Comparative Population Studies – Zeitschrift für Bevölkerungswissenschaft Vol. 35, 3 (2010): 543-568 (Date of release: 15.09.2011)

# Tempo Effects in Different Calculation Types of Period Death Rates

# **Christian Wegner**

**Abstract**: The question as to whether or not tempo effects distort the measurement of period mortality is controversial in recent demographic research. Only few publications, however, illustrate the underlying phenomenon of tempo effects, namely that the period death rate may increase although the mortality of all cohorts living during the analyzed period has fallen. Moreover, related literature only focuses on one of three methods to derive the age-specific death rate. This article primarily deals with the questions whether other methods of age-specific death rate are also affected by tempo effects in the logic of Bongaarts and Feeney and whether the tempo effect can be minimised solely by applying a specific method. The results demonstrate that all types of death rates are influenced by tempo effects and that different methods do not eliminate the influence of tempo effects. Nevertheless, it is necessary to distinguish between two types of tempo effects, which can be revealed in theoretical as well as empirical perspective.

**Keywords:** Tempo effects · Death rate · Period mortality · Age-year-method · Cohortyear-method · Age-cohort-method · Tempo-adjusted life expectancy

### 1 Introduction

The previous debate about tempo effects in period mortality has been essentially concerned with the question whether tempo effects influence period mortality indicators such as life expectancy at birth and whether current mortality conditions are therefore distorted (*Bongaarts/Feeney* 2002; *Vaupel* 2002; *Wilmoth* 2005; *Bon-gaarts/Feeney* 2008b; *Guillot* 2008; *Luy* 2008; *Rodríguez* 2008; *Vaupel* 2008; *Wachter* 2008; *Luy* 2009; *Bongaarts/Feeney* 2010). Only few publications explicitly deal with the unexpected and curious phenomenon that the trend in period death rates fluctuates despite a continuous improvement in survival conditions of all cohorts living during the analysed period (*Feeney* 2010; *Horiuchi* 2008; *Luy* 2008; *Luy*/Wegner 2009). According to the logic of *Bongaarts* and *Feeney* (2002, 2008a, 2008b), these

© Federal Institute for Population Research 2011 DOI: 10.4232/10.CPoS-2010-13en

URL: www.comparativepopulationstudies.de URN: urn:nbn:de:bib-cpos-2010-13en6

fluctuations in the death rates are caused by tempo effects. They are accompanied by a temporary change in the number of deaths within a period in which mortality conditions have changed. Although the unexpected trend in the period rates forms the basis of Bongaarts' and Feeney's methodical and empirical research, fundamental questions about tempo effects remain open until today.

A shortcoming in current research is the relation between the method to derive the death rate and the occurrence of tempo effects. Previous research merely analyses the cause of tempo effects by using the *age-year-method*, also known as type I rate. However, there are two further methods to compute the period death rate: the *cohort-year-method* (type II rate) and the *age-cohort-method* (type III rate). This leads to the question whether these two methods are also affected by tempo effects if period mortality has changed. All three methods are based on different numbers of deaths resulting from the overlap of age, time and birth intervals. Subsequently, changes in period mortality conditions have differing impacts on the respective method. Therefore, it can be assumed that the cause of tempo effects also differs. Due to the different characteristics of each method, a second important question can be raised: Does the extent of existing tempo effects depend on the selected type of death rate?

To answer these questions, the article is structured as follows: The first part of this paper introduces the different methods of deriving the death rate. The second part graphically illustrates the tempo effects in the logic of Bongaarts and Feeney by using the Lexis diagram. Their impact on age-specific mortality is explained by applying modelled numbers of living and deceased persons. A general classification of the tempo effects regardless of the model assumptions is carried out in the third section. The last part finally presents the effects of the different types of death rates and the resulting differences in tempo effects based on empirical data for 26 countries.

## 2 Methods to derive death rates

Mortality research distinguishes between three methods of deriving death rates. These methods are typified by the death counts emerging from the overlap of age, time and birth intervals. All three intervals can be presented graphically by using the Lexis diagram in Figure 1. The abscissa of the diagram shows the calendar time, whilst the age is levelled on the ordinate (*Feichtinger* 1973: 18-25). All people born in a specific year and their demographically-relevant events can then be shown diagonally to age and calendar time. The overlap of age, time and birth intervals allows to extract two triangles of events (*Becker* 1874) which are marked in the Lexis diagram by right-angled triangles.

The 1st triangle of deaths includes the number of individuals of a birth cohort c who died at age x in year t. This area is shown in Figure 1 by the triangle ABC (cf. Tab. 1.1). The 2nd triangle of deaths also contains all persons of the cohort c who died at age x, but in the following year t+1. They are presented by the triangle BCD in Figure 1.



Fig. 1: Classification modes of deaths and living persons in the Lexis diagram

The legs of each triangle present two different sets of living persons. Individuals who have reached age x in a year t are summarized as *persons living of the same age* (line AB in Fig. 1). All persons aged x who lived exactly at the beginning of year t+1 are characterised as *persons living at the same time* (line BC in Fig. 1). Both sets of living persons constitute the number of states whilst the triangles of death include number of events at a specific time or age (cf. Tab. 1.1).

The triangles of deaths and the sets of living persons define three methods for computing the death rate. The combination of both death triangles determinate three standard ways of classifying the number of deaths which form the numerator of each death rate (*Becker* 1874). The denominator contains the person-years which are estimated by the number of living persons (*Feichtinger* 1973: 55-56).

The most common procedure used to derive the death rate in official statistics is the *age-year-method*; also referred to as the *type I rate (Flaskämper* 1962: 342-391; *Wunsch/Termote* 1978: 85-87; *Caselli/Vallin* 2006: 61-63). The German Federal Statistical Office has applied this method since the General Life Table of 1970/72 (*Statistisches Bundesamt* 2006). The method is based on the *3rd class of deaths* resulting from the overlap of an age and year interval (square EFGH in Fig. 1). This classification of deaths contains the *1st triangle of deaths* of cohort *c* and of the *2nd triangle* of the previous birth cohort *c-1* at age *x* in year *t* (Tab. 1.2). The type I

Source: based on Caselli/Vallin (2006)

Tab. 1:	Classification modes of deaths and living persons and different
	methods for computing the period death rate

1.1 Events and states		Lexis-diagram in Figure 1		
$^{I}D(x,c)$	1st triangle	Area ABC		
$^{II}D(x,c)$	2nd triangle	Area BCD		
$^{t}P(x)$	Persons living at the same age	Line AB		
$^{x}P(t+1)$	Persons living at the same time	Line BC		
1.2 Classifications of deaths				
D(x,t)	3th class	Area EFGH		
D(c,t)	2nd class	Area IJKL		
D(c,x)	1st class	Area ABCD		
1.3 Death rates				

$^{I}m(x,t)$	$=\frac{D(x,t)}{0.5\cdot \left[{}^{x}P(t)+{}^{x}P(t+1)\right]}$	Type I death rate (age-year-method)
$^{II}m(c,t)$	$=\frac{D(c,t)}{0.5\cdot [x^{-1}P(t)+x^{-1}P(t+1)]}$	Type II death rate (cohort-year-method))
$^{III}m(c,x)$	$=\frac{D(c,x)}{^{x}P(t+1)}$	Type III death rate (age-cohort-method)

Source: based on Becker (1874) and Caselli/Vallin (2006)

death rate  ${}^{l}m(x,t)$  then is the quotient of the 3rd class of deaths to the person-years at age x in year t (Tab. 1.3). The average of the *living persons at the same time* at the beginning (line EG) and at the end (line FH) of year t is used as an approximation of the number of person-years.

The National Institute of Statistic of France uses the *type II rate* to determine the period survival conditions, which is also known as the *cohort-year-method* (*Flaskämper* 1962: 364-365; *Wunsch/Termote* 1978: 85-87; *Caselli/Vallin* 2006: 61-63). The type II death rate is based on the 2nd class of deaths determined by the overlap of a birth and a period interval (area IJKL in Fig. 1). This class is composed of the 1st triangle of deaths of the cohort c at age x and the 2nd triangle of the same cohort at the previous age x-1 (Tab. 1.2). The comparison with the type I rate shows that the *cohort-year-method* includes all death counts of a cohort within a year t. However, a specific age classification is not possible because the deaths are stretched over two age groups. The number of person-years is estimated from the average of the individuals living at the beginning and the end of the observed period t. In contrast to the *age-year-method*, however, the living persons are aged x-1 at the beginning of the year (line IK in Fig. 1), whilst the surviving persons are aged x at the end of the year (line JL in Fig. 1). Accordingly, the death rate type II "*Im(c,t)* is calculated from

the quotients of the *2nd class of deaths* to the approximated number of personyears (Tab. 1.3).

The last method is the *type III death rate*, which is also referred to as the *age-cohort-method* and was applied, for example, to calculate the first General Life Table of the German Reich 1871/81 (*Becker* 1874: 38-45; *Wunsch/Termote* 1978: 85-87; *Caselli/Vallin* 2006: 61-63). Determining mortality by this method is an uncommon procedure in period analysis. Due to the characteristics of the method, it is mainly used in cohort analysis (*Caselli/Vallin* 2006). The type III death rate IIIm(c,x) takes into account the total number of deaths resulting from the overlap of a cohort and age interval. This area is referred to as the *1st class of deaths* and comprises the 1st and 2nd triangle of a cohort *c* at age *x* (parallelogram ABCD in Fig. 1). This class of two periods *t* and *t*+1. The number of living persons aged *x* at the beginning of year *t*+1 (line BC in Fig. 1) is used as an approximation of the number of person-years to derive the death rate type III.

# 3 Tempo effects in different types of death rate

The presence and cause of tempo effects in each type of death rate are analysed by a modelled decline in mortality. The processes are graphically derived and explained in the Lexis diagram. The mortality model is based on the discrete models which are commonly used in literature for describing tempo effects (*Feeney* 2010; *Luy* 2008). Although all these models include simplified assumptions, they can be adjusted to a real population without modifying the underlying statements. Furthermore, other distortions, such as heterogeneity or selection effects, are excluded from the model.

The model illustrates a population in which no migration takes place and in which an annual constant number of births is distributed uniformly over the respective birth year. It is further assumed that individuals only die at a certain age x, whilst no mortality occurs in the preceding age x-1 and the next age x+1. Within age x, the number of deaths are distributed over five different dates at intervals of 0.2 years. The mortality conditions are assumed to be constant until the beginning of year t, so that the population is stationary. The decline of mortality is modelled by a linearly rising age at death at the rate of 0.2 years per year in period t. In the following year t+1, age at death remains constant at the new, higher level. The new mortality conditions stay the same but have decreased in comparison to the base level. Hence, the model presents a population with constant mortality until the beginning of year t following by a decline of mortality in year t. From year t+1 onwards mortality remains constant at a new level. There is never an observable contrary mortality-increasing effect. Further, with the help of modelled samples of living and deceased persons, the trend in mortality is illustrated. Under the constant mortality conditions, 1,000 persons alive precisely age x. Within the observed age group, 100 persons die uniformly distributed across the five dates of death.

## 3.1 The tempo effect in the age-year-method (type I rate method)

The Lexis diagram in Figure 2a shows the *3rd class of deaths* at age x for the periods t-1 to t+1. Based on the model assumptions, constant mortality conditions are prevalent until the beginning of year t. Deaths in year t-1 are spread over five times marked by vertical lines within the *3rd class of deaths* at age x. In Figure 2a, they are labelled in as a1 to a5.

The decline in mortality in year *t* goes hand in hand with a linear increase in age at death by 0.2 years per year. The increase in lifetime causes a postponement of deaths diagonally to the age and time axes. Consequently, two relevant effects occur:

(I) The first effect is an enlarged gap between the times of death (*Feeney* 2008; *Horiuchi* 2008). Under constant conditions, the range between the times a1 to a5 is 0.2 years. The gap (b1 to b4) increases to 0.25 years during the increase in age at death. Therefore, the space between times of death has widened in the year of the mortality change, which directly causes the second effect.

(II) As a result of the increased gap in the times of death, the last date b5 (and also all death counts belonging to it) is postponed into the following year t+1. Consequently, the number of times of death in year t falls from five to four, and hence the number of deceased persons. Furthermore, the increasing age at death causes a shift of deaths into the next age x+1 (area S1 in Fig. 2a). Nevertheless, these deceased persons are still covered by the observed year t.

In the next period t+1, the age at death remains constant at the new, higher level. The range between the times of death b5 to c4 has fallen from 0.25 to 0.2 years. Hence, the number of times of death has again risen to the old stationary level of five per year. However, the sequence of the times shows that the formerly constant number of deaths at age x of year t-1 has been postponed both in time and age. In year t+1, firstly those persons die who would have died in year t under the old mortality conditions (time of death b5), followed by the times c1 to c4. Moreover, time b5 as well as the subsequent time points are spread over two age groups. Within the age x, 80 % of deaths are covered in year t+1, whilst the remaining 20 % take place early at age x+1.

In a real population, the death rate would only consider those deaths which remain at age x. However, the shifted deaths from the previous age x-1 during year t (area S2 in Fig. 2a) and year t+1 (area V in Fig. 2a) would be included in the calculation of the death rate. This means that postponed deaths from previous ages during the mortality change may minimise or compensate for shifted deaths of the time b5. This is hypothetically the case if the number of deaths in area S2 is greater than the number of deaths in b5. Yet, in order to avoid the effect of postponed deaths of prior age on the derived death rate on the one hand and to analyse the overall impact of the postponed time of death b5 on the other hand, only the previously stationary number of deaths is included in the following calculation. Therefore, the age range must be extended from x to x+1.2 in order to cover the age-shifted number of deaths. Only the net effect of the number of deaths due to the increasing age at



Fig. 2a: Mortality decline in the *3rd class of deaths* and the resulting tempoeffect

**Fig. 2b:** Mortality decline in the *3rd class of deaths* and the trend in number of person-years and deaths<sup>1</sup>



<sup>1</sup> The number of person-years are bordered and the number of deaths are underlined Source: own design

death will be considered, because the model assumes no mortality at the preceding and following ages.<sup>1</sup>

The person-years (bordered) and the number of deaths (underlined) for the years t-1 to t+1 are shown in Figure 2b. The number of person-years in the initial year of the observation t-1 is 1,130.<sup>2</sup> 100 persons die during this year. The death rate for year t-1 is calculated as follows:

$$'m(t-1) = \frac{100}{1130} = 0.0885$$

The decline in mortality in year t reduces the number of deaths by 20 % caused by the postponement of deaths into the subsequent year t+1. From 80 deaths in period t, 72 occur at age x and the remaining 8 at the next age x+1. At the same time, the number of person-years increases because both the number of persons living at the same time as well as the lifetime of deceased persons has increased slightly as a result of the rising age at death. The death rate in year t declines to:

$$'m(t) = \frac{80}{1139} = 0.0702$$

During the year t+1, the number of deaths reaches the stationary level of 100 persons because the times of death have increased to five again. However, the deaths occur 0.2 years later in age than at the initial level, so that the number of person-years increases slightly further. The death rate under the new constant level equals to:

$$'m(t+1) = \frac{100}{1159} = 0.0863$$

The increase in type I rate between year t and t+1 suggests an increase in mortality. However, Figure 2a illustrates that an increase in mortality did not actually occur for any observed person. Moreover, the number of person-years steadily increased over time. Only the number of deaths falls briefly in year t because of the decline in mortality and the resulting postponement of deaths. According to the argument of *Bongaarts* and *Feeney*, the decline and the subsequent unexpected

An adequate model for the change in mortality over several age groups and the resulting trend in age-specific mortality rates was also simulated and led to identical tempo effects in the respective period mortality rate. Corresponding model calculations can be provided by the author by request.

<sup>&</sup>lt;sup>2</sup> The person-years are calculated from the survivors and the age of those who died in the respective interval. In the year t-1, survivors contribute 925 person-years at age x and another 180 years until age x+1.2. The person-years of the deceased are 25.

increase in the death rate are caused by a tempo effect (*Bongaarts/Feeney* 2002: 18-19; 2008b: 35-38). The tempo effect here primarily describes the disproportionate decline in the number of deaths in ratio to the person-years caused by a rising age at death. Hence, the increase in the death rate between year t and t+1 is not the consequence of an actual increase in mortality, but of the temporary strong decline and resurgence of death counts due to the mortality change.

### 3.2 Tempo effect in the cohort-year-method (type II rate method)

The question now arises as to whether the same tempo effect as with the age-yearmethod also occurs with the cohort-year-method. In both methods, death counts within a one-year interval form the basis for deriving death rate. Hence, a rising age at death also shifts deaths of the *2nd class* out of the analysed period. The extent to which this process also causes tempo effects in the type II rate is analysed by using the simple mortality model same as in the previous section.

The Lexis diagram in Figure 3a again illustrates the five stationary times of death (a1 to a5) in the year t-1. In order to cover all deaths at age x, both cohorts c-2 and c-1 must be considered in this year. Although the shaded lines of the times of death contain only half of both 2nd classes of deaths, they can be conceptually expanded in order to consider all deaths of the cohorts in each year. The simplified model, however, does not influence the causes and impact of tempo effects in the type II rate.

As with the age-year-method, the increasing age at death in year *t* shifts the last time of death *b5* into the following year *t*+1. Accordingly, the number of deaths in year *t* is again temporarily reduced. The mortality conditions in the model remain constant in the year *t*+1, whereas deaths occur 0.2 years later in age because of the risen age at death. Although the total number of deaths in year *t*+1 is identical to that of the previous stationary level, deaths are now stretched over three cohorts from *c*-1 to *c*+1. This expansion is caused by the postponed time point *b5*. The hatched area T1 in Figure 3a refers to the deaths of cohort *c* which under the former mortality conditions would have been covered in the age interval [*x*-1/*x*]. Due to the reduced mortality, these deaths now occur in the next age interval [*x*/*x*+1]. Furthermore, the shifted time of death *b5* also contains deaths of the previous cohort *c*-1 (black area T2). These deaths now take place within the next age group [*x*+1/*x*+2]. The remaining times of death *c1* to *c4* are comparable with the old stationary times *a1* to *a4*, whilst the deaths have shifted by 0.2 years over time and age.

The effect of the risen age at death on the type II rate is illustrated by using the model populations in Figure 3b. As with the type I rate, the focus of the model calculation lies on the trend in death rate, based on the constant number of deaths at the initial level. It is interesting here that in a real population the postponed deaths of the time b5 cannot be compensated for by shifted deaths from previous age groups in year *t*. The *2nd class of deaths* covers two age groups. Therefore, deaths which are postponed to age *x* (area S) are still examined in the analysed class.

The following model considers the age range from x-1 to x+2.2 in order to take the shifted deaths of area T2 in Figure 3a into account. In comparison to the age-



Fig. 3a: Mortality decline in the 2nd class of deaths and the resulting tempoeffect  $^{1}$ 

**Fig. 3b**: Mortality decline in the *2nd class of deaths* and the trend in number of person-years and deaths<sup>1,2</sup>



<sup>1</sup> The brackets indicate two considered age groups
 <sup>2</sup> The number of person-years are bordered and the number of deaths are underlined
 Source: own design

year-method, the number of person-years is higher in the cohort-year-method because in the 2nd class of deaths only half of the deaths are modelled.<sup>3</sup> Therefore, the individuals of cohort c-1 at the beginning of the year t-1 are not exposed to mortality risk until they have reached age x. The situation is similar for individuals born at c-2 who are not exposed to mortality after age x+1. The death rate in year t-1 then equals to:

$$''m(t-1) = \frac{100}{2080} = 0.0481$$

In year t, the number of deaths decreases by 20 % due to the rising age at death and the postponement of deaths to the next period. The deaths of cohort c decline more strongly (from 50 to 32 deaths) in comparison to the older cohort c-1 (from 50 to 48). At the same time, the number of person-years increases slightly as a result of the shifted deaths. The death rate in year t is:

$$''m(t) = \frac{80}{2090} = 0.0383$$

The postponed deaths of b5 occur in the next year t+1 and hence increase the total number to the previously stationary base level of 100 deaths. At the same time, the number of person-years increases slightly because all deaths now occur at a higher age. The new stationary death rate is calculated as:

$$''m(t+1) = \frac{100}{2100} = 0.0476$$

The results show that the type II rate is also affected by tempo effects if period mortality changes. The rate also falls between year t-1 and t, whilst it rises again in the transition to the new stationary level in year t+1. In comparison to the base level, the rate is lower in the new constant level because the individuals survive 0.2 years longer. As in the case of the type I rate, the increase in death rate during the transition to the new stationary level is the consequence of the tempo effect, caused by the temporary and disproportional fall in deaths in relation to the person-years in year t. Since both methods have identical period intervals, the tempo effects have a comparable effect on both death rates.

# 3.3 Tempo effect in the age-cohort-method (type III rate method)

The reduction in the number of deaths during the mortality transition leads to an increase in the death rate in both the age-year- and cohort-year-method, although

<sup>&</sup>lt;sup>3</sup> In the first year, the survivors contribute to a total of 2,030, and the deceased 50 person-years, over the analysed age interval.

mortality has continually fallen among the population. The significant reduction was caused by those deaths which were postponed from year t into the next year t+1 because of the risen age at death. In the age-cohort-method, however, the number of the deaths is defined by age intervals and not by calendar years. A lower number of deaths in the 1st class can only occur via a postponement of deaths to the next age interval.

Figure 4a shows the 1st class of deaths at age x to x+1.2 for the period from t-2 to t+3. Each class embraces two calendar years. To determine the period mortality in the period [t-2/t-1], the death counts of cohort c-2 at age x (area D in Fig. 4a) and of birth cohort c-3 at age x+1 (area E in Fig. 4a) are needed. In order to distinguish more easily between the periods, they were visualised in Figures 4a and 4b by alternate grey shading.

The death counts of cohort c-2 at age x still experience the constant mortality level. Since the 1st class of deaths stretches over the years t-2 and t-1, a total of ten times of death (5 per year) are considered at intervals of 0.2 years. In the period [t-1/t], the age at death rises within the cohort c-1 at age x. Whilst the times of death a1to a5 are still characterised by the old mortality level, the range between the following times b1 to b5 expand, as it was already demonstrated in the previous sections. The number of deaths, however, no longer declines because of the postponement of the time b5 but because of the age increase in b1 to b5. All deceases of cohort c-1 which are postponed to the following age level x+1 (area T1 in Fig. 4a) are not considered within the observed period [t-1/t]. Consequently, the number of the remaining deaths at age x has hence fallen by 10 % compared to the initial stationary level.

In the next period [t/t+1], the age at death increases until the beginning of year t+1 and remains constant at the new level afterwards. The 1st triangle of cohort c at age x is still affected by the age-shifted deaths of b1 to b5. In contrast, the new stationary mortality level is already prevalent in the 2nd triangle. Although at the base level, the deaths only occurred at age x, the deaths of cohort c are now spread over age x and x+1. For quantifying the mortality conditions in period [t/t+1], however, the deaths of cohort c at age x as well as the age-shifted deaths of the earlier cohort c-1 at age x+1 (area T1) are taken into account. The deaths of cohort c which have been postponed to the next age group x+1 (area T2) are not considered until the following period [t+1/t+2]. In a real population, the postponed deaths of cohort c from the previous age x-1 would additionally be accommodated in the measurement (area S). As in the age-year-method, the deaths from previous ages can therefore reduce or even compensate for the postponing effect (area T2). However, in the model used here – analogous to the approach in the other death rate types – these shifted deaths are ignored in order to illustrate the net effect of the rising age at death.

The new stationary mortality conditions can be observed in the period [t+1/t+2] for the first time. The deaths are now spread over two age groups and come from both cohorts c+1 and c due to the improvement in survival conditions. The reduction in mortality in year t hence causes a postponement of deaths in the 1st class of deaths to the next age intervals.



Fig. 4a: Mortality decline in the *1st class of deaths* and the resulting tempo-effect<sup>1</sup>

**Fig. 4b**: Mortality decline in the *1st class of deaths* and the trend in number of person-years and deaths<sup>1,2</sup>



<sup>1</sup> The brackets indicate two considered periods

<sup>2</sup> The number of person-years are bordered and the number of deaths are underlined Source: own design

Whether this delay causes a tempo effect in the death rate, as with the other two rate types, can be tested by using the model populations in Figure 4b. Again, the focus of the model lies on the impact of the risen age at death. 100 deaths occur in the stationary level of the period [*t*-2/*t*-1]. The number of person-years at age *x to* x+1.2 is 1,130.<sup>4</sup> The type III death rate is then calculated from:

$$m(t-2/t-1) = \frac{100}{1130} = 0.0885$$

Caused by the rising age at death in the period [t-1/t], the deaths at age x decrease by 10 %. The formerly 50 deaths reduce to 40, whilst the 10 death counts are postponed to the following age level x+1, and hence not considered in the analysed period. The number of person-years increases slightly by the gained lifetime caused by the increased age at death. The death rate in the period [t-1/t] is:

$${}^{\prime\prime\prime}m(t-1/t)=\frac{90}{1133}=0.0794$$

The increase in the age at death also affects the death rate in the period [t/t+1]. Although the age-shifted deaths of cohort *c*-1 (area T1 in Fig. 4a) are taken into account, however, the deaths of cohort *c*, which occur at the next age level (area T2 in Fig. 4a) are missing. The number of person-years increases once more as a result of the rising age at death. The death rate then emerges from:

$$^{\prime\prime\prime}m(t/t+1)=\frac{90}{1147}=0.0785$$

The new constant mortality conditions in the year [t+1/t+2] include the number of deaths of cohorts c+1 at age x and cohort c at age x+1. Despite the different cohorts, the number of deceased is again 100, 80 % of which occur at age x and 20 % at age x+1. The number of person-years is 1,150. Hence, the new constant death rate is derived as follows:

$${}^{\prime\prime\prime}m(t+1/t+2)=\frac{100}{1150}=0.0870$$

In comparison to the previous period, the death rate increases slightly in the transition to the new stationary level. Therefore, the trend in the death rate type III is also influenced by tempo effects. However, the age-shifted and not period-shifted deaths influence the trend of the death rate by the disproportional decline in the number of deaths.

The surviving persons together lived 1,080 person years, whilst the deceased contributed 50 person years to the total.

### 3.4 Comparison of the tempo effect in the three calculation procedures

The previous models have shown that tempo effects influence the trend in mortality in each of the methods of deriving the death rate. Figure 5 shows the relative change in all three death rates in the course of the linear increasing age at death compared to the constant level at the beginning. It is evident that the relative trend in the type I and type II death rates is almost identical. Both death rates decline by 20 % in the year of the mortality reduction *t* although both classes of death and hence the levels of the rates differ. Even if all deaths of the *2nd class* were considered in the death rate type II, the relative decline would also be identical because the number of times of death in year *t* also falls by 20 %. The rate at the new constant level is less than two percent in comparison to the old stationary level. The relative level of the tempo effect in year *t* is consequently identical in both procedures.

A different picture emerges following the trend of the type III death rate. The rate declines disproportionately quickly over two consecutive periods as a result of the tempo effect. As a result of the spread of the tempo effect over two periods, the impact is somewhat less pronounced in relative terms (10 % in the period [t-1/t] and 12 % in the period [t/t+1]) than in the other two methods. The relative deviation at the new stationary level is reduced to less than two percent as in the type I and type II death rates.



**Fig. 5:** Trend in modeled death rate by different computation methods in relation to a constant starting level<sup>1</sup>

<sup>1</sup> Type I and type II death rate relate to the number of deaths in a year (bordered time), and the type III death rate covers two periods

Source: author's calculation

## 4 Tempo effects of the 1st and 2nd kind

The differences in tempo effects between the types of death rate are partly dependent on the respective model, in particular on the linear increase in the age at death. A non-linear increase in the age at death in the model, by contrast, would lead to differing amounts of tempo effects, and thereby possibly cause a more pronounced tempo effect in the type III death rate. Hence, the model does not permit any universal statements regarding the degree of the tempo effect in all three methods. However, differences between the causes of the respective tempo effects can be demonstrated. Tempo effects accordingly result from two different effects. Based on different classes of deaths, the number of deaths is reduced by the postponement of deaths over either the *period* or *age interval*.

The postponement of deaths over the *period interval* can be referred to as a tempo effect of the 1st kind. As a result, the number of deaths of the 1st triangle (area ABC in Fig. 1) decreases at a certain age if the age at death increases. The occurrence of this tempo effect influences the death rate type I and II. In both methods, the number of deaths in an analysed period can only decline if deaths have been shifted to the next calendar year. Consequently, the total number of postponed deaths is identical in both methods although the level of death rates differs due to the different classes of deaths.

The change in the 2nd triangle of deaths (area BCD in Fig. 1) and the concomitant postponement of deaths beyond the age interval can also cause tempo effects, which are referred to as tempo effects of the 2nd kind. This kind is exclusively relevant for the age-cohort-method because only this method is influenced by age-shifted deaths. Within the type I and II death rate, the age-postponed deaths are still considered in the next age group within the observed year.

The two kinds of tempo effect differ according to whether the increase in the age at death – regardless of the nature of the increase – causes a postponement of deaths to the next period or age interval. When it comes to the practical application, the question arises whether the tempo effect can be minimised by applying a specific type of mortality. This question can only be answered by looking at two highly-simplified scenarios:

(I) In the mortality models which are commonly used in the literature (*Feeney* 2010; *Horiuchi* 2008; *Luy* 2008), the number of deaths of the *1st class* remains constant during the mortality change. Therefore, the change in mortality only causes a postponement of deaths to the next period but not the next age interval. In these models, consequently, the resulting tempo effect of the 1st kind only influences the death rate derived from the age-year-method and the cohort-year-method. The trend of the type III death rate does not indicate any tempo effect. The decline in this death rate is entirely caused by the increase in the number of person-years, whilst the number of deaths remains constant.

(II) In a second theoretically-conceivable scenario the 2nd class of deaths contains constant number of death during a change in mortality. In this scenario, reduced mortality can only occur through a postponement of deaths to the next age interval, but not by period-shifted deaths. In the cohort-year-method, the death rate decreases by the gain in person-years but the number of deaths remains unchanged. In the age-year-method, the number of deaths also changes at the respective age level, however, the age-postponed deaths still occur within the analysed period. Hence, the death rates of type I and of type II would not contain any tempo effect. Only the trend of the type III death rate fluctuates due to the tempo effect of the 2nd kind.

Finally, only these two extreme scenarios can demonstrate a significant difference between both kinds of tempo effect because only one kind of tempo effect can occur in each case. In all other cases (combinations of the two scenarios), tempo effects have a permanent influence on each type of death rate. The intensity depends on the increase in the age at death and the resulting shift of deaths to the next period and age interval. If more deaths would be postponed to the next age than to the next period, the tempo effect in the age-cohort-method would be greater than in the other two methods. The opposite is the case if more deaths are postponed in the following period than in the next age group.

### 5 The relevance of tempo effects in empirical data

The last section analyses the differences of tempo effects and their impact on life expectancy by using mortality data of the *Human Mortality Database (HMD 2010)* for 26 countries. The HMD contains the two triangles of deaths as well as the number of persons living at the same time. Thus, all three types of death rate can be derived. The amount of the tempo effect corresponds to the difference between the conventional life expectancy and the tempo-adjusted life expectancy at age 50 for the year 2005. The reason for analysing life expectancy at age 50 is, firstly, that in developed countries, approximately 95 % of mortality occurs above this age and, secondly, that the mortality conditions follow the methodical requirements of tempo adjustment. As proposed by *Bongaarts* and *Feeney (Bongaarts 2008; Bongaarts/Feeney 2008b)*, tempo adjustment was carried out on the basis of the Total Mortality Rate (TMR) (see Annex for a more detailed description).

The degree of the tempo effect (measured in years) is shown in Figure 6 for women and in Figure 7 for men. The first two bars present the tempo effect of the 1st kind in the type I and type II death rate. For men as well as for women, there are either no or only marginal differences between both methods. The average tempo effect is 1.41 years among women ( $\pm$ 0.43 years) and 1.87 years among men ( $\pm$ 0.69 years). The lowest tempo effect is shown for Bulgaria, at 0.01 years among men and 0.76 years among women. This difference is presumably caused by the constant trend in life expectancy at age 50. Until 1996 the standardised death rate from cardiovascular disease continually increased in Bulgaria, whereas the death rate in e.g.



\* Difference between type I/II and type III death rate is lower than the mean - standard deviation

\*\* Difference between type I/II and type III death rate is higher than the mean - standard deviation

Source: Human Mortality Database, author's calculation

560 • Christian Wegner



Tempo effects in life expectancy at age 50 by different types of death rate, males, 2005 Fig. 7:

\*\* Difference between type I/II and type III death rate is lower than the mean - standard deviation \*\* Difference between type I/II and type III death rate is biober than the mean - standard deviation

\*\* Difference between type //II and type III death rate is higher than the mean - standard deviation Source: Human Mortality Database, author's calculation

Hungary, Poland and Romania either remained constant or declined (*Meslé* 2004). The low tempo effect in Bulgaria in 2005 suggests that life expectancy is only marginally increasing because mortality caused by cardiovascular disease could have reached a constant level.

In contrast, women in Eastern Germany with 2.11 years and men in Ireland with 3.01 years have the highest amount of tempo effect. The high tempo effect in Eastern Germany is the consequence of the rapid mortality decline since reunification, which can also be observed among East German men (*Luy* 2009). In Ireland, on the other hand, the significant increase in male life expectancy and the resulting high tempo effect can presumably be ascribed to the rapid drop in smoking-attributed mortality since the early 1990s (*Peto et al.* 2006).

The right-hand bar in Figures 6 and 7 presents the tempo effect according to the age-cohort-method. In the previous models, the tempo effects of the 1st and 2nd kinds differed considerably in quantitative terms. The empirical data, however, only show slight deviations. The tempo effect in the type III death rate is even slightly stronger than in the two other types of death rate. Among men and women, the average difference between the tempo effects of the 1st and 2nd kind is around -0.13 years ( $\pm$ 0.16 years among women and  $\pm$ 0.14 years among men). Differences outside the standard error can be observed for Portugal (-0.51 years among women and -0.44 years among men) and Spain (-0.51 year among both genders), as well as among Canadian women (-0.31 years). A lower tempo effect of the 2nd kind is shown for Australia ( $\pm$ 0.07 years), Northern Ireland ( $\pm$ 0.05 years) and Sweden ( $\pm$ 0.10 years among women and  $\pm$ 0.3 years among men), as well as among Finnish ( $\pm$ 0.12 years) and Irish men ( $\pm$ 0.07 years).

The empirical data, therefore, do not permit any unambiguous statements which type of death rate can minimise the degree of the tempo effect. The dynamics of the mortality change in each population rather determine the impact of the respective tempo effect as it was described in theoretic terms in the previous chapter. Consequently, the slightly higher tempo effect of the 2nd kind is ascribed to a higher proportion of age-shifted deaths, whilst a higher tempo effect of the 1st kind results from postponements of deaths into the next period.

## 6 Summary and discussion

In demographic research, the age-specific death rate is an important indicator to analyse the period mortality conditions. It also constitutes the key variable in the construction of life tables. Various recent publications have shown that the type I death rate is affected by tempo effects if the mortality conditions change during an analysed period. This paper demonstrates that all types of death rates are affected by tempo effects.

The modelled reduction in mortality causes a fall and subsequent increase in all death rate types during the transition to the new stationary level. This fluctuation is not caused by an increase in mortality, but by a temporary, disproportionate decline in the number of deaths if mortality changes in the respective period. *Bongaarts* and

*Feeney* (1998, 2002) introduced the term "*tempo effect*" to describe this phenomenon.

The tempo effects that were revealed in all three methods can be divided in two different kinds according to their origin. The tempo effect of the 1st kind is caused by the postponement of deaths to the next period interval whereas the tempo effect of the 2nd kind is generated by the age-shifted deaths. All three methods of deriving the death rate are influenced in different ways by these kinds of tempo effect. Both the age-year-method and the cohort-year-method are affected by the tempo effect of the 1st kind. The age-cohort-method is influenced by the tempo effect of the 2nd kind if mortality changes within a period.

The modelled trends in the rates in chapter 3 present major differences in the scope of the tempo effect between each computation method. However, these differences resulted from the assumption that the age at death increases linearly and hence causes a higher tempo effect of the 1st kind. The decline in mortality in real populations, however, is taking place in period as well as in age. Thus, neither the tempo effect of the 1st kind only determines the mortality trend. This is finally shown in the empirical calculations for the 26 countries. The tempo effects in life expectancy at age 50 for the year 2005 only show marginal differences between the death rates of type I and II on the one hand and of type III on the other. Moreover, it is not possible to make any unambiguous statement whether the death rate type III or the other two methods minimise the impact of the tempo effect. The empirical data show higher as well as lower tempo effects for the age-cohortmethod (type III rate).

This leads to several important questions which must be studied in greater detail in further research. The currently available methods to adjust the life expectancy by tempo effects are based on the assumption that the age-specific pattern of period mortality changes proportionally (*Bongaarts* 2008; *Bongaarts/Feeney* 2008b). It is assumed that tempo effects occur in all age-specific death rates and that these are more pronounced the higher the death rate is. To what degree the strict proportionality assumption is actually reflected in a real population, or whether different tempo effects are available in the age-specific death rates, are two methodicallyand empirically-relevant questions for future research.

Despite the unresolved questions, the results of this paper confirm that it can certainly be expedient and helpful to adjust the death rates of all types by the tempo effect if period mortality changes. Without this adjustment, the trend in the death rates would suggest overestimated or nonexistent changes in period mortality. Moreover, other articles show that differences in period mortality between two populations may be heavily influenced by tempo effects and that the lack in using tempo-adjusted rates may lead to incorrect conclusions about the dynamic of mortality trends (vgl. *Luy* 2008, 2009; *Luy/Wegner* 2009; *Luy et al.* 2011). Future analyses of period mortality can therefore only become more authoritative through an additional tempo adjustment of conventional indicators. Especially as period death rates and measures derived from them, such as life expectancy, are the most frequently used indicators to assess changes or differences in mortality between certain populations or sub-populations.

#### References

- Becker, Karl 1874: Zur Berechnung von Sterbetafeln an die Bevölkerungsstatistik zu stellende Anforderungen. Berlin: Verlag des Königlichen Statistischen Bureaus. http:// gdz.sub.uni-goettingen.de/dms/load/img/?PPN=PPN527528277
- Bongaarts, John 2008: Five period measures of longevity. In: Barbie, Elisabetta; Bongaarts, John; Vaupel, James W. (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 237-245 [doi: 10.1007/978-3-540-78520-0 12].
- *Bongaarts, John; Feeney, Griffith* 1998: On the quantum and tempo of fertility. In: Population and Development Review 24,2: 271-291.
- Bongaarts, John; Feeney, Griffith 2002: How long do we live? In: Population and Development Review 28,1: 13-29 [doi: 10.1111/j.1728-4457.2002.00013.x].
- Bongaarts, John; Feeney, Griffith 2008a: Estimating mean lifetime. In: Barbie, Elisabetta; Bongaarts, John; Vaupel, James W. (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 11-27 [doi: 10.1007/978-3-540-78520-0\_2].
- *Bongaarts, John; Feeney, Griffith* 2008b: The quantum and tempo of life-cycle events. In: *Barbie, Elisabetta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 29-65 [doi: 10.1007/978-3-540-78520-0 3].
- *Bongaarts, John; Feeney, Griffith* 2010: When is a tempo effect a tempo distortion? In: Genus 66,2: 1-15 [doi: 10.4402/genus-188].
- *Caselli, Graziella; Vallin, Jacques* 2006: From situation events in time to the Lexis diagram and the computing of rates. In: *Caselli, Graziella; Vallin, Jacques; Wunsch, Guillaume* (Eds.): Demography: Analysis and Synthesis. Amsterdam et al.: Elsevier Academic Press: 55-68.
- *Feeney, Griffith* 2008: Increments of life and mortality tempo. In: *Barbie, Elisabeta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 153-165 [doi: 10.1007/978-3-540-78520-0 8].
- *Feeney, Griffith* 2010: Mortality Tempo: A Guide for the Skeptic. In: Comparative Population Studies – Zeitschrift für Bevölkerungswissenschaft 35,3: 483-496 [doi: 10.4232/10. CPoS-2010-12en].
- Feichtinger, Gustav 1973: Bevölkerungsstatistik. New York: Walter de Gruyter.
- *Flaskämper, Paul* 1962: Grundriß der Sozialwissenschaftlichen Statistik. Volume 1: Bevölkerungsstatistik. Hamburg: Verlag von Richard Meiner.
- *Guillot, Michel* 2008: Tempo effects in mortality: An appraisal. In: *Barbie, Elisabeta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 129-152 [doi: 10.1007/978-3-540-78520-0\_7].

HMD 2010: Human Mortality Database. URL: www.mortality.org, 02.11.2010.

*Horiuchi, Shiro* 2008: Tempo effect on age-specific death rates. In: *Barbie, Elisabeta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 191-201 [doi: 10.1007/978-3-540-78520-0\_10].

- *Luy, Marc* 2008: Mortality tempo-adjustment: theoretical considerations and an empirical application. In: *Barbie, Elisabeta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 203-233 [doi: 10.1007/978-3-540-78520-0 11].
- Luy, Marc 2009: Der Einfluss von Tempo-Effekten auf die ost-west-deutschen Unterschiede in der Lebenserwartung. In: Cassens, Ina; Luy, Marc; Scholz, Rembrandt (Eds.): Die Bevölkerung in Ost- und Westdeutschland. Demografische, gesellschaftliche und wirtschaftliche Entwicklung seit der Wende. Wiesbaden: VS Verlag für Sozialwissenschaften: 140-168.
- *Luy, Marc; Wegner, Christian* 2009: Conventional versus tempo-adjusted life expectancy which is the more appropriate measure for period mortality? In: Genus 65,2: 1-28.
- Luy, Marc; Wegner, Christian; Lutz, Wolfgang 2011: Comparative International Trends: Europe. In: Rogers, Richard G.; Crimmins, Eileen M. (Eds.): International Handbook of Adult Mortality. Dordrecht et al.: Springer: 49-81 [doi: 10.1007/978-90-481-9996-9 3].
- *Meslé, France* 2004: Mortality in Central and Eastern Europe: Long-term trends and recent upturns. In: Demographic Research Special Collection 2, Article 3: 45-70 [doi: 10.4054/DemRes.2004.S2.3].
- *Peto, Richard et al.* 2006: Mortality from smoking in developed countries 1950-2000, 2nd edition. URL: www.ctsu.ox.ac.uk/~tobacco/index.htm.
- *Rodríguez, Germán* 2008: Demographic translation and tempo effects: An accelerated failure time perspective. In: *Barbie, Elisabeta; Bongaarts, John; Vaupel, James W.* (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 69-92 [doi: 10.1007/978-3-540-78520-0 4].
- *Sardon, Jean-Paul* 1994: A Period Measure of Mortality: The Example of France. In: Population: An English Selection 6: 131-150.
- Statistisches Bundesamt 2006: Generationensterbetafeln für Deutschland. Modellrechnungen für die Geburtsjahrgänge 1871-2004. Wiesbaden.
- *Vaupel, James W.* 2002: Life expectancy at current rates vs. current conditions: A reflexion stimulated by Bongaarts and Feeney's "How long do we live?". In: Demographic Research 7, Article 8: 365-377 [doi: 10.4054/DemRes.2002.7.8].
- Vaupel, James W. 2008: Lifesaving, lifetimes and lifetables. In: Barbie, Elisabeta; Bongaarts, John; Vaupel, James W. (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 93-107 [doi: 10.1007/978-3-540-78520-0 5].
- Wachter, Kenneth W. 2008: Tempo and its tribulations. In: Barbie, Elisabeta; Bongaarts, John; Vaupel, James W. (Eds.): How long do we live? Demographic models and reflections on tempo effects. Berlin et al.: Springer: 109-128 [doi: 10.1007/978-3-540-78520-0 6].
- *Wilmoth, John R.* 2005: On the relationship between period and cohort mortality. In: Demographic Research 13, Article 11: 231-280 [doi: 10.4054/DemRes.2005.13.11].
- Wunsch, Guillaume J.; Termote, Marc G. 1978: Introduction to demographic analysis. Principles and methods. New York & London: Plenum Press.

Date of submission: 05.12.2010

Date of acceptance: 21.03.2011

Christian Wegner (🖂). Vienna Institute of Demography of the Austrian Academy of Sciences, Wittgenstein Centre for Demography and Global Human Capital, Wohllebengasse 12-14, 6th floor, A-1040 Vienna, Austria. E-Mail: christian.wegner@oeaw.ac.at URL: www.oeaw.ac.at

Translated from the original text by the Federal Institute for Population Research. The reviewed and author's authorised original article in German is available under the title "Tempoeffekte in der Periodensterberate bei alternativen Berechnungsverfahren", DOI 10.4232/10.CPoS-2010-13de or URN urn:nbn:de:bib-cpos-2010-13de5, at http://www.comparativepopulationstudies.de.

### Annex – Total mortality rate and tempo adjustment

The total mortality rate (TMR) is a rarely used measure to determine the mortality condition of a period (*Sardon* 1994; *Bongaarts/Feeney* 2008b). The calculation is based on the age-specific death rates of the 2nd kind. These state the proportion of persons of a birth cohort who died at age *x* at time *t*.

(1) Death rate 2nd kind 
$$m(x, t)_2 = \frac{D(x, t)}{B(t - x)}$$
  
 $D(x,t)$  Number of deaths at age x of year t  
 $B(t - x)$  Number of persons born t-x years ago

The TMR is then calculated from the sum of the age-specific death rates of the 2nd kind:

(2) Total mortality rate 
$$TMR(t) = \sum_{x=0}^{\infty} m(x, t)_2$$

Similar to the total fertility rate (TFR), the TMR is a "quantum measure" as it states the average number of events per individual for a hypothetical cohort of a year *t*. Since each person can only die once, the expected value of the TMR is always one. However, the TMR is below one if the average age at death increases during an analysed period or is bigger than one if the age at death declines (*Bongaarts/Feeney* 2008b; *Luy/Wegner* 2009). The difference of the TMR of one is regarded as being the indicator of the presence of tempo effects. In order to adjust the conventional death rates by the tempo effect, Bongaarts and Feeney work on the assumption that the effect is constant at all ages. The tempo-adjusted death rate  $m(x,t)^*$  is then determined from the ratio between the conventional death rate and the TMR:

(3) Tempo-adjusted death rate

$$m(x,t)^* = \frac{m(x,t)}{TMR(t)}$$

## Comparative Population Studies – Zeitschrift für Bevölkerungswissenschaft

www.comparativepopulationstudies.de ISSN: 1869-8980 (Print) – 1869-8999 (Internet)

Published by / Herausgegeben von Prof. Dr. Norbert F. Schneider

Layout and print: Federal Institute for Population Research, Wiesbaden (Germany)

Managing Editor / Redaktion Frank Swiaczny

**Copy Editor** / **Schlussredaktion** Dr. Evelyn Grünheid

## Scientific Advisory Board / Wissenschaftlicher Beirat

Jürgen Dorbritz (Wiesbaden) Paul Gans (Mannheim) Johannes Huinink (Bremen) Marc Luy (Wien) Clara H. Mulder (Groningen) Notburga Ott (Bochum) Peter Preisendörfer (Mainz)

### **Board of Reviewers / Gutachterbeirat**

Martin Abraham (Erlangen) Laura Bernardi (Lausanne) Hansjörg Bucher (Bonn) Claudia Diehl (Göttingen) Andreas Diekmann (Zürich) Gabriele Doblhammer-Reiter (Rostock) Henriette Engelhardt-Wölfler (Bamberg) E.-Jürgen Flöthmann (Bielefeld) Alexia Fürnkranz-Prskawetz (Wien) Beat Fux (Zürich) Joshua Goldstein (Rostock) Karsten Hank (Mannheim) Sonja Haug (Regensburg) Franz-Josef Kemper (Berlin) Michaela Kreyenfeld (Rostock) Aart C. Liefbroer (Den Haag) Kurt Lüscher (Konstanz) Dimiter Philipov (Wien) Tomáš Sobotka (Wien) Heike Trappe (Rostock)

 $\ensuremath{\mathbb{C}}$  Federal Institute for Population Research 2011 – All rights reserved