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Soundness of Formal Systems for Relational Database Dependencies: Application of Tableau Deductive System

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The theory of relational database dependencies is introduced. It is shown that formal systems for functional, multivalued, and subset dependencies are sound. The application of tableau deductive system is presented.

1. Introduction

Logical design of relational database schemes is based on the constraints that data must satisfy to model correctly the part of the world under consideration. Of particular importance are the constraints called dependencies. Examples of such constraints are functional and multivalued dependencies ([Armstrong and Delobel 80], [Beeri 80], [Beeri and Vardi 84], [Ginsburg and Zaiddan 82], [Honeyman 82], [Maier 83], [Saxena and Tripathi 89], [Ullman 88], and [Vardi 88]), and subset dependencies ([Sagiv and Walecka 82]).

The purpose of this paper is to present a theory of relation database dependencies and to use tableau deductive system in proving the soundness of formal systems for relational dependencies. Tableau deductive system is "machine-oriented" (it is suitable for implementation in an automatic or interactive computer theorem proving program). This work is the first step in designing the strategic (or heuristic) component of the resolution deductive tableau system for reasoning about the database dependencies. In this way, we can extend the deductive abilities of database systems.

The paper is organized as follows. Section 2 contains the basic concepts of the dependency theory (a conventional approach) and the tableau deductive system. In Section 3, we present our theory of relational database dependencies (a logical approach). The theory consists of axioms for the theory of finite sets (implicitly) and five axioms for predicate $E(X,t_1,t_2)$, where $E(X,t_1,t_2)$ means that tuples t_1 and t_2 are equal on a set of attributes X. In Section 4, we give the proofs of soundeness of formal systems for functional, multivalued, and subset dependencies; the proofs are based on application of tableau deductive system. Conclusions are discussed in Section 5.

2. Basic concepts

In this section, we describe the basic concepts of dependency theory (a conventional approach) and tableau deductive system.

2.1 The dependency theory

A relational scheme R is a finite (nonempty) set of attributes, $R = \{A,..,A_n\}$. We use the letters U, V, W, X, Y, Z (possibly with subscripts) to indicate subsets of R. The union of X and Y is denoted by XY. We assume that with each attribute A_i there is associated a (nonempty) set, called its domain, denoted by $dom(A_i)$.

Let $R = \{A ,..,A_n\}$ be a relational scheme, and $D=\bigcup_{A_i \in R}(A_i)$ be a domain of R.

A tuple on scheme R is a mapping $t: R \rightarrow D$, such that $t(A_i) \in dom(A\ )$ for all $A_i \in R$.

A relation on relation scheme R is an ordered pair (R,r) , where r is a finite set of tuples on R.

We denote tuples by the letters t, u, v (possibly with subscripts).

Functional dependencies

Let $U,V \subseteq R$ be subsets of a relational scheme R.

A functional dependency is an expression of the form $U \rightarrow V$

$U \rightarrow V$ holds in (R,r) if

(FD) $(\forall t_1 ,t_2 \in r)[t_1[U] = t_2[U] \Rightarrow t_1[V] = = t_2[V]]$, where $t_i[Z]$ is a restriction of a tuple t_i on a set of attributes Z.

Intuitively speaking, $U \rightarrow V$ holds in (R,r) if values for the attributes of U (U-value) uniquely determine values for the attributes of V (V-value).

Multivalued dependencies

Let $U,V \subseteq R$ be subsets of a relational scheme R.

A multivalued dependency is an expression of the form $U \rightarrow\rightarrow V$.

$U \rightarrow\rightarrow V$ holds in (R,r) if

(MD) $(\forall t_1 ,t_2 \in r)\ [t_1\ [U] = t_2[U] \Rightarrow\ (\exists t_3 \in r)\ [t_3[UV] = t_1[UV] \wedge t_3[R \backslash UV] = t_2[R \backslash UV]]]$

Intuitively speaking, $U \rightarrow\rightarrow V$ holds in (R,r) if, given values for the attributes of U, there is a set of zero or more associated values for the attributes of V, independently of values for the attributes in $R \backslash UV$.

Subset dependencies

Let $U,V,W \subseteq R$ be subsets of a relational scheme R.

A subset dependency is an expression of the form $W(U) \subset W(V)$

$W(U) \subset W(V)$ holds in (R,r) if

(SD) $(\forall t_1 ,t_2 \in r)\ [t_1\ [U] = t_2[U] \Rightarrow\ (\exists t_3 \in r)\ [t_3[V] = t_1[V] \wedge t_3[W] = t_2[W]]]$

The definition (SD) says that $\prod_W(\sigma_{U\text{-value}}(r)) \subseteq \prod_W(\sigma_{V\text{-value}}(r))$, where $\prod$ and σ are project and select operators, respectively.

For the details of the dependency theory see [Ullman 88].

2.2 Tableau deductive system

Let T be a finite theory whose axioms are $A_1,..,A_k$ (axioms are closed sentences).

If each axiom A_i of the theory T is true under an interpetation I, then we say that the interpretation I is a model for the theory T.

The closed sentence F of a theory is valid in the theory if F is true under every model for the theory.

A closed sentence G is implied by a set of closed sentences $F_1,..,F_k$ in the theory T if, whenever each F_i is true under model I for the theory T, G is also true under the model I. It is denoted by

$F_1,..,F_k \overset{T}{\models} G$

Note that $F_1,..,F_k \overset{T}{\models} G$ if and only if $F \wedge .. \wedge F \Rightarrow G$ is valid in the theory T .

A theory T_1 is an augmentation of a theory T_2 if the vocabulary of the theory T_2 is a subset of the vocabulary of the theory T_1 and each axiom of T_2 is also axiom of T_1.

In solving the implication problem $F_1,..,F_k \overset{T}{\models} G$, we shall use tableau deductive system. The basic structure of a tableau deductive system is a tableau. The tableau consists of a collection of rows of two columns each. Each row contains a sentence, either an assertion A or a goal G. The assertions appear in the first column and the goals in the second column. An assertion and a goal may not both appear in the same row.

A tableau T_1 with assertions $A_1,..,A_m$ and goals $G_1,..,G_n$ assumes the following form:

assertions	goals
A_1	
.	
.	
A_m	
	B_1
	.
	B_n

The tableau T_1 represents the sentence

F: $(\forall *)A_1 \wedge .. \wedge (\forall *)A \Rightarrow (\exists *)G_1 \vee .. \vee (\exists *)G_n$, where

$(\forall *)A_i$ and $(\exists *)G_j$ are universal closure of A_i and existential closure of G_j . We say that F is an associated sentence of the tableau T_1 . The order in which the rows occur in the tableau has no significance.

If m = 0 (i.e., T_1 has no assertions), then F is

$(\exists *)G_1 \vee .. \vee (\exists *)G_n$. If n=0 (i.e., T_1 has no goals), then F is $\neg((\forall *)A_1 \wedge .. \wedge (\forall *)A_m)$.

A tableau is valid if its associated sentence is valid. We say that two tableaux T_1 and T_2 have the same meaning if

[T_1 is valid] if and only if [T_2 is valid].

Suppose we wish to prove $F_1,..,F_k \models^{T} G$ in the finite theory T whose axioms are Ax1,..,Axm. The deductive process is as follows:

(1) We form the initial tableau

assertions	goals
Ax1	
.	
.	
Am	
F_1	
.	
.	
F_k	
	G

(2) The initial tableau is developed by applying deduction rules succesively (each rule adds one or more rows to the tableau, in such a way that validity is preserved).

(3) The process continues until the final goal TRUE or the final assertion FALSE appears in the tableau.

The final tableau with the goal TRUE or the assertion FALSE is valid. Because the deduction rules preserve validity, we have thus obtained the validity of the initial tableau. Therefore,

$F_1,..,F_k \models^{T} G$ holds.

We will apply the following deduction rules:

R1: The resolution rule (the rule performs a case analysis on the truth of a subsentence).
R2: The quantifier-elimination rules (skolemization). ($\forall,\exists$- elimination rules remove quantifiers from the assertions or goals).
R3: The splitting rules (and, or, if-split rules break a row down into its logical components).
R4: The rewriting rule (the rule replaces a subsentence with an equivalent sentence).

For a complete description of resolution tableau deductive system, the reader should consult [Manna and Waldinger 90] .

3. The theory of relational database dependencies

The theory of dependencies, T_D , is an augmentation of the theory of finite sets (as described in [Manna and Waldinger 85]), whose vocabulary contains a ternary predicate symbol $E(X,t_1,t_2)$.

Under the intended models for the theory, the relation $E(X,t_1,t_2)$ is true if tuples t_1 and t_2 are equal on a set of attributes X .

The Axioms

The theory of dependencies, T_D, is a theory whose axioms include those of the theory of the finite sets, and the following axioms:

Ax1: $(\forall X,Y)(\forall t_1,t_2)[Y \subseteq X \Rightarrow [E(X,t_1,t_2) \Rightarrow E(Y,t_1,t_2)]]$ (triviality)

Ax2: $(\forall X,Y)(\forall t_1,t_2)[E(X,t_1,t_2) \wedge E(Y,t_1,t_2) \Rightarrow E(XY,t_1,t_2)]$ (union)

Ax3: $(\forall X)(\forall t1,t2)[E(X,t1,t2) \Rightarrow E(X,t2,t1)]$ (symmetry)

Ax4: $(\forall X)(\forall t)[E(X,t,t)]$ (reflexivity)

Ax5: $(\forall X)(\forall t1,t2,t3)[E(X,t1,t2) \wedge E(X,t2,t3) \Rightarrow E(X,t1,t3)]$ (transitivity)

Now, we introduce functional, multivalued, and subset dependencies. Definitions are analogous to the definitions (FD), (MD), and (SD) from 2.1.

Functional dependency

The sentence

F: $(\forall t1,t2)[E(U,t1,t2) \Rightarrow E(V,t1,t2)]$

is a functional dependency, and is denoted by $U \rightarrow V$.

Multivalued dependency

The sentence

G: $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists t3)[E(UV,t3,t1) \wedge E(c(UV),t3,t2)]]$,

where c(UV) is a complement of UV (with respect to the relational scheme R), is a multivalued dependency. It will be denoted by $U \twoheadrightarrow V$.

Subset Dependency

The sentence

H: $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists t3)[E(V,t3,t1) \wedge E(W,t3,t2)]]$

denoted $W(U) \subset W(V)$, is a subset dependency.

4. Application of tableau deductive system in formal system soundness proving

4.1 Soundness of Armstrong's formal system for functional dependencies

Armstrong's formal system contains three rules:

fd1: $\vdash U \rightarrow V$ if $V \subseteq U$ (reflexivity)

fd2: $U \rightarrow V \vdash UW \rightarrow VW$ (augmentation)

fd3: $U \rightarrow V, V \rightarrow W \vdash U \rightarrow W$ (transitivity)

In the next proposition we express that the formal system {fd1, fd2, fd3 } is sound.

Proposition 1

Armstrong's formal system {fd1, fd2, fd3} is sound, that is,

FD1: $\models^{T_D} U \rightarrow V$ if $V \subseteq U$

FD2: $U \rightarrow V \models^{T_D} UW \rightarrow VW$

FD3: $U \rightarrow V, V \rightarrow W \models^{T_D} U \rightarrow W$

Proof

We give the proof of FD2. The proofs of the other two parts, FD1 and FD3, are similar to that of FD2, and are omitted.

Well, we would like to show that $UW \rightarrow VW$ is valid in the theory T_D. For this purpose it suffices to prove the tableau

assertions	goals
Ax1	
Ax2	
.	
.	
Ax5	
A1. $U \rightarrow V$	
	G1. $UW \rightarrow VW$

where each assertion Axi, $1 \leq i \leq 5$, is an axiom in the theory T_D.

Hereafter we shall not represent the axioms of T_D explicitly in the initial tableau, that is, the initial tableau mentioned above will have the following form

assertions	goals
A1. $U \rightarrow V$	
	G1. $UW \rightarrow VW$

If we represent the sentences $U \rightarrow V$ and $UW \rightarrow VW$, we obtain

A1. $(\forall t1,t2)[E(U,t1,t2) \Rightarrow E(V,t1,t2)]$	
	G1. $(\forall t1,t2)[E(UW,t1,t2) \Rightarrow E(VW,t1,t2)]$

By the $\forall,\exists$ - elimination rules, we have

A2. $[E(U,t1,t2)]^{+} \Rightarrow E(V,t1,t2)$	
	G2. $E(UW,a,b) \Rightarrow E(VW,a,b)$

By the if-split rule, applied to goal G2., we obtain

A3. $[E(UW,a,b)]^{-}$	
	G3. $[E(VW,a,b)]^{+}$

Because $U \subseteq UW$ and $W \subseteq UW$, we have

A4. $[U \subseteq UW]^{-}$	
A5. $[W \subseteq UW]^{-}$	

By the resolution rule, applied to assertion A4. and the triviality axiom Ax1 (after outermost skolemization), denoted A6.

A6. $[Y \subseteq X]^{+} \Rightarrow E(X,t1,t2) \Rightarrow E(Y,t1,t2)]$	

with { X / UW , Y / U }, we obtain

A7. $[E(UW,t1,t2)]^{+} \Rightarrow E(U,t1,t2)$	

By the resolution rule, applied to A5. and the triviality axiom Ax1 (assertion A6.), with { X / UW, Y / W }, we obtain

A8. $[E(UW,t1,t2)]^{+} \Rightarrow E(W,t1,t2)$	

By the resolution rule, applied to A2. and A7., we obtain

A9. $[E(UW,t1,t2)]^{+} \Rightarrow E(V,t1,t2)$	

By the resolution rule, applied to A3. and A9., with { t1 / a , t2 / b } , we obtain

A10. $[E(V,a,b)]^{-}$	

By the resolution rule, applied to A3. and A8., with { t1 / a , t2 / b } , we obtain

A11. $[E(W,a,b)]^{-}$	

By the resolution rule, applied to A10. and the union axiom Ax2 (after outermost skolemization), denoted A12.,

A12. $[E(X,t1,t2)]^{+} \wedge E(Y,t1,t2) \Rightarrow E(XY,t1,t2)$	

with { X / V , t1 / a , t2 / b } , we obtain

A13. $[E(Y,a,b)]^{+} \Rightarrow E(VY,a,b)]$	

By the resolution rule, applied to A11. and A13., with { Y / W } , we obtain

A14. $[E(VW,a,b)]^{-}$	

Finally, by the resolution rule, applied to assertion A14. and goal G3., we obtain the goal

	G4. TRUE

4.2 Soundness of formal system for multivalued dependencies

The formal system for multivalued dependencies contains five rules:

mvd1: $U \twoheadrightarrow V \vdash U \twoheadrightarrow c(UV)$ (complementation)

mvd2: $U \twoheadrightarrow V \vdash UW1 \twoheadrightarrow VW2$ if $W2 \subseteq W1$ (augmentation)

mvd3: $U \twoheadrightarrow V, V \twoheadrightarrow W \vdash U \twoheadrightarrow (W \backslash V)$ (transitivity)

mvd4: $U \rightarrow V \vdash U \twoheadrightarrow V$ (translation)

mvd5: $U \twoheadrightarrow V, W \rightarrow W1 \vdash U \rightarrow W1$ if $W1 \subseteq V$ and $W \cap V = \varnothing$ (coalescence rule)

Proposition 2

The formal system {mvd1, mvd2, mvd3, mvd4, mvd5 } is sound, that is,

MVD1: $U \twoheadrightarrow V \overset{T_D}{\models} U \twoheadrightarrow c(UV)$

MVD2: $U \twoheadrightarrow V \overset{T_D}{\models} UW1 \twoheadrightarrow VW2$ if $W2 \subseteq W1$

MVD3: $U \twoheadrightarrow V, V \twoheadrightarrow W \overset{T_D}{\models} U \twoheadrightarrow (W \backslash V)$

MVD4: $U \to V \overset{T_D}{\models} U \twoheadrightarrow V$

MVD5: $U \twoheadrightarrow V, W \to W1 \overset{T_D}{\models} U \to W1$ if $W1 \subseteq V$ and $W \cap V = \varnothing$

Proof

We give the proof of MVD1.

We begin with the tableau

assertions	goals
A1. $U \twoheadrightarrow V$	
	G1. $U \twoheadrightarrow c(UV)$

After representation of the sentences $U \twoheadrightarrow V$ and $U \twoheadrightarrow c(UV)$, we have

A1. $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists t3)[E(UV,t3,t1) \wedge E(c(UV,t3,t2)]\,]$	
	G1. $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists t3)[E(Uc(UV),t3,t1) \wedge E(c(Uc(UV)),t3,t2)]\,]$

By the $\forall,\exists$ - elimination rules, we have

A2. $[E(U,t1,t2)]^{+} \Rightarrow [E(UV,f(t1,t2),t1) \wedge E(c(UV),f(t1,t2),t2)]$	
	G2. $E(U,a,b) \Rightarrow [E(Uc(UV),t3,a) \wedge E(c(Uc(UV)),t3,b)]]$

By the if-split rule, applied to goal G2., we obtain

A3. $[E(U,a,b)]^{-}$	
	G3. $[E(Uc(UV),t3,a)]^{+} \wedge E(c(Uc(UV)),t3,b)$

By the resolution rule, applied to A3. and the symmetry axiom Ax3 (after outermost skolemization), denoted A4.,

A4. $[E(X,t1,t2)]^{+} \Rightarrow E(X,t2,t1)$	

with { X / U , t1 / a , t2 / b } , we obtain

A5. $[E(U,b,a)]^{-}$	

By the resolution rule, applied to A5. and A2., with { t1 / b , t2 / a } , we obtain

A6. $E(UV,f(b,a),b) \wedge E(c(UV),f(b,a),a)$	

By the and-split rule, applied to A6., we have

A7. $[E(UV,f(b,a),b)]^{-}$	
A8. $[E(c(UV),f(b,a),a)]^{-}$	

Because $c(Uc(UV)) = c(U) \cap V \subseteq UV$, we have

A9. $[c(Uc(UV)) \subseteq UV]^{-}$	

By the resolution rule, applied to A9. and the triviality axiom Ax1 (after automatic outermost skolemization) denoted A10.,

A10. $[Y \subseteq X]^{+} \Rightarrow [E(X,t1,t2) \Rightarrow E(Y,t1,t2)]$	

with { X / UV , Y / c(Uc(UV))} , we obtain

A11. $[E(UV,t1,t2)]^{+} \Rightarrow E(c(Uc(UV)),t1,t2)$	

By the resolution rule, applied to A11. and A7., with { t1 / f(b,a) , t2 / b}, we obtain

A12. $[E(c(Uc(UV)),f(b,a),b)]^{-}$	

Because $U \subseteq UV$, we have

A13. $[U \subseteq UV]^{-}$	

By the resolution rule, applied to A13. and the triviality axiom Ax1 (assertion A10.), with { X / UV , Y / U }, we obtain

A14. $[E(UV,t1,t2)]^{+} \Rightarrow E(U,t1,t2)$	

By the resolution rule. applied to A14. and A7., with { t1 / f(b,a) , t2 / b }, we obtain

A15. $[E(U,f(b,a),b)]^-$	

By the resolution rule, applied to A15. and the transitivity axiom Ax5 (after automatic outermost skolemization) denoted A16.,

A16. $[E(X,t1,t2)]^+ \wedge E(X,t2,t3) \Rightarrow E(X,t1,t3)$	

with { X / U , t1 / f(b,a) , t2 / b }, we obtain

A17. $[E(U,b,t3)]^+ \Rightarrow E(U,f(b,a),t3)$	

By the resolution rule, applied to A17. and A5., with { t3 / a }, we obtain

A18. $[E(U,f(b,a),a)]^-$	

By the resolution rule, applied to A8. and the union axiom Ax2 (after automatic outermost skolemization) denoted A19.,

A19. $[E(X,t1,t2) \wedge E(Y,t1,t2)]^+ \Rightarrow E(XY,t1,t2)$	

with { Y / c(UV) , t1 / f(b,a) , t2 / a }, we obtain

A20. $[E(X,f,(b,a),a)]^+ \Rightarrow E(X,c(UV),f(b,a),a)$	

By the resolution rule, applied to A20. and A18., with { X / U }, we obtain

A21. $[E(Uc(UV),f(b,a),a)]^-$	

By the resolution rule, applied to A21. and G3., with { t3 / f(b,a)}, we obtain

	G4. $[E(c(Uc(UV)),f(b,a),b)]^+$

By the resolution rule, applied to A12. and G4., we obtain the final goal

	G5. TRUE

The proofs of MVD2, MVD3, and MVD5, are similar, and are omitted.

4.3 Soundness of formal system for subset dependencies

The formal system for subset dependencies consists of two rules:

sd1: $\vdash W(U) \subset W(V)$ if $V \subseteq U$ (reflexivity)

sd2: $W(U) \subset W(V), W(V) \subset W(V1) \vdash W(U) \subset W(V1)$ (transitivity)

Proposition 3

The formal system { sd1,sd2 } is sound, that is,

SD1: $\models^{T_D} W(U) \subset W(V)$ if $V \subseteq U$

SD2: $W(U) \subset W(V), W(V) \subset W(V1) \models^{T_D} W(U) \subset W(V1)$

Proof

We shall prove SD2 . The proof of SD1 is similar.

We begin with the tableau

assertions	goals
A1. $W(U) \subset W(V)$	
A2. $W(V) \subset W(V1)$	
	G1. $W(U) \subset W(V1)$

After representation of the sentences $W(U) \subset W(V)$, $W(V) \subset W(V1)$, and $W(U) \subset W(V1)$, we have

A1. $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists\ t3)[E(V,t3,t1) \wedge E(W,t3,t2)]]$	
A2. $(\forall t1,t2)[E(V,t1,t2) \Rightarrow (\exists\ t3)[E(V1,t3,t1) \wedge E(W,t3,t2)]]$	
	G1. $(\forall t1,t2)[E(U,t1,t2) \Rightarrow (\exists\ t3)[E(V1,t3,t1) \wedge E(W,t3,t2)]]$

By the ∀,∃ - elimination rules, and renaming common free variables rules, we have

A3. $[E(U,t1,t2)]^{+} \Rightarrow E(V,f(t1,t2),t1 \wedge E(W,f(t1,t2),t2)$		
A4. $[E(V,x1,x2)]^{+} \Rightarrow E(V1,g(x1,x2),x1) \wedge E(W,g(x1,x2),x2)$		
	G2. $E(U,a,b) \Rightarrow E(V1,t3,a) \wedge E(W,t3,b)$	

By the if-split rule, applied to G2., we obtain

A5. $[E(U,a,b)]^{-}$	
	G3. $[E(V1,t3,a)]^{+} \wedge E(W,t3,b)$

By the resolution rule, applied to A5. and A3., with { t1 / a , t2 / b }, we obtain

A6. $E(V,f(a,b),a) \wedge E(W,f(a,b),b)$	

By the and-split rule, applied to A6., we have

A7. $[E(V,f(a,b),a)]^{-}$	
A8. $[E(W,f(a,b),a)]^{-}$	

By the resolution rule, applied to A7. and the symmetry axiom Ax3 (after outermost skolemization), denoted A9.,

A9. $E(X,t1,t2) \Rightarrow E(X,t2,t1)$	

with { X / V , t1 / f(a,b) , t2 / a }, we obtain

A10. $[E(V,a,f(a,b))]^{-}$	

By the resolution rule, applied to A10. and A4., with { x1 / a , x2 / f(a,b)}, we obtain

A11. $E(V1,g(a,f(a,b)),a) \wedge E(W,g(a,f(a,b)),f(a,b))$	

By the and-split rule, applied to A11., we have

A12. $[E(V1,g(a,f(a,b)),a)]^{-}$	
A13. $[E(W,g(a,f(a,b)),f(a,b))]^{-}$	

By the resolution rule, applied to A13. and the transitivity axiom Ax5 (after outermost skolemization), denoted A14.

A14. $[E(X,t1,t2)]^{+} \wedge E(X,t2,t3) \Rightarrow E(X,t1,t3)$	

with { X / W , t1 / g(a,f(a,b)) , t2 / f(a,b)}, we have

A15. $[E(W,f(a,b),t3)]^{+} \Rightarrow E(W,g,(a,f(a,b)),t3)$	

By the resolution rule, applied to A15. and A8., with { t3 / b }, we obtain

A16. $[E(W,g(a,f(a,b)),b)]^{-}$	

By the resolution rule, applied to A12. and G3. with { t3 / g(a,f(a,b)) }, we obtain

	G4. $[E(W,g(a,f(a,b)),b)]^{+}$

Finally, by the resolution rule, applied to A16. and G4., we obtain

	G5. TRUE

5. Conclusions

The theory of relational database dependencies is presented. The axioms of the theory include those of the theory of finite sets (implicitly) and five axioms for predicate E(X,t1,t2).

We showed by the application of tableau deductive system that the formal systems for functional, multivalued and subset dependencies are sound. In fact, we showed the soundness of the following rules: FD2 (augmentation for functional dependencies), MVD1 (complementation), and SD1 (reflexivity for subset dependencies). The proofs of soundness of the other rules are similar, and are omitted.

In the next paper, we will design the strategic (or heuristic) component of the resolution deductive tableau system for reasoning about the database dependencies. In this way, we will be able to extend the deductive abilities of database systems.

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