
	2	66.67	80	2	66.67	50
	3	100.00	100	3	100.00	50
	4	41.67	800	4	41.67	80
	5	10.00	800	5	10.00	400
B	6	20.00	100	6	20.00	0
	7	66.67	80	7	66.67	50
	8	15.63	400	8	15.63	80
	9	42.86	800	9	42.86	100
	10	6.67	1000	10	6.67	400

6. Conclusions

Interplant water integration across two RANs is discussed in this paper. Considering union of all individual plants as a single plant, the minimum resource requirement can

be determined. A mathematically rigorous algorithm is presented to determine the minimum cross flow requirement.
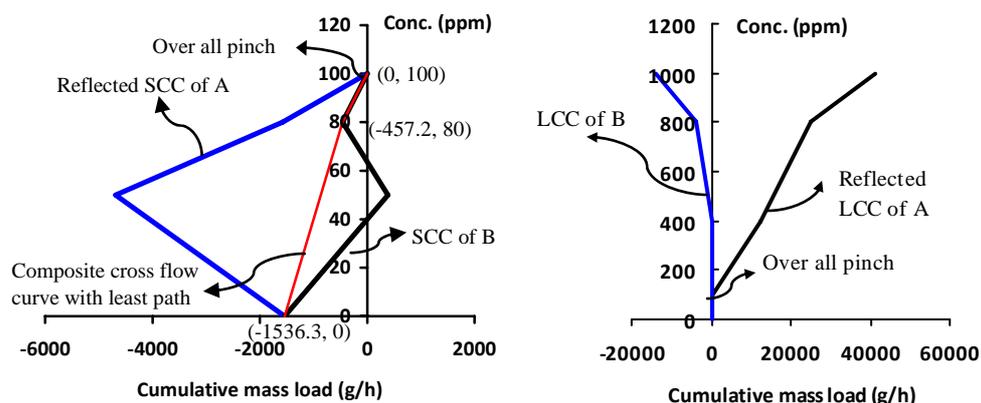
Figure 1: The reflected SCC of A and SCC of B as well as LCC of B and reflected LCC of A for the illustrative example.

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