

# Optimal Heat Exchanger Network Synthesis with Detailed Shell and Tube Heat Exchanger Design Based on Superstructure Model

Lin Sun\*, Bo-Shi Zhao, Xiong-Lin Luo

Department of Automation, China University of Petroleum, Beijing, 102249, China  
[sunlin@cup.edu.cn](mailto:sunlin@cup.edu.cn)

The shell and tube heat exchanger (SHE) is the most common type of heat transfer equipment used in heat exchanger networks (HENs) in the field of chemical process industries. Both counter-current flow and co-current flow may be involved in the shell and tube heat exchangers. To calculate the temperature difference, the correction factor  $F_T$  is generally used for multi-pass heat exchanger optimization. For synthesis of heat exchanger networks, a lot of researchers use the correction factor  $F_T$  as a constraint condition, and the HEN with minimum number of shells is optimized by iterative calculation based on a stage-wise superstructure model. However, in these studies the heat transfer temperature difference correction factor  $F_T$  are used to avoid temperature crossing without considering the optimization of shell and tube number. In this paper, a method of HEN synthesis with the optimization of the number of shells and tubes for each SHE based on superstructure model is presented. Firstly, the heat transfer process of counter-current and co-current flow in SHE is studied. The correction factor of heat transfer temperature difference  $F_T$  is calculated based on the mechanism of heat transfer process. Then, each tube of heat exchanger instead of a SHE is defined as a single unit and the energy balance function is established for each unit to minimize the total cost of HEN in the proposed superstructure model of HEN. By using the methodology of mixed integer nonlinear program (MINLP), the HEN synthesis and the correction factor of heat transfer temperature difference  $F_T$  are optimized simultaneously. The proposed methodology allows for proper handling of the trade-offs involving energy consumption, number of units, number of shells and tubes, and network area to provide a network with the minimum total annual cost. Finally, the case study results demonstrate the effectiveness of this proposed method, and the total cost of HEN are lowered as well as the number of tubes and shells for each SHE is optimized simultaneously.

## 1. Introduction

The shell and tube heat exchanger (SHE) is the most common type of heat transfer equipment used in heat exchanger networks in the field of chemical process industries. Different methods for optimization and design of the SHE have been used for the economic factors. Some scholars (Selbas et al., 2006) took the allowable pressure drop as the constraint and designed shell and tube heat exchanger based on the genetic algorithm. Patel et al. (2010) applied particle swarm optimization to design shell-and-tube heat exchangers from the perspective of economics. In recent years, the generalized disjunctive programming (GDP) was used for optimization problem formulation and the mixed-integer nonlinear programming (MINLP) was used for its solution (Mizutani et al. and Pessoa, 2003). However, all of these studies were focused on the design of heat exchangers without considering the optimization of the tubes and shells for multi-pass SHE.

In terms of the multi-pass heat exchangers, the flow arrangement involves part counter-current and part co-current flow. The effective temperature difference for heat exchanger is reduced compared with a counter-current device, which is accounted for in design by the introduction of the  $F_T$  factor into the basic heat exchanger design equation. Ahmad et al. (1988) proposed a non-interactive algebraic solution for the number of shell side passes and the  $X_P$  parameter. As an alternative approach, some equations were reported to estimate  $X_P$  for different values of  $R$  (Shenov, 1996). Although Moita (2014) proposed approaches to solve this

problem, the algorithm could sometimes lead to suboptimal designs and was difficult to use for optimum HEN synthesis. Though these researchers have proposed various methods to optimize the design of heat exchanger, it does not involve the effect of a single heat exchanger for the heat exchanger network (HEN) optimal integration.

Recently, there are also a lot of studies which presented about the synthesis of the HEN with multi-pass SHE. Kravanja et al. (2002) considered different types of the heat exchangers in the superstructure model and optimized the HENs and the heat exchangers simultaneously. Scholars computed the number of shells in a HEN firstly and the design began by assuming a specific number of shells, then the  $F_T$  value was evaluated (Wang and Sundén, 2001). A mixed integer non-linear programming formulation for this optimization problem which used the correction factor as a constraint to calculate the minimum shell number was proposed (Ponce-Ortega et al., 2006). Sun et al. (2011) proposed a methodology to analysis the number of tube passes and the minimum temperature difference by using the composite curves and problem table. These studies focused on the synthesis of HEN with the shell and tube optimization as a constraint to avoid temperature cross.

Polley and Shahi (1991) took account of the pressure drop in network while designing heat exchanger networks. Some scholars took account of the selection of heat exchanger and heat transfer enhancement equipment selection during the integration optimization of heat exchanger network. Ebert and Panchal (1995) used simulated annealing algorithm and MILP optimization methods to optimize the performance considerations of heat transfer enhancement of the HEN. In recent years, some researchers have carried out the design of heat exchanger based on the optimal synthesis of HENs (Ravagnani et al., 2005). In these studies, the pressure drop and the transfer coefficient are mainly considered.

In this work, the synthesis of HEN with the optimum number of shells and tubes for each SHE based on superstructure model is presented. Firstly, the correction factor of heat transfer temperature difference  $F_T$  is calculated based on the mechanism of heat transfer process. Then, to minimize the total cost based on the proposed superstructure model of a HEN, the heat transfer unit is represented as the heat transfer process in one tube and one shell of the SHE. Finally, the HEN synthesis and the correction factor of heat transfer temperature difference  $F_T$  are optimized simultaneously.

## 2. Superstructure model for multi-pass HEN synthesis

Based on the stage-wise superstructure model of heat exchanger network with no split streams proposed by (Yee and Grossmann, 1990), this paper builds a stage-wise superstructure model of multi-pass heat exchanger network with no split streams. Not only must the number of the shells and the tubes but also the optimization results of the HEN be calculated. At the same time, the proposed superstructure model is modified. The tubes are defined as the separate units instead of the heat exchangers, then the superstructure model is established. At last, this paper uses the formulation of MINLP to solve the model.

As Figure 1 shows, the modified stage-wise superstructure, and it consists of  $k$  stages. In the Figure1, if the tubes are existed, they are showed as '○', while they are showed as '○' when they are not existed. The number of tubes  $n_t$  can be obtained by the optimization of the HEN.

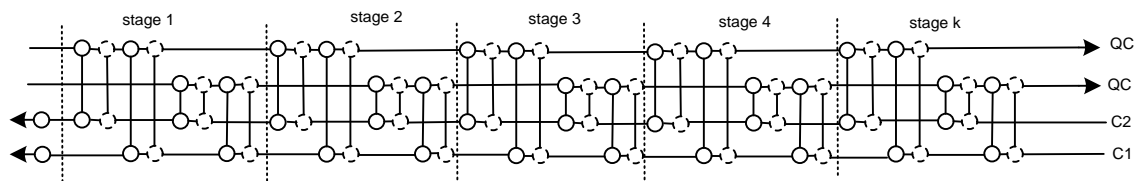


Figure 1: Stage-wise superstructure of multi-pass HEN with no split streams

Here,  $n_t$  is the number of tubes, and  $n_s$  is the number of shells. The note  $i$  indicates the  $i$ th heat flow while  $j$  indicates the  $j$ th cold flow. The number of stages is  $k$ . The specific heat capacity is  $C_p$ . The balance equations of the tube pass are,

$$q_t = C_{pi}(T_{it,in} - T_{it,o}) \quad (1)$$

$$q_{jk} = C_{pj}(T_{jk} - T_{jk+1}) \quad (2)$$

Where,  $q$  means the heat loads, and  $T$  indicates the temperature for the inlet and outlet of the tube side. The energy balance equation of the multi-pass SHE is

$$q_{ijk} = \sum_{s=1}^{n_s} q_{s,ijk} = \sum_{s=1}^{n_s} \sum_{t=1}^{n_t} q_{t,ijk} \quad (3)$$

Where,  $s$  indicates the shell and  $t$  indicates the tube. The  $q_{ijk}$  means the heat transfer loads for the  $i$ th heat flow and the  $j$ th cold flow in the  $k$ th stage.

The heat transfer area  $A$  for each SHE is calculated by using following equation,

$$A_{ijk} = \frac{q_{ijk} n_{t,ijk}}{K_{ijk} \sum_{t=1}^{n_{t,ijk}} \Delta_{LM} T_{ijk}} \quad (4)$$

Where,  $K$  notes the heat transfer coefficient. Where  $\Delta_{LM} T_{ijk}$  indicates the logarithmic mean temperature difference for the  $t$ -th tube with the  $s$ -th shell in the  $k$ th stage.

For the counter-current flow,

$$\Delta_{LM} T_{ijk}^{cu} = \frac{(T_{h,ijk}^{in} - T_{c,ijk}^{out}) - (T_{h,ijk}^{out} - T_{c,ijk}^{in})}{\ln \frac{T_{h,ijk}^{in} - T_{c,ijk}^{out}}{T_{h,ijk}^{out} - T_{c,ijk}^{in}}} \quad (5)$$

For the co-current flow,

$$\Delta_{LM} T_{ijk}^{co} = \frac{(T_{h,ijk}^{in} - T_{c,ijk}^{in}) - (T_{h,ijk}^{out} - T_{c,ijk}^{out})}{\ln \frac{T_{h,ijk}^{in} - T_{c,ijk}^{in}}{T_{h,ijk}^{out} - T_{c,ijk}^{out}}} \quad (6)$$

Where,  $\Delta_{LM} T_{ijk}^{cu}$  is the countercurrent flow logarithmic mean temperature difference for the  $t$ -th tube with the  $s$ -th shell in the  $k$ -th stage, and where  $T_{h,ijk}^{in}$  and  $T_{h,ijk}^{out}$  denote the inlet and outlet temperature of the tube;  $T_{c,ijk}^{in}$  and  $T_{c,ijk}^{out}$  are the shell inlet and outlet temperature.  $\Delta_{LM} T_{ijk}^{co}$  is the co-current flow logarithmic mean temperature difference for the  $t$ -th tube with the  $s$ -th shell in the  $k$ -th stage.

Use the binary system to show the existence of the tube,

$$z_{ijk} = \begin{cases} 1, & q_{ijk} > 0 \\ 0, & q_{ijk} = 0 \end{cases}, \quad i \in NH, j \in NC, k \in NK \quad (7)$$

Based on this proposed superstructure model of the multi-pass HEN, the number of tubes and the HEN are synthesized simultaneously. By calculating the logarithmic mean temperature difference  $\Delta_{LM} T_{ijk}$  for co-current side and countercurrent side separately, the  $F_T$  factor is also calculated.

### 3. Heat Exchanger Network synthesis with detailed equipment design

Based on the provided superstructure model of multi-pass HEN, a two-stage strategy is used to resolve this problem. Firstly, the HEN is synthesized with the optimization of the number of tubes and shells based on the proposed superstructure model. In the second stage, the SHEs are designed in detail based on the TEMA standard.

In order to calculate the  $F_T$  of the heat exchangers and the heat transfer area of the heat exchangers, the main steps of the algorithm are as shown in Figure 2 based on the presented superstructure model and the energy balance.

As shown in Figure 2 to synthesize the multi-pass HEN, an initial heat transfer coefficient  $K_0$  is defined, and then the superstructure model of HEN established. Based on this proposed model, the HEN is synthesized and the number of shells and tubes is calculated. The temperature difference correction factor  $F_T$  is also calculated and verified. Based on this first stage, the detailed equipment design can be calculated based on the TEMA standards. Consequently, by the iterative calculation the optimal HEN with detailed equipment design is obtained.

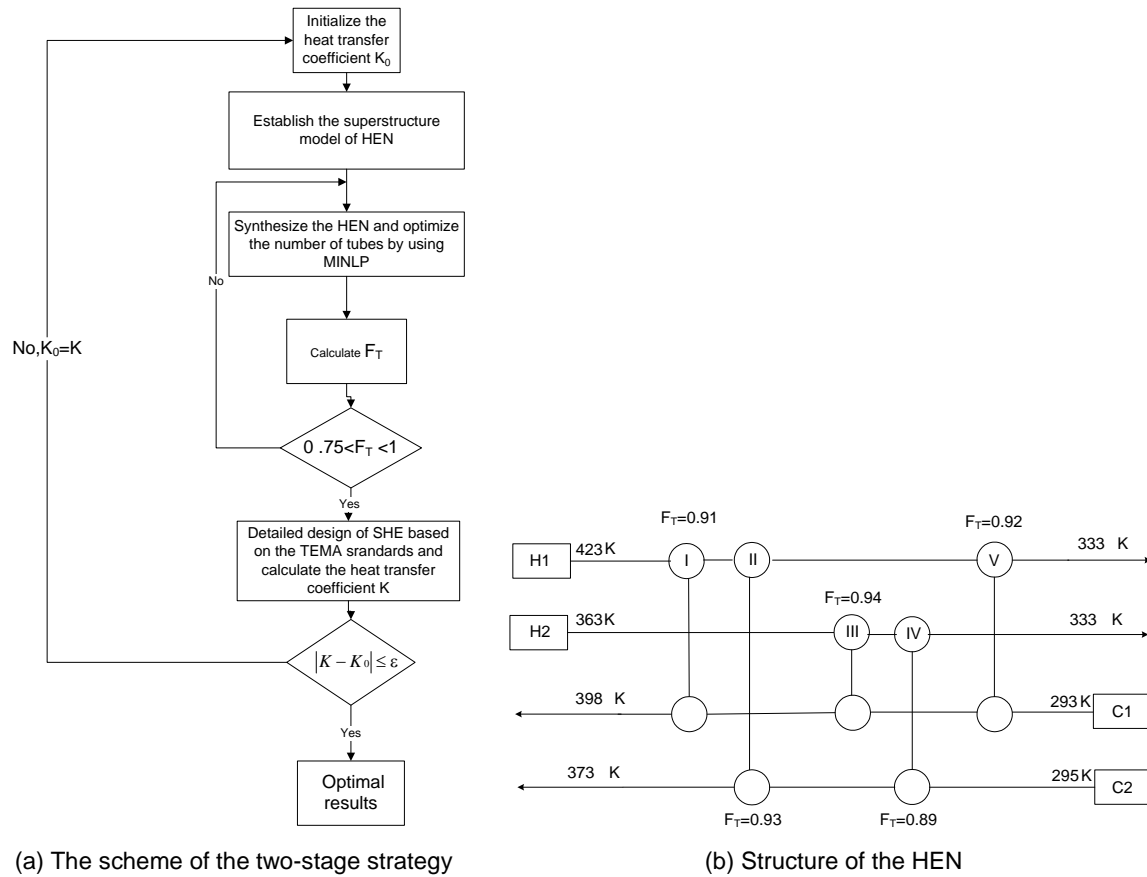


Figure 2: Scheme of the strategy and the structure

#### 4. Case Study

A case study is used and the basic stream data is listed in Table 1 (Osman et al., 2014). The case consists of two hot streams, two cold streams, one hot utility and one cold utility. Note that in this case it is assumed that all heat exchangers have only one shell, and optimize the number of the tubes of the whole network. Then the  $F_T$  of each heat exchanger can be calculated simultaneously based on the proposed equations.

Table 1: Basic data of streams and heat exchangers

Stream	H1	H2	C1	C2
$T_{in} / K$	423	363	293	295
$T_R / K$	333	333	398	373
Density / $kg \cdot m^{-3}$	850	850	995	995
Heat capacity / $kJ \cdot (kg \cdot K)^{-1}$	4	4	2.5	2.5
Flowrate / $kg \cdot s^{-1}$	5	20	10	12
Viscosity / $10^{-3} pa \cdot s$	0.40	0.40	0.358	0.358
Heat conductivity / $W \cdot (m \cdot s)^{-1}$	0.13	0.13	0.13	0.13
Fouling factor / $10^{-3} m^2 \cdot K \cdot W^{-1}$	0.61	0.61	0.61	0.61

Table 2 shows the comparison between case study and reference. It is obvious that the new method costs less than the previous one. Table 3 shows the design of the heat exchangers in detail.

Many papers calculated the number of the shells of the multiple heat exchangers and regarded the  $F_T$  factor as a constraint condition. Compare with the above method, this paper proposes a method which optimizes the number of the tubes of the HEN and calculates the  $F_T$  factor of each heat exchanger and designs heat

exchanger in detail simultaneously. The result of the method shows that the new method is more accurate than the other one.

Table 2: Comparison between case study and reference

Heat exchanger No.	Area /m <sup>2</sup>		F <sub>T</sub>		Cost /10 <sup>5</sup> \$	
	Case study	Osman et al. (2014)	Case study	Osman et al. (2014)	Case study	Osman et al. (2014)
I	272.903	134.79	0.91	0.95		
II	291.09	197.525	0.93	0.93		
III	260.12	297.465	0.94	0.95	7.298	8.256
IV	562.54	566.2	0.96	0.98		
V	109.39	115.19	0.91	0.95		

Table 3: Results of heat exchanger detailed design

Heat Exchangers	I	II	III	IV	V
A / m <sup>2</sup>	272.903	291.09	260.12	562.54	109.39
N <sub>s</sub>	1	1	1	1	1
N <sub>t</sub>	2	2	8	4	2
D <sub>s</sub> / m	0.716	0.737	0.723	1.01	0.465
arr	Squ	Squ	Squ	Squ	Squ
d <sub>o</sub> / m	0.025	0.025	0.025	0.025	0.025
L / m	6.096	6.096	6.096	6.096	3.658
B / m	0.143	0.147	0.144	0.202	0.093
n	750	800	715	1546	290
δ / m	0.002	0.002	0.002	0.002	0.002
P <sub>t</sub> / m	0.025	0.025	0.025	0.025	0.025
h <sub>t</sub> / W·m <sup>-2</sup> ·K <sup>-1</sup>	98.73	99.92	259.80	192.70	431.46
h <sub>s</sub> / W·m <sup>-2</sup> ·K <sup>-1</sup>	1,452	1,960	1,638	1,003	2,212
K / W·m <sup>-2</sup> ·K <sup>-1</sup>	68.18	69.51	145.2	102.76	174.3

Table 2 shows the comparison between case study and reference. It is obvious that the new method costs less than the previous one. Table 3 shows the design of the heat exchangers in detail.

Many papers calculated the number of the shells of the multiple heat exchangers and regarded the F<sub>T</sub> factor as a constraint condition. Compare with the above method, this paper proposes a method which optimizes the number of the tubes of the HEN and calculates the F<sub>T</sub> factor of each heat exchanger and designs heat exchanger in detail simultaneously. And the result of the method shows that the new method is more accurate than the other one.

## 5. Conclusions

In this paper, by using the superstructure model, a multi-pass heat exchanger network synthesis methodology is presented in order to minimize total cost and achieve energy saving. Based on the proposed superstructure model, the number of tubes and shells are optimized and the HEN is synthesized simultaneously. By using the TEMA standards the detailed SHE design is obtained finally and the transfer coefficient is calculated. In this paper, the proposed method is applied to a classic example, and the contrastive analysis results demonstrate the effective of this proposed method.

## Acknowledgments

Chapter 2 The authors are pleased to acknowledge the financial support by the National Natural Science Foundation of China (NO. 21676295), the Science Foundation of China University of Petroleum, Beijing (2462015YQ0511).

### Nomenclature

A	Heat transfer area, m <sup>2</sup>	n	Number of tubes
B	Bare module factor	Q	Heat duty, w
D <sub>S</sub>	Shell diameter, m	P <sub>t</sub>	Tube pitch, m
d <sub>o</sub>	Tube outside diameter, m	h <sub>t</sub>	Tube side heat transfer coefficient, W·m <sup>-2</sup> ·K <sup>-1</sup>
K	Heat transfer coefficient, W·m <sup>-2</sup> ·K <sup>-1</sup>	h <sub>s</sub>	Shell side heat transfer coefficient, W·m <sup>-2</sup> ·K <sup>-1</sup>
L	Tube length, m	ΔT	Temperature difference, K

### References

- Ahmad S., 1985, Heat exchanger networks: Cost trade-offs in energy and capital. PhD thesis, University of Manchester Institute of Science and Technology, UK.
- Ebert W., Panchal C.B., 1995, Analysis of Exxon crude-oil-slip stream coking data[R]. Argonne National Lab, IL, US.
- Gulyani B.B., Khanam S., Mohanty B., 2009, A new approach for shell targeting of a heat exchanger network. *Computers and Chemical Engineering*, 33, 1460-1467.
- Jegade F.O., Polley G.T., 1992, Optimum heat exchanger design: process design. *Chemical Engineering Research and Design*, 70, 133-141.
- Kravanja, 2002, Simultaneous MINLP synthesis of heat exchanger networks comprising different exchanger types. *Computers & Chemical Engineering*, 26, 599-615.
- Mizutani, Pessoa., 2003, Mathematical programming model for heat-exchanger network synthesis including detailed heat exchanger designs. *Ind Eng Chem Res*, 42, 4009-4018.
- Moita R.D., Fernandes C., Matos H.A., Nunes C.P., 2004, A cost-based strategy to design multiple shell and tube heat exchangers, *ASME J. Heat Trans*, 126, 119-130.
- Osman A., Zhang J., Sun L., 2014, Research advances in the design for shell-and-tube heat exchangers of heat exchanger networks. *Computers and Applied Chemistry*, 32, 34-36.
- Patel V.K., Rao R.V., 2010, Design optimization of shell-and-tube heat exchanger using particle swarm optimization technique. *Applied Thermal Engineering*, 30, 1417-1425.
- Polley G.T., Shahi M.H., 1991, Interfacing heat exchanger network synthesis and detailed heat exchanger design. *Chemical Engineering Research and Design*, 69, 445-457.
- Ponce-Ortega J.M., Serna-González M., Salcedo-Estrada L.I., Jiménez-Gutiérrez A., 2006, Minimum-investment design of multiple shell and tube heat exchangers using a MINLP formulation, *Chemical Engineering Research & Design*, 84, 905-910.
- Ravagnani M.A.S.S., Silva A.P., Arroyo P.A., Constantino A.A., 2005, Heat exchanger network synthesis and optimization using genetic algorithm. *Applied Thermal Engineering*, 25, 1003-1017.
- Selbas R., Kizilkan O., Reppich M., 2006, A new design approach for shell-and-tube heat exchangers using genetic algorithms from economic point of view. *Chemical Engineering and Processing*, 45, 268-275.
- Shenoy, 1995, Heat Exchanger Network Synthesis—Process Optimization by Energy and Resources Analysis, Gulf Publishing Company, USA.
- Sun L., Luo X.L., Zhao Y., 2015, Synthesis of multi-pass heat exchanger network with the optimal number of shells and tubes base on pinch technology, *Chemical Engineering Research Design*, 93, 185-193.
- Wang L., Sundén B., 2001, Detailed simulation of heat exchanger networks for flexibility consideration. *Applied Thermal Engineering*, 21, 1175-1184.
- Yee T.F., Grossmann I.E., 1990, Simultaneous optimization models for heat Integrations II: heat exchanger network synthesis. *Computers & Chemical Engineering*, 14, 1165-1184.