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Research Article

Applying the Bayesian Technique in Designing a Single Sampling Plan

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ABSTRACT

The Bayesian sampling plans for production inspection are considered a technique of sampling inspection techniques for determining the characteristics of the sampling plan based on the assumption that the rate of defectives is a random variable that varies from one production batch to the next, resulting in a probability distribution $f(p)$ that could be determined based on experience and the available quality information available. As part of this study, the parameters of a single Bayesian sampling plan (n,c) were derived using the beta-binomial distribution and compared with those of other single sampling plans. Researchers have identified (ALA company for soft drinks), which handles product quality control. One hundred and twenty production batches were selected, and the size of the batch and the number of defective items were used to determine the proportion of defective items, given that the variable varies randomly from one production batch to the next. Bayesian and decision-making models can be implemented to create a single sampling inspection process that is close to the actual quality level. The researchers discovered that when the decision-making model was used, the sample size was minimal compared to other inspection plans, leading to a low inspection cost.

Keywords: Statistical quality control, average sample number, acceptance quality level, operating characteristic, Bayesian sampling plans

INTRODUCTION

Among the statistical tools used to control and monitor the quality of production are control charts and sampling inspection plans. Regarding sample testing plans, they are an accurate and appropriate method to obtain an estimate of the presence of one or more characteristics among the produced units. This is achieved by examining a small percentage of the production, which is randomly selected to determine whether to accept or reject production based on the results of the sample drawn. There are two methods of examination using the sampling method: The discriminatory examination method (by attributes), where the produced units are classified into defective and good units, or by the variable examination method (by variables) based on a standard such as height or weight.

The examination may be based on the researcher's or examiner's experience, in addition to any other available information about the production process (prior information). This approach leads to the use of Bayesian theory and decision theory, and the adoption of Bayesian estimation for the quality parameter of production, depending on the loss function, to avoid making wrong decisions regarding the production process.[1,2]

Due to the effort, cost, and time required to conduct a comprehensive examination, as well as cases known to statisticians where the above examination can be applied, the sampling examination method was used to evaluate the performance of the quality control department in the company. Since most of the staff working in this department are not statistical experts and are not familiar with statistical methods in the field of quality control, they rely on the accumulated experience of their members, examiners, and laboratory workers.

For the aforementioned reasons, this research aims to shed light on the methods of sample examination and their use in estimating sample size and designing sampling examination plans that can minimize spoilage and reduce costs by making the right decision to accept or reject the produced batch.

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This research aims to investigate the method of sampling and decision-making regarding the acceptance or rejection of batches. This will be achieved through the design of a discriminatory sampling examination plan using the Bayesian method and applying it to the production of Pepsi Ala Soft Drinks Company to make informed decisions on whether to accept or reject the produced batch.[3,4]

BAYESIAN SAMPLING PLANS

One of the methods of sample examination is the Bayesian sampling plan, which is an alternative to the comprehensive (classic) examination, as well as the single, double, and sequential sampling plans. However, Bayesian sampling plans differ from single, double, and sequential sampling plans in that they deal with specifying plan parameters based on the percentage of defects in the product with random variation. This percentage changes from one production batch to another, as it has a probability distribution that can be determined from previous experience and available information on quality.[1,5]

Bayesian plans have emphasized the importance of utilizing available information on quality, and this information is referred to as prior distributions. The prior distributions are used to obtain the post-quality posterior distribution, assuming that the type of sampling distribution being studied may be binomial with parameters (n,p), poisson with parameter (λ), or normal with parameters (μ,σ). Bayesian plans are of great importance in examining products, and it is necessary to provide a brief explanation of this topic to reach a Bayesian sampling plan (n, c) that reduces the total cost function of quality control. The cost function is the sum of inspection costs, costs of rejecting non-defective units (good), and costs of accepting defective units (bad).[6]

Bayesian Plans Concept for Product Inspection

Bayes' plans have been named by this name in relation to the scientist (Thomas Bayes) (1702–1761), where Bayes is the first to use the prior distribution for defective percentages in statistical inference, and since 1960, attention began to focus on Bayes' plans to test the product.

Bayesian plans are named after the scientist Thomas Bayes (1702–1761), who was the first to use prior distributions for defective percentages in statistical inference. Since the 1960s, attention has been focused on Bayesian plans for testing products. In 1964, the scientist Hald (1981) was able to develop a model for the total cost function of quality control. Through this model, the parameters of the sampling plans are determined when the quality is fixed or the random variable has a prior distribution (prior distribution).[7,8]

In 1968, the scientist Hald presented a model for Bayesian plans to examine products. This model is used to obtain the parameters of the individual Bayesian plan (n, c) by reducing the standard cost function of quality control, which is:

$$R(N,n,c) = n(p_s - p_m + (N-n)\left[\int_0^{pr}(p_r - p)Q(p)w(p)dp + \int_{pr}^{1}(p-p_r)P(p)w(p)dp\right] \quad (1)$$

Where ($R = P+Q$), denotes sample costs.

P_m-: The standard cost in cases of rejection and acceptance.

P_s: - The average cost of examination per unit in both cases of rejection and acceptance, and that: -

$P_s \geq P_r \geq P_m$, $0 < P_r < 1$, $0 < P_s < 1$

On equation (1) If the value of (P) is very small, we use the poisson distribution, that is,:

$$b(x,np) = \frac{e^{-np}(np)^x}{x!} \quad (2)$$

And we change

$$z = \frac{p}{p_r}, m = np_r, \quad M = NP_r$$

The form of the function will be:

$$B(c,np) = B(c,mz) = P(z) \quad (3)$$

Total Cost Function for Quality Control

The scientist Hald (1981) developed a model that included the total costs of examination, acceptance, and rejection. The model aims to determine the individual sampling plan (n, c) for examining batch N of the product of quality p by applying filtering examination. He expressed these costs in the following formula:

$$h(x,X,p,N,n,c) = nS_1 + nS_2 + (N-n)A_1 + (X-x)A_2, x \leq c \quad (4)$$

$$h(x,X,p,N,n,c) = nS_1 + nS_2 + (N-n)R_1 + (X-x)R_1 \quad x > c \quad (5)$$

The examination and repair costs, in addition to the costs resulting from accepting the quantity (N-n) remaining after drawing the sample, represent the first part. The examination and repair costs, in addition to the costs resulting from rejecting the quantity (N–n), represent the second part. Equating equation (4) with equation (5) results in:

$$P_r = \frac{X-x}{N-x} = \frac{R_1 - A_1}{A_2 - R_2}$$

The average cost in both cases of acceptance and rejection is equal to:

In the event of acceptance

$$n(S1+S2p)+(N–n)(A_1+A_2p)$$

in case of refusal

$$n(S_1+S_2p)+(N–n)(R_1+R_1p)$$

And by entering the probability of acceptance of the product, $P(p)$ and the probability of rejection, $Q(p)$ since:-

$$P(p) = p_r(x \leq c) = \sum_{x=0}^{c} b(x,n,p) = B(c,n,p)$$

$$Q(p) = 1 - P(p) = p_r(x > c) = \sum_{x=c+1}^{n} b(x,n,p) = E(c+1,n,p)$$

Thus, the cost rate (p) is obtained in the cases of acceptance and rejection as follows:

$$K(p) = n(S_1 + S_2 p) + (N-n)$$
$$\{(A_1 + A_2 p)P(p) + (R_1 + R_2 p)Q(p)\}$$

$$K(p) = nk_s(p) + (N-n)\{k_a(p)P(p) + k_r(p)Q(p)\} \quad (6)$$

Where $Q(p) = 1–P(p)$ becomes clear from equation (6), then:

$$K(p) = nk_s(p) + (N-n)\{k_a(p)P(p) + k_r(p)(1-P(p))\}$$
$$= nk_s(p) + (N-n)\{k_a(p)P(p) + k_r(p) - k_r(p)P(p)\}$$
$$= nk_s(p) + (N-n)\begin{Bmatrix} k_a(p)P(p) \\ +(N-n)k_r(p) - (N-n)k_r(p)P(p) \end{Bmatrix}$$
$$= nk_s(p) + (N-n)\begin{Bmatrix} k_a(p)P(p) + Nk_r(p) \\ -nk_r(p) - (N-n)k_r(p)P(p) \end{Bmatrix}$$
$$= nk_s(p) - nk_r(p) + (N-n)k_a$$
$$(p)P(p) - (N-n)k_r(p)P(p) + Nk_r(p)$$
$$K(p) = n[k_s(p) + k_r(p)] + (N-n)\left[k_a(p) - k_r(p)\right]P(p) + Nk_r(p) \quad (7)$$

The values of $k_r(p)$, $k_a(p)$, $k_s(p)$ express the rates of examination, acceptance, and rejection costs per unit, respectively.[9-12]

FINDING THE SINGLE BAYESIAN PLAN (*n*, *C*)

This method relies on the iterative method for determining the parameters of the single sampling plan (n, c). The method involves minimizing equation (7) $k(p)$ subject to certain conditions of the operating curve function (OC). We find the smallest value of the function at $k(p)$ ($c = 0$), then at ($c = 1$), then at ($c = 2$), and so on. Once the absolute minimum value is obtained, we stop and fix the parameters of the single sampling plan (n, c) at this value. It is important to note that the cost function model shown in equation (7) assumes a constant level of quality, but the quality level may change from one production batch to another due to random and attritional reasons that lead to qualitative deviations. Therefore, it is necessary to estimate the level of quality in the future by taking advantage of all available information about previous tests and information about the production process, which is supposed to be within control limits. Furthermore, natural qualitative changes that occur due to market competition should also be taken into account.

It is worth noting that incorporating previous information available on quality levels, as well as estimated information from samples, leads to more accurate decisions regarding quality level estimates. The scientist Hald has emphasized this point and relied on the following expected total cost function K relative to the previous distribution of defective ratios $f(p)$.

$$K = \int_{-\infty}^{\infty} K(p) f(p) dp$$

BAYESIAN PLAN RELATIVE TO THE BETA DISTRIBUTION

The beta distribution is considered one of the important statistical distributions in Bayesian plans. In this distribution, the percentage of defects is a random variable with a beta distribution, and its parameters can be estimated using the moment method. The probability function of the random variable p with (α, β) parameters takes the following form:

$$f(p,\alpha,\beta) = \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1} \qquad 0 < p < 1$$
$$= 0 \qquad O.W$$

In many cases, it cannot be assumed that p remains constant from one batch to another. For example, consider a machine used to produce a specific unit. After producing a batch, the machine is checked and put back into operation. For each batch, p can be determined according to certain distributions, which are referred to as the initial distributions of p. For this reason, the beta distribution was chosen as the initial distribution of p with (α, β) parameters. The expected value of the random variable for the beta distribution is $\alpha/(\alpha+\beta)$. This means that (β) must be greater than (α), and when estimating (α, β), it must be rounded to the nearest integer. If the estimated value lies between 0 and 1; then, it must be rounded to the expected value of p, to the nearest integer. This method allows us to use standard tables to determine the sampling plan and also shows us why the plans are not clearly sensitive to small changes in the parameters without taking into account the appropriate initial distributions.[13-15]

Direct Formulas for Determining Individual Base Plan Parameters by Decision Model

To define the parameters of the single Bayesian Economic Statistical (BES) plan (n, c) for product inspection, a formula for the expected risk must be developed and specified directly for the parameters of the sampling plan. We can rely on the definition of risk provided by Guthrie and Johns, who defined it as the sum of examination costs plus the loss resulting from wrong decisions. It is well-known that examination costs depend on the volume. It represents the loss resulting from accepting defective units and the loss resulting from rejecting good units. The expected risk formula, under the conditions of binomial sampling, takes into account the distributions of continuous defective percentages, which are as follows:

$$R[f(p),n,c] = n\{(S_2 - R_2)\overline{P} + (S_1 - R_1) + (A_2 - R_2)$$
$$\int_0^{P_r} (P_r - P)Q(P)f(p)dp\} +$$
$$N\{R_2\overline{P} + R_1 + (A_2 - R_2)\int_0^{P_r} (P - P_r)f(P)dp$$
$$(N-n)\{\frac{1}{2n}(R_1 - A_1)(1-p_r)f(p_r) \quad (8)$$

When neglecting the upper limits, equation (8) is reduced to the following form:

$$R[f(p),n,c] = An + BN + C(N-n)\frac{1}{n} \quad (9)$$

So:

$$A = [(S_2 - R_2)\overline{P} + (S_1 - R_1) + (R_2 - A_2)\int_0^{P_r} (P - P_r) f(P) dp$$

$$B = R_2\overline{P} + R_1 + (A_2 - R_2)\int_0^{P_r} (P - P_r) f(P) dp$$

$$C = \frac{1}{2}(R_1 - A_1)(1 - P_r) f(p = p_r)$$

$$\int_0^{P_r} (P - P_r) f(P) dp = \overline{P} IB_{P_r}(\alpha + 1, \beta) - P_r IB_{P_r}(\alpha, \beta)$$

When we derive equation (9) with respect to n and equate the derivative to zero, we obtain the optimal value of n (n^*). To find the sample size necessary to examine batch N of the product, assuming binomial sampling and that the defective percentages change from one production batch to another, we can use the following formula: The random variable has a prior distribution equal to beta with (α, β) parameters, and the optimal value of n can be obtained from the following equation.

$$n^* = \left[\frac{(R_1 - A_1) P_r^{\alpha-1} (1 - P_r)^{\beta} N}{2B(\alpha, \beta)[S_2 - R_2)\overline{P} + (S_1 - R_1) + (R_2 - A_2)} \right]^{\frac{1}{2}}$$
$$(\overline{P} IB_{P_r}(\alpha + 1, \beta) - P_r IB_{P_r}(\alpha, \beta))]$$

When the values of (α, β) are integers, the relationship between the incomplete beta function and the cumulative binomial function can be derived. This relationship can be used to extract the following:

$$IB_{pr}(\alpha, \beta) = \sum_{x=\alpha}^{\alpha+\beta-1} C_x^{\alpha+\beta-1} P_r^x (1 - p_r)^{\alpha+\beta-x-1}$$

$$IB_{pr}(\alpha + 1, \beta) = \sum_{x=\alpha+1}^{\alpha+\beta} C_{\alpha+1}^{\alpha+\beta} P_r^x (1 - p_r)^{\alpha+\beta-x}$$

The number of acceptance can be obtained from the following relationship:

$$c \simeq n\theta_{\circ} - \frac{2}{3}$$

$\theta_{\circ}$: Critical quality level.

Direct Formulas for Determining Single Bayes Plan Parameters from Hald Model

Determining the parameters of individual Bayesian plans (n, c) for product inspection requires lengthy iterative calculations to find the values (n, c) that achieve the smallest expected total cost or the smallest standard cost. Research in this area has focused on finding formulas that efficiently and quickly obtain optimal parameter values. Continuous studies and research have led to formulas used directly for large production batches where the quality of the batches is a random variable with a prior distribution $f(p)$, which is continuous and differentiable at points adjacent to a point (p = pr). The discontinuous distribution of defective percentages was discussed by the scientist Hald in 1965, and direct formulas were developed by Hald in 1968, which were supported by auxiliary tables. These formulas can be used to extract Bayesian plans in distributions such as gamma-poisson and beta-binomial.[8,16-19]

It is then transformed into the standard cost function (R, N, n, c), which, in turn, requires defining each of the standard loss rates for acceptance d_a, rejection d_r, and examination d_s as follows:

$$d_a = \frac{(K_a - K_m)}{(A_2 - R_2)} \quad (10)$$

$$d_r = \frac{(K_r - K_m)}{(A_2 - R_2)}$$

$$d_s = \frac{(K_s - K_m)}{(A_2 - R_2)}$$

If all units of the product are classified correctly, the value of K_m is the smallest cost per unit, that is, in the case of acceptance when ($P \le P_r$) and rejection d_r when ($P > P_r$) (). In other words, Km represents.

$$K_m = \int_0^{pr} (A_1 + A_2 P) f(p) dp + \int_{pr}^1 (R_1 + R_2 P) f(p) dp$$

$$K_m = \int_0^{pr} (A_1 + A_2 P) f(p) dp +$$
$$\int_0^1 (R_1 + R_2 P) f(p) dp - \int_0^{pr} (R_1 + R_2 P) f(p) dp$$

$$K_m = K_r - (A_1 + R_2)\int_0^{pr} (P_r - P) f(p) dp$$

Among them, we find that:

$$d_r = \int_0^{pr} (P_r - P) f(p) dp$$

The value of d_a,d_s in terms of d_r, as:

$$d_a = d_r - P_r + \overline{P}$$

$$d_s = d_r + \{(S_1 - R_1) + (S_2 - R_2)\overline{P}\} + (A_2 - R_2)$$

$$K_m = K_r - (A_2 + R_2) d_r$$

So it is:

$$NK_m = K_r - (A_2 + R_2) d_r \quad (11)$$

equations (11), (10) are then substituted into equation (12) and the equation of the standard cost function defined in equation (11) is obtained.

$$R(N,n,c)=\frac{K(N,n,c)-K_m}{A_2-R_2} \quad (12)$$

$$R(N,n,c)=nds+(N-n)d(n)$$

$$d(n)=d_r+\int_0^1 (P-P_r)B(c,n,p)f(p)dp$$

Depending on the equation

$$\lambda_1^2=\frac{p_r^{\alpha}q_r^{\beta}(A_2-R_2)}{2B(\alpha,\beta)(k_s-k_m)}$$

$$\lambda_1^2=\frac{3(\alpha,\beta)^2-11(\alpha,\beta)-2-(3\alpha-1)}{p_r}-\frac{(3\beta-1)\beta}{q_r}-\frac{1}{p_rq_{r-1}}$$

The value of n necessary to check the batch (N) is the value defined by the following relation:

$$n^*=\lambda_1\sqrt{N}+\lambda_2$$

As for the number of acceptance of c, it is extracted from the following relationship:

$$c^*=n^*p_r+\beta_1$$

Whereas:

$$\beta_1=\hat{\beta}p_r-\hat{\alpha}q_r-\frac{1}{2}$$

To find the value of the standard cost function $R_0(N)$ corresponding to the optimal sampling plan (n^*, c^*), which we will symbolize $R_0(N)$, the following equation will be relied on:

$$R_0(N)=(2n^*-\lambda_1^2-\lambda_2)ds \quad and \quad d_s=\frac{k_s-k_s}{(A_2-R_2)}$$

Therefore, the value $K(P)$ of the expected total cost of quality control is equal to:

$$k(p)=R_0(N)(A_2-R_2)+NK_m$$

RESULTS

The process of improving product quality requires relying on modern scientific methods. By utilizing modern practical methods and adhering to standard specifications, products can be made suitable and conform to the desired specifications of consumers. This not only elevates the status of the production facility in local and global markets but also enhances the value of these products in these markets. Therefore, it is essential to establish quality control requirements fully. This includes prioritizing standard and manufacturing specifications for input, process, and output elements. One of these requirements is to identify and provide a scientific method for examining materials and products to ensure the reduction of damage and to ensure the regular flow and handling of circulation during production processes in the facility.

Application of the Decision-Making Model

Based on the decision-making model, a set of Bayesian economic-statistical (BES) plans was developed to test the product, depending on the previous distribution of defective percentages (Beta-Prior). The parameters of the individual BES plan were determined according to the decision-making model to obtain the values of n and the acceptance number c. The inspection plans for this product are shown in Table 1, taking into account the levels of quality and sizes of production batches.

The parameters of the sampling plan required to check the daily production of the 1.5-L (Pepsi Ala) product of quality (X_p = 0.005349) and value (P_r = 0.00617) were extracted using this model. The values obtained were (n,c) = (1495, 14), and the expected risk value for the sampling plan was also determined to be (1495, 14). The value of $R\{f(p), n, c\}$ was found to be equal to $43,210.

Hald Model Application

Since the distribution of the defective percentages $f(p)$ is one of the continuous and derivable distributions at point ($P = P_r$), a set of Bays plans will be extracted from the direct formulas created by (Hald), as the set of Bays plans necessary to check the product, which is defined from equation (14), must be calculated before that each of: -

Examination cost rate per unit

(1) $k_s=S_1+S_2\bar{p}=0.0011+(0.03)(0.005349)=0.00126$ \$

Rejection cost rate per unit

(2) $k_r=R_1+R_2\bar{p}=k_s$

Examination cost rate per unit

$k_s=A_1+A_2\bar{p}$

(3) $0+0.208(0.005349)=0.0011\$$

As well as the values of the ingredients (4, 5, 6, and 7):

(4) $\int_0^{p^r}(p_r-p)f(p)dp$

$=p_rIB_{pr}(\alpha,\beta)-\bar{p}IB_{pr}(\alpha+1,\beta)$

$=(0.006179)(0.789972)$

$-(0.005349)(0.734239)=0.000954$

(5) $k_m=k_r-(A_2-R_2)\int_0^{p_r}(P_r-P)f(p)dp$

$=(0.00126)-(0.178)(0.0009544)=0.0010901)$

(6) $\lambda_1^2=\frac{p_r^{\alpha}q_r^{\beta}(A_2-R_2)}{2B(\alpha,\beta)(k_s-k_m)}$

$\lambda_1^2=148.25059$

Whereas:

(7)

$p_r=0.0061, \quad A_2=0.208, \quad R_2=0.03, \quad \hat{\alpha}=25, \quad \hat{\beta}=4648$

$$\lambda_2=3\frac{(\alpha+\beta)^2-11(\alpha+\beta)-2-(3\alpha+1)}{p_r}-\frac{(3\beta-1)\beta}{q_r}-\frac{1}{p_rq_{r-1}}$$

Table 1: Bayesian plans to test the product according to the distribution of beta-prior extracted from the decision-making model

Quality level 0.005		Quality level 0.004		Quality level 0.003		Quality level 0.002		Quality level 0.001		Batch size
Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	
c	n	c	n	c	n	c	n	c	n	
6	669	5	527	4	449	3	398	7	753	10,000
6	701	5	553	4	471	4	418	7	790	11,000
7	733	5	578	4	492	4	436	8	825	12,000
7	763	5	602	4	512	4	454	8	859	13,000
7	791	6	624	5	532	4	471	8	891	14,000
8	819	6	646	5	550	4	488	9	923	15,000
8	846	6	667	5	569	4	504	9	953	16,000
8	872	6	688	5	586	5	519	9	982	17,000
8	897	6	708	5	603	5	534	9	1011	18,000
9	922	7	727	6	620	5	549	10	1038	19,000
9	946	7	746	6	636	5	563	10	1065	20,000
9	969	7	765	6	651	5	577	10	1092	21,000
9	992	7	783	6	667	5	591	11	1117	22,000
9	1014	7	800	6	682	5	604	11	1143	23,000
10	1036	8	817	6	696	6	617	11	1167	24,000
10	1058	8	834	6	711	6	630	11	1191	25,000
10	1079	8	851	7	725	6	642	11	1215	26,000
10	1099	8	867	7	739	6	654	12	1238	27,000
11	1119	8	883	7	752	6	666	12	1261	28,000
11	1139	8	899	7	766	6	678	12	1283	29,000
11	1159	8	914	7	779	6	690	12	1305	30,000
11	1178	9	929	7	792	6	701	13	1327	31,000
11	1197	9	944	7	804	6	712	13	1348	32,000
11	1215	9	959	8	817	7	724	13	1369	33,000
12	1233	9	973	8	829	7	734	13	1389	34,000
12	1251	9	987	8	841	7	745	13	1410	35,000
12	1269	9	1001	8	853	7	756	14	1430	36,000
12	1287	9	1015	8	865	7	766	14	1449	37,000
12	1304	10	1029	8	876	7	776	14	1469	38,000
13	1321	10	1042	8	888	7	787	14	1488	39,000
13	1338	10	1055	8	899	7	797	14	1507	40,000
13	1355	10	1069	8	910	7	807	15	1526	41,000
13	1371	10	1082	9	921	7	816	15	1544	42,000
13	1387	10	1094	9	932	8	826	15	1562	43,000
13	1403	10	1107	9	943	8	836	15	1581	44,000
14	1419	11	1120	9	954	8	845	15	1598	45,000
14	1435	11	1132	9	964	8	854	15	1616	46,000
14	1450	11	1144	9	975	8	864	16	1634	47,000
14	1466	11	1156	9	985	8	873	16	1651	48,000
14	1481	11	1168	9	995	8	882	16	1668	49,000
14	1496	11	1180	9	1005	8	891	16	1685	50,000

Table 2: Bayesian plans to test the product extracted from the (Hald) model

Quality level 0.005		Quality level 0.004		Quality level 0.003		Quality level 0.002		Quality level 0.001		Batch size
Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	Acceptance number	Sample size	
c	n	C	n	c	n	c	n	c	n	
10	1162	8	798	7	671	3	588	6	529	10,000
11	1222	8	839	7	707	3	620	6	557	11,000
11	1278	8	879	8	741	3	650	7	584	12,000
11	1333	9	917	8	773	3	679	7	611	13,000
12	1385	9	954	8	804	3	706	7	636	14,000
12	1436	9	989	8	835	3	733	7	660	15,000
12	1485	9	1023	8	864	3	759	7	683	16,000
12	1532	10	1057	9	892	3	784	7	706	17,000
13	1578	10	1089	9	920	3	808	8	728	18,000
13	1623	10	1120	9	946	3	832	8	750	19,000
13	1667	10	1151	9	972	3	855	8	771	20,000
14	1709	10	1181	9	998	3	878	8	791	21,000
14	1751	10	1210	9	1022	3	900	8	811	22,000
14	1791	11	1238	9	1047	3	921	8	830	23,000
14	1831	11	1266	10	1070	3	942	8	849	24,000
15	1870	11	1293	10	1094	3	963	8	868	25,000
15	1908	11	1320	10	1116	3	983	8	886	26,000
15	1945	11	1346	10	1139	3	1003	9	904	27,000
15	1982	11	1372	10	1161	3	1022	9	922	28,000
15	2018	12	1397	10	1182	3	1041	9	939	29,000
16	2054	12	1422	10	1203	3	1060	9	956	30,000
16	2088	12	1446	11	1224	3	1078	9	973	31,000
16	2123	12	1470	11	1245	3	1096	9	989	32,000
16	2156	12	1494	11	1265	3	1114	9	1006	33,000
17	2190	12	1517	11	1285	3	1132	9	1022	34,000
17	2222	13	1540	11	1304	3	1149	9	1037	35,000
17	2255	13	1563	11	1323	3	1166	10	1053	36,000
17	2287	13	1585	11	1342	3	1183	10	1068	37,000
17	2318	13	1607	11	1361	3	1200	10	1083	38,000
18	2349	13	1629	12	1380	3	1216	10	1098	39,000
18	2380	13	1650	12	1398	3	1232	10	1113	40,000
18	2410	13	1672	12	1416	3	1248	10	1127	41,000
18	2440	13	1693	12	1434	3	1264	10	1142	42,000
18	2469	14	1713	12	1452	3	1280	10	1156	43,000
18	2499	14	1734	12	1469	3	1295	10	1170	44,000
19	2527	14	1754	12	1486	3	1310	10	1184	45,000
19	2556	14	1774	12	1503	3	1325	10	1197	46,000
19	2584	14	1794	12	1520	3	1340	10	1211	47,000
19	2612	14	1813	13	1537	3	1355	11	1224	48,000
19	2640	14	1833	13	1553	3	1370	11	1237	49,000
19	2667	14	1852	13	1570	3	1384	11	1251	50,000

And I extracted my value (λ_1,λ_2), so the value of n necessary to check the batch is N = 39388, which is the value specified by the following relationship:

$$n^* = \lambda_1\sqrt{N} + \lambda_2$$

$$n^* = (12.17582)\sqrt{39388} + (55.4084) = 2361 units$$

$c^* = n^* p_r$ As for the acceptance number c, it is extracted from the relationship, so that

$$\beta_1 = \hat{\beta} p_r - \hat{\alpha} q_r - \frac{1}{2}$$

Thus, the value of the acceptance number corresponding to the sample size (n = 2361) is equal to:

$$\beta_1 = (4648)(0.0061) - (25)(0.9939) - 0.5$$

$$\beta_1 = 3.0053$$

Accordingly:

$$c^* = 2361(0.061) + 3.0053 = 17 units$$

Therefore, the single sampling plan necessary to check the production rate is N = 39388, and extracted from the (Hald) model is (2361.17), and this plan means that the examination of a random sample of (2361) is invalid. If the number of defective (damaged) units in the sample is equal to (17) champion or less all units are accepted, otherwise, the batch is rejected.

As for the total cost of quality control resulting from the sampling plan (2361.17), it will be extracted based on the smallest standard cost $R_0(N)$ achieved in the optimal sampling plan (2361.17), as

$$R_0 = (2n^* - \lambda_1^2 - \lambda_2)ds \quad ds = \frac{(k_s - k_m)}{(A_2 - R_2)} \quad ds = 0.000954$$

$$R_0 = [2(2361) - 148.2506 + 55.4084](0.00954)$$

$$R_0 = 4.4162$$

Therefore, the value of the expected total cost $k(p)$ of quality control is equal to:

$$k(p) = R_0(A_2 - R_2) + NK_m$$

$$= 4.4162(0.178) + 49388(0.001091)$$

$$= 42.9731 \ \$$$

Table 2 includes all the results of Bayesian plans to test the 1.5-L liquid Pepsi product extracted from the (Hald) model according to the previous distribution (Beta), classified according to quality levels X_p = 0.001(0.001)0.005 and batch sizes N = 10,000(1000)50,000.

Table 3: Bayesian plans to test the product according to the decision-making model and the (Hald) model

Hald model			Decision-making model			Batch size
Pn (*c*)	c	*n*	*Pn* (*c*)	c	*n*	
0.005998286	10	1162	0.00580307	6	669	10,000
0.00610687	11	1222	0.005768515	6	701	11,000
0.006049403	11	1278	0.005919349	7	733	12,000
0.005994006	11	1333	0.005886681	7	763	13,000
0.006107626	12	1385	0.005856515	7	791	14,000
0.006056638	12	1436	0.00600874	8	819	15,000
0.006008444	12	1485	0.005979344	8	846	16,000
0.005962933	12	1532	0.005951307	8	872	17,000
0.006079027	13	1578	0.005924596	8	897	18,000
0.006035578	13	1623	0.006076854	9	922	19,000
0.005993691	13	1667	0.006050899	9	946	20,000
0.006110937	14	1709	0.006026232	9	969	21,000
0.006070984	14	1751	0.006001765	9	992	22,000
0.006033416	14	1791	0.005978548	9	1014	23,000
0.00599631	14	1831	0.006130671	10	1036	24,000
0.006113404	15	1870	0.006107137	10	1058	25,000
0.006078104	15	1908	0.00608484	10	1079	26,000
0.006044122	15	1945	0.006063756	10	1099	27,000
0.006010518	15	1982	0.00621547	11	1119	28,000
0.00597818	15	2018	0.006194081	11	1139	29,000
0.006094842	16	2054	0.00617284	11	1159	30,000
0.006064192	16	2088	0.006152794	11	1178	31,000
0.006032961	16	2123	0.006132879	11	1197	32,000
0.006003807	16	2156	0.00611413	11	1215	33,000
0.006119773	17	2190	0.006264815	12	1233	34,000
0.006091371	17	2222	0.00624578	12	1251	35,000
0.006062356	17	2255	0.00622686	12	1269	36,000
0.006034483	17	2287	0.006208054	12	1287	37,000
0.006007724	17	2318	0.006190397	12	1304	38,000
0.006123612	18	2349	0.006339673	13	1321	39,000
0.006096696	18	2380	0.006321743	13	1338	40,000
0.006070874	18	2410	0.006303915	13	1355	41,000
0.006045269	18	2440	0.006287227	13	1371	42,000
0.006020722	18	2469	0.006270627	13	1387	43,000
0.005995538	18	2499	0.006254115	13	1403	44,000
0.006111111	19	2527	0.006401838	14	1419	45,000
0.006086596	19	2556	0.006385069	14	1435	46,000
0.006063111	19	2584	0.006369427	14	1450	47,000
0.006039808	19	2612	0.006352826	14	1466	48,000
0.006016683	19	2640	0.006337342	14	1481	49,000
0.00599455	19	2667	0.006321932	14	1496	50,000

Calculating the Value of the Defective Fraction in the Unexamined Quantities

According to the decision-making models and the (Hald) model, a group of Bayesian plans have been extracted to test the product. After this extraction, it is necessary to determine the expected value of the expected fraction of the defective

fraction in the unexamined quantities (N–n) which will be accepted based on the acceptance of the sample, and then, we will depend on the value of the average of the subsequent distribution for the defective lineage ($E(p/x)$) when ($X = c$), that is, ($Pn(c)$) and it has become clear to us that the subsequent distribution $f(p/x)$ is also a house with features ($\alpha+\beta+n, x+\alpha$), and therefore, it is

$$E(p \mid x) = P_n(x) = \frac{x+\alpha}{\alpha+\beta+n}$$

And when $X = c$ is

$$P_n(c) = \frac{c+\alpha}{\alpha+\beta+n}$$

The following Table 3 includes a comparison of BIS plans according to the (Hald) model and the decision-making model and at the level of quality ($X_p = 0.005349$) and the values of ($Pn(c)$) and for each of the plans of the decision-making model and the (Hald) model.

Table 3 shows that the expected value of the fraction of defective items in the accepted quantities (N–n) is 0.6% according to both the decision-making model and the (Hald) model. This value corresponds to the permissible percentage of defective items (LTPD) approved by Pepsi Company (ALA) for soft drinks. The correspondence of the average value of ($Pn(c)$) with the value of (LTPD) indicates the efficiency of the BIS plans, which take into account all available information about the quality when estimating the quality of subsequent production batches. This congruence underscores the importance and efficiency of the BIS plans. Moreover, the actual production quality level ($\underline{X}_p = 0.005349$) shows that the sample size for different batch sizes is smaller compared to other sampling inspection plans, which reduces examination costs and total costs.

CONCLUSION

1. The appropriate probability distribution to represent the defective percentages of the actual production is the beta-binomial distribution with a rate of (0.005349)
2. After applying the Bayes model in designing the sampling inspection plan, it was found that the parameters of this plan are (n = 1495) and (c = 14), and for the quality level of the actual production ($\underline{X}_p = 0.005349$), we find that the sample size for the different batch sizes is small compared to other sampling plans, which means reducing examination costs and therefore the total costs.

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