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On graphs that have a unique least common multiple

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ABSTRACT

A graph G without isolated vertices is a least common multiple of two graphs H_1 and H_2 if G is a smallest graph, in terms of number of edges, such that there exists a decomposition of G into edge disjoint copies of H_1 and there exists a decomposition of G into edge disjoint copies of H_2. The concept was introduced by G. Chartrand *et al.* and they proved that every two nonempty graphs have a least common multiple. Least common multiple of two graphs need not be unique. In fact two graphs can have an arbitrary large number of least common multiples. In this paper graphs that have a unique least common multiple with $P_3 \cup K_2$ are characterized.

RESUMEN

Un grafo G sin vértices aislados es un mínimo común múltiplo de dos grafos H_1 y H_2 si G es uno de los grafos más pequeños, en términos del número de ejes, tal que existe una descomposición de G en copias de H_1 disjuntas por ejes y existe una descomposición de G en copias de H_2 disjuntas por ejes. El concepto fue introducido por G. Chartrand *et al.* donde ellos demostraron que cualquera dos grafos no vaciós tienen un mínimo común múltiplo. El mínimo común múltiplo de dos grafos no es necesariamente único. De hecho, dos grafos pueden tener un número arbitrariamente grande de mínimos comunes múltiplos. En este artículo caracterizamos los grafos que tienen un único mínimo común múltiplo con $P_3 \cup K_2$.

Keywords and Phrases: Graph decomposition, common multiple of graphs.

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1 Introduction

All graphs considered in this paper are assumed to be simple and to have no isolated vertices. The number of edges of a graph G denoted by $e(G)$, is called the size of G. $\delta(G)$ and $\Delta(G)$ respectively denote the minimum and maximum of the degrees of all vertices in G. $\chi'(G)$ denotes the edge chromatic number of G, the minimum number of colors needed to color the edges of G, so that no two adjacent edges in G have the same color. An odd component of a graph is a maximal connected subgraph of the graph with odd number of edges. Two graphs G and H are said to be isomorphic, denoted as $G \cong H$ if there exists a bijection between the vertex sets of G and H, $f\colon V(G) \to V(H)$ such that two vertices u and v of G are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H. For graphs G_1 and G_2, their union $G_1 \cup G_2$ is the graph with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set consisting of all the edges in G_1 together with all the edges in G_2. If k is a positive integer, then kG is the union of k disjoint copies of G.

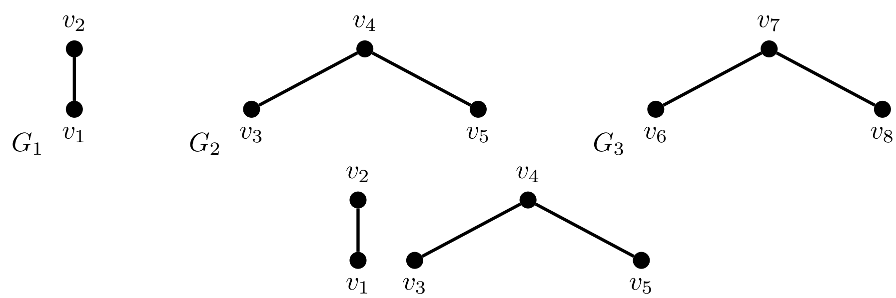


Figure 1: $G_1 \cup G_2$

Let $G = G_2$. Then $G \cong G_3$ and $2G$ is shown in Figure 2.

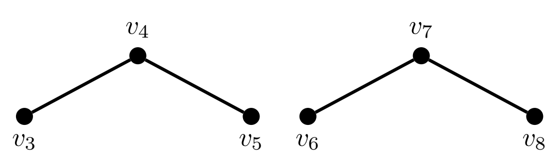


Figure 2: $2G$

A vertex u of a graph G is said to cover an edge e of G or e is covered by u, if e is incident with u. Let u, w be two vertices of a graph G and take two copies of G : G_1, G_2. The graph H obtained by identifying the vertex u in G_1 with the vertex w in G has vertex set $V(H) = V(G_1) \cup V(G_2) - \{w\}$ and edge set $E(H) = E(G_1) \cup E(G_2)$, where the edges in G_2 incident with w are now incident with u.

A graph H is said to divide a graph G if there exists a set of subgraphs of G, each isomorphic to H, whose edge sets partition the edge set of G. Such a set of subgraphs is called an *H-decomposition* of G. If G has an *H-decomposition*, we say that G is *H-decomposable* and write $H|G$.

A graph is called a common multiple of two graphs H_1 and H_2 if both $H_1|G$ and $H_2|G$. A graph G is a least common multiple of H_1 and H_2 if G is a common multiple of H_1 and H_2 and no other common multiple has a smaller positive number of edges. Several authors have investigated the problem of finding least common multiples of pairs of graphs H_1 and H_2; that is graphs of minimum size which are both H_1 and H_2 decomposable. The problem was introduced by Chartrand *et al.* in [5] and they showed that every two nonempty graphs have a least common multiple. The problem of finding the size of least common multiples of graphs has been studied for several pairs of graphs: cycles and stars [5, 13, 14], paths and complete graphs [11], pairs of cycles [10], pairs of complete graphs [4], complete graphs and a 4-cycle [1], pairs of cubes [2] and paths and stars [8]. Least common multiple of digraphs were considered in [7].

If G is a common multiple of H_1 and H_2 and G has q edges, then we call G a *(q, H_1, H_2) graph*. An obvious necessary condition for the existence of a (q, H_1, H_2) graph is that $e(H_1)|q$ and $e(H_2)|q$. This obvious necessary condition is not always sufficient. Therefore, we may ask: Given two graphs H_1 and H_2, for which value of q does there exist a (q, H_1, H_2) graph? Adams, Bryant and Maenhaut [1] gave a complete solution to this problem in the case where H_1 is the 4-cycle and H_2 is a complete graph; Bryant and Maenhaut [4] gave a complete solution to this problem when H_1 is the complete graph K_3 and H_2 is a complete graph. The problem to find least common multiples of two graphs H_1 and H_2 is to find all (q, H_1, H_2) graphs G of minimum size q. We denote the set of all least common multiples of H_1 and H_2 by $LCM(H_1, H_2)$. The size of a least common multiple of H_1 and H_2 is denoted by $lcm(H_1, H_2)$. Since every two nonempty graphs have a least common multiple, $LCM(H_1, H_2)$ is nonempty. For many pairs of graphs the number of elements of $LCM(H_1, H_2)$ is greater than one. For example both P_7 and C_6 are least common multiples of P_4 and P_3. In fact Chartrand *et al.* [6] proved that for every positive integer n there exist two graphs having exactly n least common multiples. In [11] it was shown that every least common multiple of two connected graphs is connected and that every least common multiple of two 2-connected graphs is 2-connected. But this is not the case for disconnected graphs. For example if we take $H_1 = 2K_2$, $H_2 = C_5$, then $G_1 = 2C_5$ and G_2 - the graph obtained by identifying two vertices in two copies of C_5, are in $LCM(H_1, H_2)$ of which G_1 is disconnected while G_2 is connected.

As two graphs can have several least common multiples, it is interesting to search for pairs of graphs that have a unique least common multiple. Pairs of graphs having a unique least common multiple were investigated by G. Chartrand *et al.* in [6] and they proved the following results.

Theorem 1.1. *A graph G of order p without isolated vertices and the graph P_3 have a unique least common multiple if and only if every component of G has even size or $G \cong K_p$, where $p \equiv 2$ or 3 (mod 4).*

Theorem 1.2. *A nonempty graph G without isolated vertices and the graph $2K_2$ have a unique* *least common multiple if and only if* $G \cong K_2, G \cong K_3$ *or* $2K_2|G$.

Theorem 1.3. *Let* r *and* s *be integers with* $2 \leq r \leq s$. *Then the stars* $K_{1,r}$ *and* $K_{1,s}$ *have a unique least common multiple if and only if* $gcd(r,s) \neq 1$.

A result proved by N. Alon [3] on tK_2-decomposition of a graph is used to find those graphs that have a unique least common multiple with tK_2.

Theorem 1.4. *For every graph* G *and every* $t > 1$, $tK_2|G$ *if and only if* $t|e(G)$ *and* $\chi'(G) \leq \dfrac{e(G)}{t}$.

We will also make use of a result proved by O. Favaron, Z. Lonc and M. Truszczynski [9] to characterize those graphs that have a unique least common multiple with $P_3 \cup K_2$.

Theorem 1.5. *If* G *is none of the six graphs* G_1 *to* G_6 *listed below, then* G *is* $P_3 \cup K_2$ *decomposable if and only if*

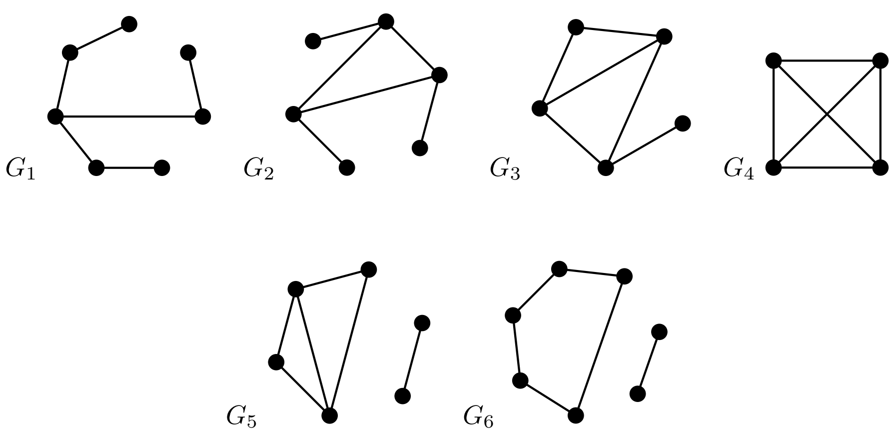
(1) $e(G) \equiv 0 \pmod 3$,

(2) $\Delta(G) \leq \frac{2}{3}e(G)$,

(3) $c(G) \leq \frac{1}{3}e(G)$, *where* $c(G)$ *denote the number of odd components of* G,

(4) *the edges of* G *cannot be covered by two adjacent vertices;*

where,



2 Main results

In this section we are characterizing those graphs that have a unique least common multiple with tK_2 and $P_3 \cup K_2$.

2.1 On graphs that have a unique least common multiple with tK_2

Theorem 2.1. *A nonempty graph G without isolated vertices and the graph tK_2 have a unique least common multiple if and only if $tK_2|G$ or $\delta(G) > \dfrac{lcm(tK_2, G)}{2t}$.*

Proof. Consider the graph tG. Clearly tG is both G and tK_2 decomposable. Let $q = e(G)$. Since $e(tG) = tq$, we have $lcm(tK_2, G) \leq tq$. But $lcm(tK_2, G)$ is a multiple of q. So $lcm(tK_2, G) = ql$, where $l \leq t$. This implies $\frac{lcm(tK_2,G)}{t} = \frac{ql}{t}$. Let H be a least common multiple of G and tK_2.

Case 1. $l > 1$.

Since H is tK_2-decomposable, by Theorem 1.4, $\chi'(H) \leq \frac{ql}{t}$. Since $G|H$, $\chi'(G) \leq \chi'(H) \leq \frac{ql}{t}$. Thus $\Delta(G) \leq \frac{ql}{t}$.

Subcase (i): $\delta(G) \leq \frac{ql}{2t}$.

Consider the graph $G \circ G$, which is obtained by identifying two vertices of least degree in G. In this subcase $\Delta(G \circ G) \leq \frac{ql}{t}$, since $\Delta(G) \leq \frac{ql}{t}$. $\chi'(G) \leq \frac{ql}{t}$ implies $\chi'(G \circ G) \leq \frac{ql}{t}$. Color G_1, a copy of G in $G \circ G$, with $k \leq \frac{ql}{t}$ colors. This is possible, since $\chi'(G) \leq \frac{ql}{t}$. Let v be the identified vertex in $G \circ G$. Since $\delta(G) \leq \frac{ql}{2t}$, the edges adjacent to v in G_1 are colored using at most $\frac{ql}{2t}$ colors. Color G_2, the copy of G in $G \circ G$ other than G_1, with the same k colors as follows. Color the edges adjacent to v in G_2 using colors different from those which were used to color the edges adjacent to v in G_1. The remaining colors used in the coloring of G_1 can be used to color other edges of G_2. Thus $\chi'(G \circ G) = k \leq \frac{ql}{t}$.

Let $H_1 = lG$, the union of l disjoint copies of G and $H_2 = G \circ G \cup (l-2)G$. Clearly H_1 and H_2 are divisible by G. Since $\chi'(H_1) = \chi'(G) \leq \frac{ql}{t}$, H_1 is tK_2-decomposable. $\chi'(H_2) = \chi'(G \circ G) \leq \frac{ql}{t}$, H_2 is tK_2-decomposable by Theorem 1.4. Thus $H_1, H_2 \in LCM(tK_2, G)$. $e(H_1) = e(H_2) = ql$, where $q = e(G)$. Since $lcm(tK_2, G) = ql$, H_1 and H_2 are two non-isomorphic least common multiples of tK_2 and G.

Subcase (ii): $\delta(G) > \frac{ql}{2t}$.

In this case $l > 1$ and $lcm(tK_2, G) = ql$, where $q = e(G)$. Thus $H \in LCM(tK_2, G)$, should be decomposed into at least two copies of G. If H is different from lG, then $\Delta(H) > \frac{ql}{t}$ which implies $\chi'(H) > \frac{ql}{t}$ and hence by Theorem 1.4, H is not tK_2-decomposable. Thus lG is the unique least common multiple of tK_2 and G.

Case 2. $l = 1$.

In this case $lcm(tK_2, G) = q$. Thus $tK_2|G$ and G is the unique least common multiple. □

Remark 2.2. *The result in the above theorem, Theorem 2.1, appeared in [12]. We are giving the proof of this result here since the result is needed for proving Theorem 2.3. The result was proved* *by the first author of this manuscript.*

2.2 On graphs that have a unique least common multiple with $P_3 \cup K_2$

Theorem 2.3. *A nonempty graph G without isolated vertices and the graph $P_3 \cup K_2$ have a unique least common multiple if and only if $G = K_2$ or $P_3 \cup K_2 \mid G$.*

Proof. Let $q = e(G)$.

Case 1. G is a connected graph.

If $G = K_2$, then $G \mid P_3 \cup K_2$. Thus $LCM(P_3 \cup K_2, K_2) = \{P_3 \cup K_2\}$ and hence their least common multiple is unique. So we are going to analyse the case where $G \neq K_2$.

Consider the graph $3G$, a union of three disjoint copies of G. Then

(1) $e(3G) \equiv 0 \pmod 3$.

(2) $\Delta(3G) = \Delta(G) \leq q = \frac{1}{3}(3q) \leq \frac{2}{3}(3q) = \frac{2}{3}e(3G)$.

(3) $c(3G) \leq 3 \leq \frac{1}{3}(3q) = \frac{1}{3}e(3G)$, if $e(G) \geq 3$. If $e(G) = 2$, then $c(3G) = 0 \leq \frac{1}{3}e(3G)$.

(4) The edges of $3G$ cannot be covered by two adjacent vertices, since the graph is disconnected.

Thus by Theorem 1.5, $3G$ is $P_3 \cup K_2$-decomposable. Clearly $3G$ is G-decomposable. Hence $lcm(P_3 \cup K_2, G) \leq 3q$.

Subcase (i): $lcm(P_3 \cup K_2, G) = 3q$.

Consider the graph $H = G \circ G \cup G$, where $G \circ G$ is the graph obtained by identifying a least degree vertex in two copies of G. Then

(1) $e(H) \equiv 0 \pmod 3$.

(2) $\Delta(H) \leq 2q = \frac{2}{3}(3q) = \frac{2}{3}e(H)$.

(3) $c(H) \leq 1 \leq \frac{1}{3}(3q) = \frac{1}{3}e(H)$.

(4) Since H is disconnected, edges of H cannot be covered by two adjacent vertices.

Thus by Theorem 1.5, H is $P_3 \cup K_2$- decomposable. Clearly H is G- decomposable. Hence in this case both H and $3G$ are elements of $LCM(P_3 \cup K_2, G)$ and hence their least common multiple is not unique.

Subcase (ii): $lcm(P_3 \cup K_2, G) = 2q$.

In this case there exists a graph H such that $e(H) = 2q$ and $H \in LCM(P_3 \cup K_2, G)$. Consider the graph $2G$.

(1) Since $H \in LCM(P_3 \cup K_2, G)$ we get $3 \mid e(H) = 2q = e(2G)$ and hence $e(2G) \equiv 0 \pmod 3$.

(2) Since H is G-decomposable and $\Delta(G) = \Delta(2G)$, $\Delta(2G) \leq \Delta(H)$. H is $P_3 \cup K_2$-decomposable and so by Theorem 1.5, $\Delta(H) \leq \frac{2}{3}e(H) = \frac{2}{3}e(2G)$. Thus $\Delta(2G) \leq \frac{2}{3}e(2G)$.

(3) In this case $q \geq 3$ $\big($if $q = 1$, then $G = K_2$ and if $q = 2$, then $e(2G) = 4 \not\equiv 0 \pmod 3\big)$. So $c(2G) \leq 2 \leq \frac{1}{3}2q = \frac{1}{3}e(2G)$.

(4) Since $2G$ is disconnected, the edges of $2G$ cannot be covered by two adjacent vertices.

By applying Theorem 1.5, $2G$ is $P_3 \cup K_2$-decomposable. $2G$ is clearly G-decomposable. Thus $2G \in LCM(P_3 \cup K_2, G)$.

We can also prove that $G \circ G \in LCM(P_3 \cup K_2, G)$.

(1) $e(G \circ G) = e(2G) \equiv 0 \pmod 3$.

(2) In order to prove that $\Delta(G \circ G) \leq \frac{2}{3}e(G \circ G)$ it is enough to prove that $\Delta(G)$ and $2\delta(G)$ are less than or equal to $\frac{2}{3}e(G \circ G)$, since $G \circ G$ is obtained by identifying a vertex of least degree in two copies of G.

Since $H \in LCM(P_3 \cup K_2, G)$, $\Delta(G) \leq \Delta(H) \leq \frac{2}{3}e(H) = \frac{2}{3}e(G \circ G)$.

$2\delta(G) \leq \frac{2}{3}e(G \circ G) \iff \delta(G) \leq \frac{2q}{3}$. Suppose $\delta(G) > \frac{2q}{3}$. Then $2q = \sum_{v \in V(G)} d(v) \geq \sum_{v \in V(G)} \delta(G) = n\delta(G) > n\frac{2q}{3}$, where $n = |V(G)|$. This implies $n < 3$. G is a connected graph without isolated vertices and $G \neq K_2$. Thus $n \geq 3$ and so $\delta(G) \leq \frac{2q}{3}$.

(3) $c(G \circ G) = 0 < \frac{1}{3}e(G \circ G)$.

(4) The edges of $G \circ G$ cannot be covered by two adjacent vertices. Suppose the edges of $G \circ G$ can be covered by two adjacent vertices, then the identified vertex is one such vertex, since in $G \circ G$, no two vertices are adjacent except the identified vertex. This implies using the identified vertex and one other vertex it is possible to cover all the edges of $G \circ G$. This is possible only if G is a star with the identified vertex as the center of the star. This is a contradiction, since to construct $G \circ G$ a vertex of least degree in G is identified.

Applying Theorem 1.5, $G \circ G$ is $P_3 \cup K_2$- decomposable and it is clearly G-decomposable. So $G \circ G \in LCM(P_3 \cup K_2, G)$.

We have proved that $2G$ and $G \circ G \in LCM(P_3 \cup K_2, G)$ and hence their least common multiple is not unique.

Subcase (iii): $lcm(P_3 \cup K_2, G) = q$.

In this subcase G is the unique least common multiple, since $q = e(G)$.

Case 2. G is disconnected.

As in the first case, assume that $G \neq tK_2$. Then at least one component of G has more than one edge. We construct a graph of size $3q$, which is a $(3q, G, P_3 \cup K_2)$-graph, where $q = e(G)$. The construction is as follows. Take a least degree vertex from each component of G. Let H be the connected graph obtained by identifying all these vertices together. Take a least degree vertex in H. Denote by $H \circ H \circ H$, the graph obtained by identifying this least degree vertex in three copies of H.

(1) $e(H \circ H \circ H) = e(3H) = 3e(G) \equiv 0 \pmod 3$.

(2) $\Delta(H \circ H \circ H) \leq 2\Delta(H) \leq 2e(G) = \frac{2}{3}e(3G) = \frac{2}{3}e(H \circ H \circ H)$.

(3) $c(H \circ H \circ H) \leq 1 \leq \frac{1}{3}e(H \circ H \circ H)$.

(4) As in Subcase *(ii)* of the previous case, the edges of $H \circ H \circ H$ cannot be covered by two adjacent vertices.

By Theorem 1.5, $H \circ H \circ H$ is $P_3 \cup K_2$-decomposable and obviously it is G-decomposable. Thus $lcm(P_3 \cup K_2, G) \leq 3q$.

Subcase (i): $lcm(P_3 \cup K_2, G) = 3q$.

From the above discussion $H \circ H \circ H$ is a least common multiple in this subcase. Consider the graph $H \circ H \cup H$.

(1) $e(H \circ H \cup H) = 3e(G) \equiv 0 \pmod 3$.

(2) $\Delta(H \circ H \cup H) \leq 2\Delta(H) \leq 2e(G) = \frac{2}{3}e(3G) = \frac{2}{3}e(H \circ H \cup H)$.

(3) $c(H \circ H \cup H) \leq 1 = \frac{1}{3}e(H \circ H \cup H)$.

(4) Since $H \circ H \cup H$ is disconnected, the edges of $H \circ H \cup H$ cannot be covered by two adjacent vertices.

Applying Theorem 1.5, $H \circ H \cup H$ is $P_3 \cup K_2$-decomposable and by construction it is G-decomposable. Thus both $H \circ H \circ H$ and $H \circ H \cup H$ are in $LCM(P_3 \cup K_2, G)$ and hence their least common multiple is not unique.

Subcase (ii): $lcm(P_3 \cup K_2, G) = 2q$.

In this subcase there exists a graph H' of size $2q$ which is both G and $P_3 \cup K_2$ decomposable. We will prove that $H \circ H$ and $H \cup H$ are in $LCM(P_3 \cup K_2, G)$.

(1) $e(H \circ H) = 2q \equiv 0 \pmod 3$, since $e(H') = 2q$ and H' is $P_3 \cup K_2$- decomposable.

(2) In order to prove that $\Delta(H \circ H) \leq \frac{2}{3}e(H \circ H)$, it is enough to prove that $2\delta(H) \leq \frac{2}{3}e(H \circ H)$. That is we need to prove $\delta(H) \leq \frac{1}{3}(2q)$, where $q = e(H) = e(G)$.

Suppose $\delta(H) > \frac{2q}{3}$. Then $2q = \sum_{v\in V(H)} d(v) \geq \sum_{v\in V(H)} \delta(H) = n\delta(H) > n(\frac{2q}{3}) \Rightarrow n < 3$. Since G is a disconnected graph without isolated vertices, $n < 3$ is not possible. Hence $\delta(H) \leq \frac{2q}{3}$. Thus $\Delta(H \circ H) \leq \frac{2}{3}e(H \circ H)$.

(3) $c(H \circ H) = 0 < \frac{1}{3}e(H \circ H)$.

(4) By the construction of $H \circ H$, the edges of $H \circ H$ cannot be covered by two adjacent vertices.

By Theorem 1.5, $H \circ H$ is $P_3 \cup K_2$-decomposable and by construction, $H \circ H$ is G-decomposable and so $H \circ H \in LCM(P_3 \cup K_2, G)$.

Also $H \cup H \in LCM(P_3 \cup K_2, G)$, since

(1) $e(H \cup H) = 2q \equiv 0 \pmod 3$, since $lcm(P_3 \cup K_2) = 2q$, where $q = e(G) = e(H)$.

(2) $\Delta(H \cup H) \leq \Delta(H \circ H) \leq \frac{2}{3}e(H \circ H) = \frac{2}{3}e(H \cup H)$.

(3) Here $c(H \cup H) \leq 2$. Thus $c(H \cup H) \leq \frac{1}{3}e(H \cup H)$ if $2 \leq \frac{2q}{3}$, that is if $q \geq 3$, where, $q = e(G) = e(H)$. Since G is a disconnected graph without isolated vertices, $q \neq 1$. Also if $q = 2$, then $2q = 4 \not\equiv 0 \pmod 3$. Thus in this subcase, $q \geq 3$ and hence $c(H \cup H) \leq \frac{1}{3}e(H \cup H)$.

Thus $H \circ H$ and $H \cup H$ belong to $LCM(P_3 \cup K_2, G)$ and hence their least common multiple is not unique. □

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