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Stability Analysis of Type-2 Fuzzy Logic Controllers

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The application of the direct Lyapunov method to the stability analysis of systems controlled by type-2 fuzzy logic controllers (FLC) is presented. The method is an extension of a method proposed for type-1 fuzzy systems. It is usually applied to systems described by state equations and controlled by fuzzy controllers using state variables as inputs but has been extended to controllers that have the error and the integral of error of the controlled variable as inputs.
The proposed method allows to modify the controller rule base so that the controlled system is stable in the operating range defined by the manipulative variable constraints.
The method is applied to the stability analysis of a bioreactor and of a CSTR controlled by type-2 FLCs.

1. Stability analysis of systems controlled by FLCs

Fuzzy controllers are mainly used for the control of non linear systems. Fuzzy controllers themselves are non linear systems. For the stability analysis of systems controlled by FLCs traditional linear stability analysis based on the frequency response methods (Bode and Nyquist criteria) or on the s-plane methods (Routh-Hurwitz and Nyquist criteria) cannot be used. It is often said that the stability analysis of fuzzy control systems is a weak point of this control technique. Actually many methods have been successfully applied to this problem.

1.1 Methods for the stability analysis of fuzzy systems

A good covering of these methods can be found in Kandel et al. (1999). Two different approaches can be used:

- a qualitative and intuitive approach
- the approach of the non-linear theory.

To the first type it belongs the energetic approach by Kinszka et. al. (1985). They use a function, representative of the energetic level of the system analyzing the behavior of the function when some parameters of the fuzzy controller change. If the energy increases the system is unstable while if it wavers the system oscillates.
Many non-linear stability analysis methods have been extended to fuzzy control systems. Kickert and Mamdani (1978) were the first to use the describing function method. Enhancements to the same method were introduced by Leephakpreeda and Batur (1997) who substituted the fuzzy controller with an appropriate describing

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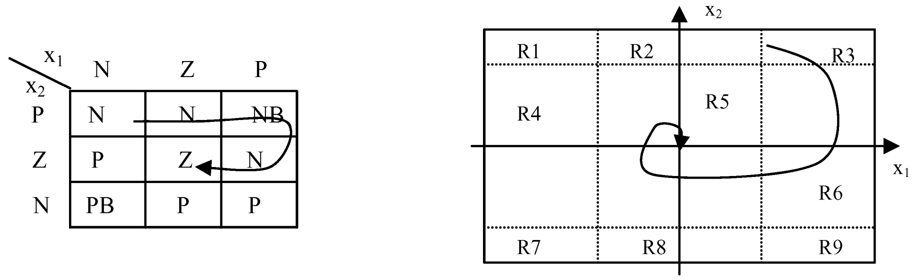

(a) (b)

Fig. 1 – Trajectory of a fuzzy control action on the rule table (a) and on the phase plane (b)

function and then used the Nyquist stability criterion. Braee and Rutherford (1979) proposed the phase plane analysis, limited to systems with no more than two inputs (state variables). The trajectory of the system in the state plane can be seen in the rule table (Fig. 1a) or in the state plane (Fig. 1b) that is segmented in areas where different rules apply. If the trajectory converges to the origin the system is stable. A geometric interpretation of the method was given by Aracil et al. (1988). The circle method was proposed by Ray and Majumder (1984) and more recently by Ban and al. (2006). The Popov method was proposed by Kandel et al. (1999).

However the attention of the majority of researchers has been paid to the Lyapunov method.

1.2 The Lyapunov method for the stability analysis of fuzzy systems

Before exposing how the Lyapunov method is applied to fuzzy control systems a brief presentation is given. We refer to the Lyapunov direct method for the global stability.

1.2.1. The Lyapunov direct method

Let consider a system defined by

$\dot{\mathbf{x}} = \mathrm{f}(\mathbf{x})$

and let be $\mathbf{x} = \mathbf{0}$ an equilibrium point. If a scalar function $\mathrm{V}(\mathbf{x})$ exists such that the following properties are verified at the same time:

1. $\mathrm{V}(\mathbf{x})$ is globally positive definite
2. $\dot{\mathrm{V}}(\mathbf{x})$ is globally negative definite
3. $\mathrm{V}(\mathbf{x})$ is radially unlimited

then the equilibrium point is globally asymptotically stable. V(x) is called Lyapunov function and the stability study consists in finding such a function.

1.2.2. Extension of the Lyapunov method to fuzzy control systems

The first to study the application of the Lyapunov stability method to fuzzy systems were Tanaka and Sano (1991). They suggested a procedure to construct a Lyapunov function. Xiu and Ren (2005) discussed the stability analysis based on the extended Lyapunov method and the design techniques for Tagaci-Sugeno fuzzy control systems. Wong et al. (2000) proposed a different procedure that is used here as basis for the application to type-2 fuzzy control systems and that will be illustrated with some detail.

A fuzzy controller consists of a set of rules. The generic rule generates a specific output u_i with a specific membership grade μ_i. The control action u will be a function of all u_i and μ_i. Let the controlled system be defined by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})g(u)$$

with **x** the state vector, **f** and **b** vectorial functions of **x**, g(u) a scalar function of the output of the fuzzy controller u (sum of all u_i μ_I products divided by the sum af all μ_I, if the defuzzification method of the average of centers is used).
Let consider an input z_0 to the controller: if the membership function of z_0 for the rule i is zero then the rule i is said to be inactive for z_0, vice versa it is active.
An active region Z_r of a fuzzy rule is defined as the region in which the rule is active for any z (input) belonging to Z_r. This means that every rule is usually active only in part of the operative range of the controller.
A fuzzy subsystem associated with the rule i is constituted by the system controlled only by u_i, the output of the rule i.
Wong et al. (2000) demonstrated that it is possible to analyze individually the Lyapunov stability of subsystems associated with every rule instead of analyzing the stability of the whole system. This extremely facilitates the analysis.
For each rule it is necessary to determine the range for which the rule is applied and the required value of the manipulative variable.
By substituting these values in the derivative of the Lyapunov function, this should be negative for all the values of the range, in order the system be stable.
The rules for which the derivative of the Lyapunov function is positive are the rules that may lead the system to the instability. They must be modified and the method may provide indications on how to modify them in order to make the system stable.

2. The Lyapunov method for the stability analysis of type-2 fuzzy systems

A type-2 FLC, just as a type-1 FLC, contains four components: Rules, Fuzzifier, Inference-engine and Output-processor. The Output-processor of a type-1 FLC is just a Defuzzifier, while the Output-processor of a type-2 FLC contains two components: the first maps a type-2 fuzzy set into a type-1 fuzzy set and the second performs the defuzzification on the latter set.
A type-2 fuzzy set $\tilde{A}$, characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ *x* and , $u \in J_x \subseteq [0,1]$, is given by

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \tag{1}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$.

In (1) $\mu_{\tilde{A}}(x,u)$ is a *secondary grade* and the domain of a secondary membership function is called the primary membership of x.
Uncertainty in the primary memberships of a type-2 fuzzy set, $\tilde{A}$, consists of a bounded region that is called the Footprint of Uncertainty (FOU). The FOU characterizes type-2 fuzzy sets and is defined as the union of all primary membership functions:

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \qquad (2)$$

The FOU is associated with the concepts of lower and upper membership functions and models the uncertainties in the shape and position of the type-1 fuzzy set.

The application of the Lyapunov stability method proposed by Wong (2000) to systems controlled by type-2 fuzzy controllers must take into account the form of the type-2 fuzzy sets considering the upper membership functions only, since the lower membership range is included in the upper one. If fuzzy sets need to be changed in order to make the system stable only the upper membership functions have to be modified.

3. Application of the method

3.1 Control of a bioreactor

The first example regards the control of a simple bioreactor with only two components: substrate (x_1) and biomass (x_1). The dynamic model and parameter values are the same used by Bequette (1998). The manipulative variable to control the substrate concentration is the dilution rate that is also a bifurcation parameter, as shown in the equilibrium curve reported in Fig. 2 .

It easy to note three regions: two regions are characterized by stable equilibrium points (low and high branch) and one (in the middle, delimited by LP (limit point) and BP (branch point) characterized by instability.

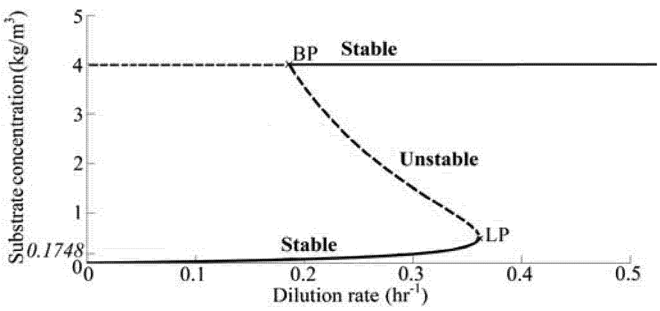

Fig. 2 - Bioreactor equilibrium curve

The proposed Lyapunov function, positive definite and radially unlimited, is given by:

$$V(x_1, x_2) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

The time derivative is :

$$\dot{V}(x_1, x_2) = \left(x_1\dot{x}_1 + x_2\dot{x}_2\right)$$

It is easy to verify that the derivative of the Lyapunov function is negative in the lower an upper branches, while it is positive in the intermediate region.

To control the bioreactor substrate a type2-FLC with zero order Sugeno inference has been designed with both the state variable as inputs. Each controller input uses 7 type-2 fuzzy sets with Gaussian membership functions. The rule set contains 49 rules.

The application of the proposed stability method requires the verification of the Lyapunov criterion for any subsystem constituted by a rule and the system to be controlled.
The derivative of the Lyapunov function is calculated for each rule i, with reference to the membership functions of the inputs used in the rule i. It is necessary to verify that the derivative is always negative between the extremes of the membership functions range in order the subsystem composed by the i-rule be stable. It was found that the derivative is negative for all the subsystems related to the 49 rules. Therefore the controlled system is globally stable.

3.2 Control of a CSTR

The second example concerns a CSTR that presents saddle-node and Hopf bifurcations. The model and all details of the reactor can be found in Perez and Albertos (2004). A type-2 FLC for the control of the reactor temperature, that uses the error and integral of error of the controlled variable, has been implemented. The controller has been optimized using a neural network algorithm to reduce the number of rules from 49 to 9. The performance of the controller is very high despite the limited number of rules.
For this system the candidate Lyapunov function and its time derivative are:

$$V(x_3) = \frac{1}{2}\left(x_3^2\right)$$

$$\dot{V}(x_3) = 0.0555x_3 - 1.5x_3^2 + 111 \cdot 10^9 x_2 x_3 e^{-\frac{1}{x_3}} - \frac{25.15 \cdot 19.79 \cdot x_5 x_3 (x_3 - 0.0353)}{19.79x_5 + 241.05}$$

with x_2 and x_3 dimensionless concentration and temperature.
The procedure for the stability analysis is similar to the one used in the previous example with only a difference: in this case, because a PI fuzzy feedback control is implemented only on x_3, the value of the state variable x_2 that corresponds to the extreme value of the membership functions relative to the errors must be determined directly from the equilibrium conditions of the system.
In this case the derivative of the Lyapunov function is negative for six rules and positive for the other three.
Despite that the control of the reactor, simulated using several different set point and disturbance changes, has proved to be stable. To be sure that the system is stable in all conditions we should anyway modify the three rules for which the derivative of the Lyapunov function is negative. This has been carried out, but the modification of rules leads to a stable system but also to a deterioration of the controller performance.

4. Conclusions

The extension of a method for the stability analysis of systems controlled by type-1 FLCs has been extended to the analysis of systems controlled by type-2 FLCs. The method usually applied to systems described by state equations and controlled by fuzzy controllers that use state variables as inputs can be applied also to controllers that have the error and the integral of error of the controlled variable as inputs.

The method allows to carry out the stability analysis of complex non linear fuzzy systems through the analysis of a number of subsystems, that although tedious it is easier than the analysis of the whole system, that may be unpractical if not impossible.
The analysis allows to identify rules that have to be modified in order to have a stable system, but does guarantee that the stabilized system has a good control performance. To obtain that it could be necessary to redesign the controller by changing all the controller rules.

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