

The Role of Shrinkage on Food Isothermal Drying: a Moving Boundary Model

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A moving-boundary model is proposed for describing food isothermal drying. The key point lies in the introduction of a point wise shrinkage velocity equal and opposite in sign to the water diffusive flux times a shrinkage factor representing the fingerprint of the food material under investigation. The model provides good results in terms of prediction of time evolution of moisture content and sample volume and furnishes an accurate and reliable estimate of water diffusivity at different operating temperatures.

1. Introduction

Mathematical modelling provides deep insights towards a better comprehension of physical processes in chemical engineering. Several new theoretical approaches now available and covering different time and length scales (Brasiello et al., 2016, 2017; Hoogerbrugge and Koelman, 1992) open new perspectives in theoretical analysis of transport phenomena (Giona et al., 2015, 2017). Food engineering is a research field in which new techniques may be promisingly applied (Brasiello et al., 2011), some of them bridging the scales between microscopic and macroscopic continuum approaches (Brasiello et al., 2010). Recently, a moving-boundary model for food dehydration has been proposed (Adrover et al., 2019a,b) that overcomes all the intrinsic limitations of empirical or semi-empirical approaches quite often adopted in food science. Sample volume reduction is modelled through the introduction of a point-wise shrinkage velocity whose dependence on water content evolution represents the constitutive equation for the food material under investigation. In this paper, a comparison between results obtained with the moving-boundary model and a standard (Fickian, fixed boundary) approach is carried out for the case of chayote isothermal drying.

2. Mathematical model

The paper presents a transport model able to accurately describe the time evolution of both the water content and the sample volume of a food matrix during isothermal drying. The food matrix, made of pulp and water, is assumed to be an isotropic and homogeneous media. Let $\mathbf{v}(\mathbf{x},t)$ be the point-wise shrinkage velocity. The water mass balance in a fixed reference frame, written in terms of the water volume fraction ϕ , reads as

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) = -\nabla \cdot \mathbf{J} = \nabla \cdot (D \nabla \phi), \quad \mathbf{x} \in V(t) \quad (1)$$

where $\nabla \cdot (\phi \mathbf{v})$ is a convective term arising from local shrinkage, and D is the water diffusivity. A functional form for \mathbf{v} , together with an equation describing the evolution of the boundary, is needed.

From Eq. (1), the macroscopic balance over the volume $V(t)$ attains the form

$$\frac{d}{dt} \int \phi dV = \oint (D \nabla \phi) \cdot \mathbf{n} dS - \oint \phi (\mathbf{v} - \mathbf{v}_s) \cdot \mathbf{n} dS \quad (2)$$

in which the boundary shrinkage velocity \mathbf{v}_s is introduced, i.e. the shrinkage velocity at any point \mathbf{x}_b on the boundary $S(t)$ of $V(t)$. By considering that the total amount of water leaving the volume $V(t)$ per unit time must

be equal to the integral of diffusive flux over the boundary $S(t)$, Eq. (2) implies that, in principle, any functional form of \mathbf{v} can be chosen but the boundary velocity has to satisfy the condition $\mathbf{v}_s = \mathbf{v}(\mathbf{x}_b)$.

By enforcing the analogy between food shrinkage and swelling of rubbery polymers (Adrover and Nobili, 2015), the following constitutive equation for \mathbf{v} is proposed (Papanu et al. 1989)

$$\mathbf{v} = -\alpha(\phi)\mathbf{J} = \alpha(\phi)D\nabla\phi \quad (3)$$

that implies that the point-wise shrinkage velocity \mathbf{v} is proportional to the water diffusive flux times a proportionality function $\alpha(\phi)$ tuning the relationship between \mathbf{J} and volume reduction. For a swelling process, $\alpha(\phi)=1$ since the solvent and the polymer are incompressible and mix with no volume change. For a food dehydration process, $\alpha(\phi)$ can be equal, greater or smaller than unity because sample volume can decrease of a quantity that can be equal, smaller or larger than the corresponding volume of released water. Moreover, $\alpha(\phi)$ is intrinsically a function of ϕ because the rate of volume reduction may change during the course of the dehydration process. In point of fact, $\alpha(\phi)$ represents the fingerprint of the food material under investigation. It can be assumed a priori or it can be determined from experimental data.

By substituting Eq. (3) into Eq. (2), the water volume fraction ϕ satisfies the following moving-boundary transport equation and boundary condition

$$\frac{\partial\phi}{\partial t} = \nabla \cdot (D\nabla\phi(1 - \alpha\phi)), \quad \mathbf{x} \in V(t) \quad (4)$$

$$-\nabla\phi \cdot \mathbf{n}|_{\mathbf{x}_b} = Bi_m(\phi|_{\mathbf{x}_b} - \phi_{eq}), \quad \mathbf{x}_b \in S(t), \quad Bi_m = \frac{h_m L_r K_p \rho_{air}}{D \rho_s} \quad (5)$$

with initial conditions $\phi(\mathbf{x},0)=\phi_0$, $V(0)=V_0$, $S(0)=S_0$. Bi_m is the mass Biot number accounting for mass transfer resistance at the solid-air interface, L_r is a reference length, h_m is a mass transfer coefficient [m/s], ρ_{air} is the air density on dry basis [kg dry air/m³], ρ_s is the solid (pulp) density [kg pulp/m³ product] and K_p is the water partition ratio between the gas and the solid phases. In order to estimate the shrinkage factor $\alpha(\psi)$, it is possible to rely on the experimental data of volume reduction V/V_0 vs moisture ratio X/X_0 on the basis of the following considerations. The macroscopic balance equation Eq.(2) can be rewritten as

$$\frac{dM_w}{dt} = \oint \rho_w \frac{\mathbf{v} \cdot \mathbf{n}}{\alpha(\psi)} dS \simeq \frac{\rho_w}{\alpha(\psi_b)} \oint \mathbf{v} \cdot \mathbf{n} dS = \frac{\rho_w}{\alpha(\psi_b)} \frac{dV}{dt} \quad (6)$$

where M_w is the amount of water in the sample at time t and $\psi_b = \phi/\phi_0$ is the normalized water volume fraction in a suitable point on the sample surface, called probe point P , whose position evolves in time together with the sample surface on which it is initially attached. The probe point P can be reasonably chosen as a point characterized by the maximum shrinkage. For example, for a discoid or a slice food sample P is a point in the center of the top (or bottom) surface sample. For a long thin cylindrical sample, P is a point on the lateral cylindrical surface at half height. Eq. (6) can be rewritten in terms of the moisture ratio X/X_0 as

$$\frac{d(X/X_0)}{dt} \simeq \frac{1}{\phi_0 \alpha(\psi_b)} \frac{d(V/V_0)}{dt} \rightarrow \alpha(\psi_b) \simeq \frac{1}{\phi_0} \frac{d(V/V_0)}{d(X/X_0)} = \frac{G'(X/X_0)}{\phi_0} \quad (7)$$

where G is the experimental curve $V/V_0=G(X/X_0)$ and G' its derivative with respect to X/X_0 . In point of fact, the derivation of $\alpha(\psi_b)$ for the experimental G curve is not so straightforward as G' actually furnishes an experimentally derived shrinkage factor $\alpha_X(X/X_0)=G'(X/X_0)/\phi_0$ that depends on the integral quantity X/X_0 and not on the point-wise concentration ψ_b . Therefore, a further step is required to relate X/X_0 to ψ_b or, in other words, to introduce a function $X/X_0=g(\psi_b, Bi_m)$ so that $\alpha(\psi_b)$ can be obtained from $\alpha_X(X/X_0)$ as

$$X/X_0 = g(\psi_b; Bi_m) \rightarrow \alpha(\psi_b) = \alpha_X(g(\psi_b; Bi_m)) \quad (8)$$

Two different approaches can be followed in order to evaluate the g function relating X/X_0 to ψ_b : (1) a fully analytical approach developed in Adrover et al., 2019a and (2) a shortcut numerical approach developed in Adrover et al., 2019b. In the next section, we apply the shortcut numerical approach to evaluate the shrinkage function for isothermal dehydration of chayote discoid samples at four different temperatures and show the capability of the moving boundary model, above described, to predict the time evolution of both moisture ratio X/X_0 and slice thickness L/L_0 .

3. Analysis of convective hot air-drying of discoid samples

We focus on literature experimental data of convective hot air drying of chayote discoid samples (data from Ruiz-López et al., 2012) with initial radius $R_0=35$ mm and initial thickness $L_0=6$ mm. We analyse four operating temperatures $T=40,50,60,70$ °C in a convective dryer (air velocity 2 m/s). Available experimental data are the moisture ratio X/X_0 vs time and the rescaled thickness L/L_0 vs time at the four temperatures. The initial water volume fraction is estimated as $\phi_0 = 0.525$ from USDA, 2019.

Given the high aspect ratio $R_0/L_0 > 5$, we can reasonably adopt the moving-boundary model in its one-dimensional formulation. This implies that (1) the water volume fraction is solely a function of the vertical coordinate z spanning the discoid thickness $\phi(\mathbf{x}) = \phi(z)$; (2) the reference length L_r is the initial thickness L_0 ; (3) radial shrinkage can be neglected; (4) water flux from the lateral cylindrical surface can be neglected; (5) volume reduction V/V_0 coincides with thickness reduction L/L_0 ; (6) the probe point P is a point attached to the top (or bottom) surface of the sample and evolves in time together with the sample thickness.

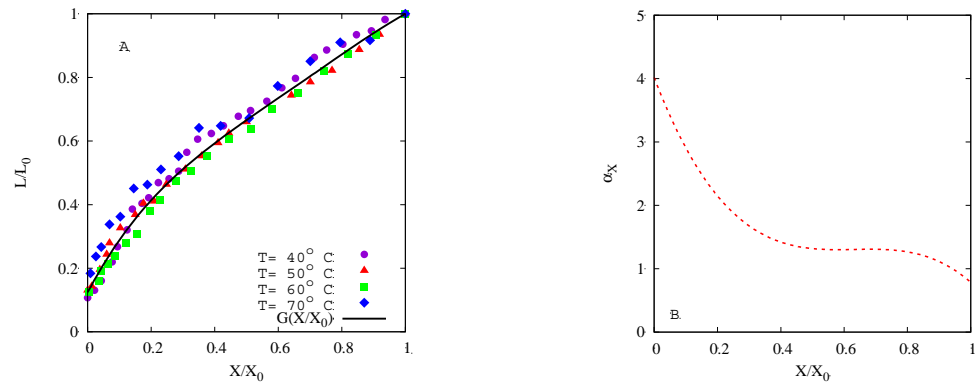


Figure 1: (A) Least-square best fit with a fourth-order polynomial function Eq.(9) of experimental data rescaled thickness vs moisture ratio of chayote discoid samples at four different temperatures. (B) Experimentally derived shrinkage factor.

Figure 1A shows the behaviour of the calibration curve L/L_0 vs X/X_0 at the four operating temperatures $T=40, 50, 60, 70$ °C. It can be observed that the system exhibits a strong thickness reduction and that a unique G curve can be adopted for the four temperatures. Figure 1B shows the corresponding behaviour for the experimentally derived shrinkage factor $\alpha_X(X/X_0)$ that clearly shows that the volume reduction is larger than the volume of released water and increases for smaller values of the moisture ratio, i.e. the more the dehydration proceeds the more the volume contraction is largely superior to the volume of water removed. This is a peculiar feature of chayote.

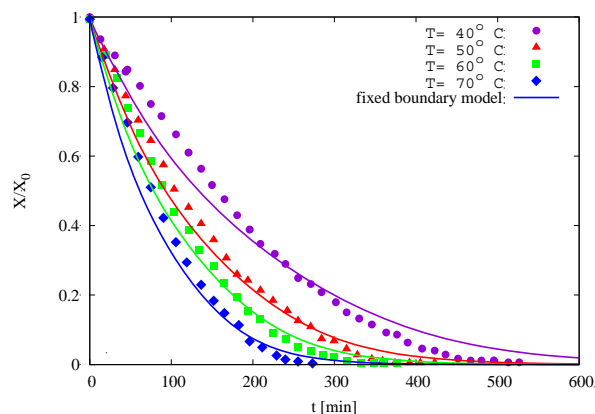


Figure 2: Experimental data for moisture ratio vs time at four different temperatures. Continuous lines represent the fixed-boundary model predictions with best-fit water diffusivity values reported in Table 1.

It is quite obvious that a transport model not accounting for thickness shrinkage is not suitable for this system. Figure 2 shows the comparison between experimental data for the moisture ratio vs time at the four temperatures and model predictions obtained with a fixed-boundary purely diffusive model. Water diffusivity values resulting from the best fit of the fixed-boundary model onto experimental data are reported in Table 1.

Table 1: Water diffusivity estimated from experimental data for L/L_0 vs X/X_0 with the fixed-boundary and the moving-boundary models.

D [m^2/s]	$D(40^\circ\text{C})$	$D(50^\circ\text{C})$	$D(60^\circ\text{C})$	$D(70^\circ\text{C})$	D_0	E/R
Fixed-boundary	$1.81 \cdot 10^{-9}$	$2.61 \cdot 10^{-9}$	$3.09 \cdot 10^{-9}$	$3.95 \cdot 10^{-9}$	$1.53 \cdot 10^{-6}$	2820
Moving-boundary	$6.71 \cdot 10^{-10}$	$9.01 \cdot 10^{-10}$	$1.06 \cdot 10^{-9}$	$1.31 \cdot 10^{-9}$	$1.28 \cdot 10^{-6}$	2358

The fixed-boundary model fails to capture the salient features of the experimental dehydration curves, especially at lower temperatures. Moreover it is reasonable that the values of water diffusivity obtained with the fixed-boundary model are largely overestimated since the model cannot take into account that the water diffusional path strongly reduces during the course of the dehydration process. This statement is supported by the direct comparison (reported in Figure 3) with the diffusivity values obtained with the moving-boundary model.

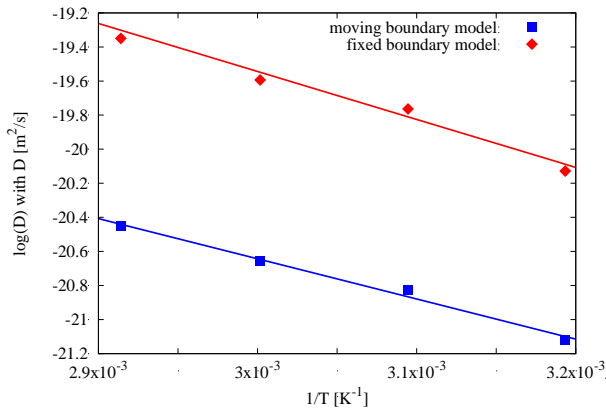


Figure 3: Arrhenius-type behaviour of water diffusivity $\log(D)$ vs $1/T$: comparison between fixed-boundary and moving-boundary models. Diffusivity values and activation energies are reported in Table 1.

In order to apply the moving-boundary model we need to evaluate the shrinkage factor $\alpha(\psi)$. Figure 4 shows the behaviour of $\alpha(\psi)$ obtained from $\alpha_X(X/X_0)$ (shown in Figure 1B) with the short-cut numerical approach, requiring the following two numerical steps. We preliminary solve the moving-boundary model with $\alpha(\psi_b) = \alpha_X(\psi_b)$, i.e. by adopting the approximation $X/X_0 = \psi_b$. This approximation tends to overestimate X/X_0 at the early stage of the drying process, while it overestimates X/X_0 on longer time scales. From this numerical solution we obtain an improved numerical estimation of the curve $X/X_0 = g(\psi_b)$ that can be used to estimate the actual $\alpha(\psi)$ function from Eq.(8).

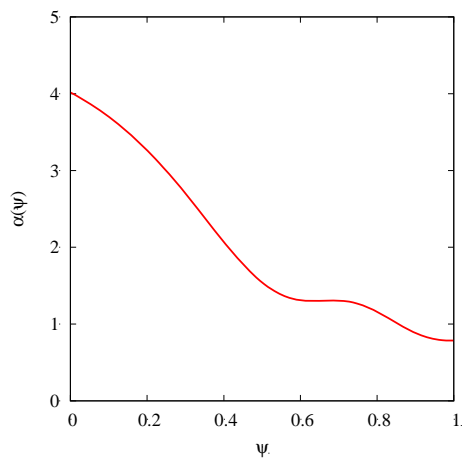


Figure 4: Shrinkage factor obtained with the shortcut approach from experimental data in Figure 1A.

Figure 5A shows the excellent agreement between moving-boundary model predictions with the shrinkage factor depicted in Figure 4 and experimental results for moisture ratio X/X_0 vs time. Therefore, the proposed model can accurately predict the temporal evolution of the water content. The resulting values of water diffusivity at different temperatures are reported in Table 1. The model is further used, in a fully predictive way, in Figure 5B, showing experimental data for the rescaled thickness L/L_0 vs time together with model predictions. Therefore, the proposed model can accurately predict also the temporal evolution of the rescaled thickness.

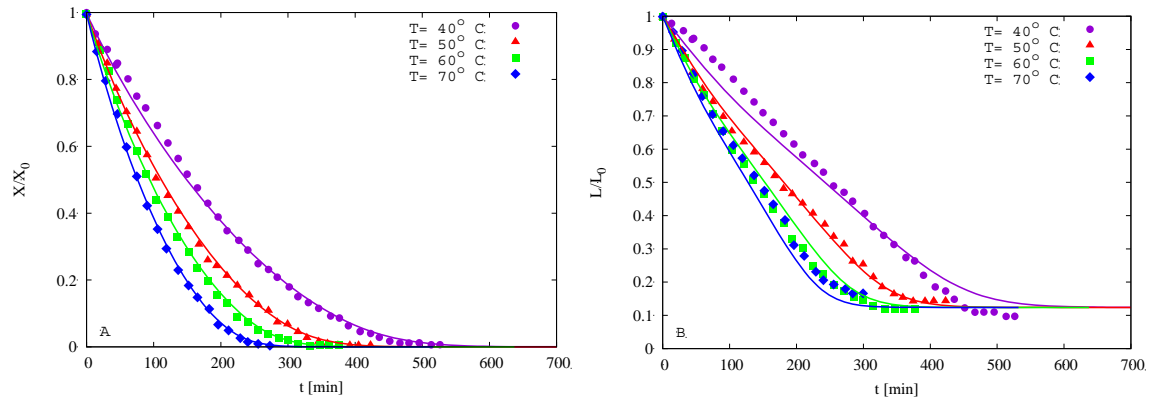


Figure 5: Comparison between moving-boundary model predictions and experimental data. (A) Moisture ratio X/X_0 vs time. (B) Rescaled thickness L/L_0 vs time.

The good agreement between the moving-boundary model predictions and experimental data confirms that there is no need to introduce a not physical time dependent diffusivity as often employed in modelling food dehydration in the presence of shrinkage (Ruiz-López et al., 2012). The time dependent diffusivity is an artefact introduced to take into account that the water diffusional path changes in time. When shrinkage is properly accounted for, a constant diffusivity can accurately describe the physics of the process.

4. Conclusions

A moving-boundary model is proposed. It takes into account volume reduction of food materials during drying. There is a fundamental difference between the present model and other models proposed in the literature taking into account volume reduction. The time evolution of the volume is not information provided to the model but it is the model itself that is able to predict volume reduction by introducing a constitutive equation, the equation for the shrinkage velocity, which relates the point-wise shrinkage rate to the point-wise water flux. The role of the shrinkage factor is that of a further "dressing" to accurately describe experimental data that quite often exhibit a rate of volume reduction that changes during the drying process.

The model, in the present paper, is adopted to accurately predict the time evolution of moisture ratio and rescaled thickness of chayote discoidal samples subjected to hot air isothermal drying in a wide range of temperatures. The paper also highlights the intrinsic limitations and risks in adopting a fixed-boundary model neglecting sample shrinkage.

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