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The Impact of the Exchange Rate Dynamics on the Dependencies in Global Stock Market[†]

A b s t r a c t. The paper addresses the question of how the exchange rate dynamics affects the analysis of linkages between national stock markets. We consider two ways of tackling the problem. The first one consists in denominating the analyzed quotations in the same currency. The second deals with a direct introducing the exchange rate into a model. Our analysis is based on the daily return series on selected stock indices from the period 1995-2010. We model the dependence structure using dynamic copulas. This allows us to separate the dynamics of dependence from the volatility dynamics.

K e y w o r d s: stock market, stock index, linkages, denomination, exchange rate, copula.

Introduction

The knowledge about linkages between stock markets is of importance in risk management and building investment strategies. Moreover, it is crucial for understanding the nature of global financial market. It is thus quite natural that there exist many papers dealing with this problem. Most of them belong to the contagion literature. The most popular approach here is to denominate the indices (or other stock market quotations) in local currencies (Eun and Shin, 1989; Koutmos, 1992; Theodossiou and Lee, 1993; Wong et al. 2004). The next popular choice is denomination in the US dollar (e.g. Karolyi and Stulz, 1996; Rodriguez, 2007)). There exist analyses performed both in a local currency and the US dollar (e.g. Lee et al., 2001). Chen and Poon (2007) use local currency for indices in the case of developed markets and for emerging market they use US dollar denominated indices. Veiga and McAleer (2004) remarked that the

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use of the US dollar as a common currency is a complicating factor. This is because in such situation the US market is always included in the empirical analysis. Changes in the US dollar are largely influenced by changes in US fundamentals, which also drive financial returns. Thus, it is likely that some of the co-movements observed among returns in different markets expressed in a common currency are caused by changes in the fundamentals driving the US dollar exchange rate. However, the findings by Veiga and McAleer (2004) based on quite extensive analysis of the sensitivity of spillover effects on denomination show that the denomination has no significant impact on the results.

In the paper, we ask how introducing the exchange rate dynamics influences the dynamics of linkages between stock indices. We consider dependencies between the elements of each pair of indices from the triple: the S&P500, the DAX and the WIG20 (the main index of the Warsaw Stock Exchange). In addition, we investigate the linkages between the DAX and the NIKKEI225. The analysis of linkages is performed by means of a DCC-copula model. We estimate dynamic copula correlations between the daily returns on the indices denominated in local currencies and in chosen alternative currencies.

The aim of the presented investigation is to analyze the sensitivity of the dynamic copula correlation estimates to the denomination of the indices in alternative currencies. In the case of the S&P500 and the DAX, the considered currencies are the US dollar and the euro. The analysis for the S&P500 (or the DAX) and the WIG20 includes denomination in the US dollar, the euro and the Polish zloty. For the pair DAX and NIKKEI225, the denominations in the US dollar, the Japanese yen and the euro are included. Moreover, for each of the considered pairs of the indices we calculate the dynamic copula correlations based on a three-dimensional DCC-copula model estimated jointly for the indices denominated in local currencies and the corresponding exchange rate (USD/EUR for SP500-DAX, USD/PLN for SP500-WIG20, EUR/PLN for DAX-WIG20, and EUR/JPY for DAX-NIKKEI225).

1. DCC-Copula Models

Modeling the dependencies between financial returns is a difficult task because of special properties of these series. Typical return series usually exhibit conditional heteroskedasticity, different types of asymmetries and structural breaks which strongly influence estimation results for models of the dependence structure. Moreover, the dynamics of dependencies significantly changes in time. For example, it is well documented in many studies that dependence between returns on different assets is usually stronger in bear markets than in bull markets (Ang and Bekaert, 2002; Ang and Chen, 2002; Patton, 2004). This example of asymmetric dependence in financial markets is of great importance for portfolio choice and risk management. The main problem connected with this phenomenon is, however, that from the theoretical point of view the mentioned asymmetry cannot be produced by a statistical model for the returns that assumes an elliptical multivariate conditional distribution, and thus applying the linear correlation is not justified. An alternative concept that allows for modeling the dependence in a general situation is copula.

Roughly speaking, a d-dimensional copula is a mapping $C:[0,1]^d \to [0,1]$ from the unit hypercube into the unit interval which is a distribution function with standard uniform marginal distributions.

Assume that $X = (X_1, \ldots, X_d)$ is a d-dimensional random vector with joint distribution F and marginal distributions F_i, $i = 1, \ldots, d$. Then, by a theorem by Sklar (1959), F can be written as:

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)). \tag{1}$$

The function C is unique if F_i are continuous. Otherwise, C is uniquely given by:

$$C(u_1, \ldots, u_d) = F(F_1^{(-1)}(u_1), \ldots, F_d^{(-1)}(u_d)), \tag{2}$$

for $u_i \in [0,1]$, where $F_i^{-1}(u) = \inf\{x : F_i(x) \geq u_i\}$. In that case, C is called the copula of F or of X. Since the marginals and the dependence structure can be separated, it makes sense to interpret C as the dependence structure of the vector X. We refer to Patton (2009) and references therein for an overview of financial time series applications of copulas. There one can also find more information about advantages and limitations of copula-based modeling.

The simplest copula is defined by $C^{\Pi}(u_1, \ldots, u_d) = u_1 \cdot \ldots \cdot u_d$, and it corresponds to independence of marginal distributions. The next two important examples are $C^{+}(u_1, \ldots, u_d) = \min(u_1, \ldots, u_d)$, and, in the two-dimensional case, $C^{-}(u_i, u_j) = \max(u_i + u_j - 1, 0)$. The first corresponds to comonotonicity or perfect dependence (one variable can be transformed almost surely into another by means of an increasing map), and the second, to countermonotonicity or perfect negative dependence of the variables X_i and X_j (one variable can be transformed almost surely into another by means of a decreasing map). In the empirical part of this paper we will use the Student t copula. It is defined as follows:

$$C_{\nu,\mathbf{R}}^{St}(u_1, \ldots, u_d) = t_{\nu,\mathbf{R}}^{d}(t_{\nu}^{-1}(u_1), \ldots, t_{\nu}^{-1}(u_d)), \tag{3}$$

where $t_{\nu,\mathbf{R}}$ denotes the d-dimensional Student's t distribution with ν degrees of freedom and correlation matrix $\mathbf{R}$, and t_{ν} stands for 1-dimensional Student's t distribution with ν degrees of freedom. In the bivariate case we will use the notation $C_{\nu,\rho}^{t}$ where ρ stands for correlation coefficient.

The density associated to an absolutely continuous copula C is a function c defined by:

$$c(u_1,\ldots,u_d) = \frac{\partial^d C(u_1,\ldots,u_d)}{\partial u_1 \cdots \partial u_d}. \tag{4}$$

For an absolutely continuous random vector, the copula density c is related to its joint density function h by the following canonical representation:

$$f(x_1,\ldots x_d) = c(F_1(x_1),\ldots,F_d(x_d)) f_1(x_1)\cdots f_d(x_d), \tag{5}$$

where $F_1,\ldots,F_d$ are the marginal distributions, and $f_1,\ldots,f_d$ are the marginal density functions.

In the case of non-elliptical distributions, measures of dependence that are more appropriate than the linear correlation coefficient are provided by two important copula-based tools known as Kendall's tau and Spearman's rho (Embrechts et al., 2002). Since the dynamics of Kendall's tau can be easily derived for the results presented in this paper, we recall suitable definitions. If (X,Y) is a random vector and $(\tilde{X},\tilde{Y})$ is an independent copy of (X,Y) then Kendall's tau for (X,Y) is defined as:

$$\tau(X,Y) = P\{(X-\tilde{X})(Y-\tilde{Y}) > 0\} - P\{(X-\tilde{X})(Y-\tilde{Y}) < 0\}. \tag{6}$$

Thus Kendall's tau for (X,Y) is the probability of concordance minus the probability of discordance. If (X,Y) is a vector of continuous random variables with copula C, then:

$$\tau(X,Y) = 4\iint_{[0,1]^2} C(u,v)\mathrm{d}C(u,v) - 1. \tag{7}$$

For the Student t copula $C^t_{\nu,\rho}$, Kendall's tau equals $\frac{2}{\pi}\arcsin(\rho)$.

A very important concept connected with copula, relevant to dependence in extreme values, is tail dependence (Nelsen, 2006). If X and Y are random variables with distribution functions F and G then the coefficient of upper tail dependence is defined as follows:

$$\lambda_U = \lim_{q\to 1^-} P(Y > G^{-1}(q) \mid X > F^{-1}(q)), \tag{8}$$

provided a limit $\lambda_U \in [0,1]$ exists. Analogously, the coefficient of lower tail dependence is defined as:

$$\lambda_L = \lim_{q\to 0^+} P(Y \le G^{-1}(q) \mid X \le F^{-1}(q)), \tag{9}$$

provided that a limit $\lambda_L \in [0,1]$ exists. If $\lambda_U \in (0,1]$ ($\lambda_L \in (0,1]$), then X and Y are said to exhibit upper (lower) tail dependence. Upper (lower) tail dependence quantifies the likelihood to observe a large (low) value of Y given a large (low) value of X. The coefficients of tail dependence depend only on the copula C of X and Y:

$$\lambda_L = \lim_{q \to 0^+} \frac{C(q,q)}{q}, \ \lambda_U = \lim_{q \to 0^+} \frac{\hat{C}(q,q)}{q} \tag{10}$$

where $\hat{C}(u,v) = u + v - 1 + C(1-u, 1-v)$. For the Student t copula $C^{St}_{\nu,\rho}$, the coefficients of upper and lower dependence are both equal to $2t_{\nu+1}\left(-\sqrt{(\nu+1)(1-\rho)/(1+\rho)}\right)$ (see McNeil et al., 2005).

Introduced by Patton (2004), the notion of conditional copula allows to apply copulas to modeling the joint distribution of $\mathbf{r}_t$ conditional on information set Ω_{t-1}, where $\mathbf{r}_t = (r_{1,t}, \ldots, r_{d,t})'$ is a d-dimensional vector of financial returns. In this paper we consider the following general conditional copula model:

$$r_{1,t} \mid \Omega_{t-1} \sim F_{1,t}(\cdot \mid \Omega_{t-1}), \ldots, r_{d,t} \mid \Omega_{t-1} \sim F_{d,t}(\cdot \mid \Omega_{t-1}), \tag{11}$$

$$\mathbf{r}_t \mid \Omega_{t-1} \sim F_t(\cdot \mid \Omega_{t-1}), \tag{12}$$

$$F_t(r_t \mid \Omega_{t-1}) = C_t(F_{1,t}(r_{1,t} \mid \Omega_{t-1}), \ldots, F_{d,t}(r_{d,t} \mid \Omega_{t-1}) \mid \Omega_{t-1}), \tag{13}$$

where the set Ω_t includes the up to time t information on the returns on both considered financial assets, and C_t is the conditional copula linking the marginal conditional distributions. Further, we assume that:

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \mathbf{y}_t, \ \boldsymbol{\mu}_t = E(\mathbf{r}_t \mid \Omega_{t-1}), \tag{14}$$

$$y_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \ \sigma^2_{i,t} = \mathrm{var}(r_{i,t} \mid \Omega_{t-1}), \tag{15}$$

$$\varepsilon_{i,t} \sim iid \ Skew\_t(0,1,\xi_i,\eta_i), \tag{16}$$

where $Skew\_t(0,1,\xi,\eta)$ denotes the standardized skewed Student t distribution with $\eta > 2$ degrees of freedom, and skewness coefficient $\xi > 0$ (Lambert and Laurent 2001). To the marginal return series $r_{i,t}$, $i = 1, \ldots, d$, we fit ARMA-GARCH models with skewed Student's t distributions for the 1-dimensional innovations.

When modeling the joint conditional distribution, the evolution of the conditional copula C_t has to be specified. Usually (Patton, 2004, 2006), the functional form of the conditional copula is fixed, but its parameters evolve through time. In this paper, we follow that approach and apply the DCC model proposed by Engle (2002), extended to Student's t copulas. Thus in our DCC-t-copula model we assume that the conditional copula C_t is a Student t Copula $C^t_{\nu,\mathbf{R}_t}$ such that:

$$\mathbf{R}_t = (\mathrm{diag}(\mathbf{Q}_t))^{-1/2}\mathbf{Q}_t(\mathrm{diag}(\mathbf{Q}_t))^{-1/2}, \tag{17}$$

$$\mathbf{Q}_t = (1-\alpha-\beta)\overline{\mathbf{Q}} + \alpha\, \tilde{\mathbf{u}}_{t-1}\tilde{\mathbf{u}}'_{t-1} + \beta\, \mathbf{Q}_{t-1}, \tag{18}$$

where $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$, $\tilde{u}_{i,t} = t_\nu^{-1}(u_{i,t})$, $u_{i,t} = F_{i,t}(r_{i,t})$, $i = 1, \ldots, d$, and $\overline{\mathbf{Q}}$ is the unconditional covariance matrix of $\tilde{\mathbf{u}}_t$.

2. The Data

In the paper we present results of analysis concerning the dependencies between the daily returns for four pairs of indices: S&P500-DAX, S&P500-WIG20, DAX-WIG20, and DAX-NIKKEI225. We have chosen three indices representing stock markets of main economies from different parts of the world. The reason for the choice of the WIG20 index is connected with the fact that the Polish stock market is significantly influenced by the US financial markets and, on the other hand, there exist strong linkages between the Polish and German economies.

As it was mentioned in Introduction, the very common approach in stock market linkages analysis is to investigate the indices of developed markets in local currency, and those from emerging markets – denominated in an alternative currency (mostly in the US dollar). We denominate the considered indices in their local currencies, in the USD, and in the euro. Moreover, the dependencies involving the WIG20 are analyzed for the indices denominated in the Polish zloty, and those involving the NIKKEI225 – in the Japanese yen. Thus our dataset contains the quotations of the considered indices and the exchange rates EUR/USD, USD/PLN, EUR/PLN USD/JPY and EUR/JPY. The quotation series were obtained from the service Stooq. The period under scrutiny is from January 3, 1995 to December 11, 2009.

Table 1. Descriptive statistics of the analyzed return series

Index	Mean	Maximum	Minimum	Stand. Dev.	Skewness	Kurtosis
S&P500	0.0238	10.957	-9.4695	1.289	-0.1783	10.9195
S&P500 in EUR	0.0190	9.5946	-8.7688	1.4663	-0.2152	7.0311
S&P500 in PLN	0.0282	10.054	-9.1886	1.4320	-0.0718	8.1544
DAX	0.0276	10.797	-9.791	1.5877	-0.0635	10.9230
DAX in USD	0.0325	13.5020	-9,4710	1.6741	0.0517	8.5868
DAX in JPY						
DAX in PLN						
WIG20	0.0302	13.709	-14.161	1.9544	-0.1548	6.7066
WIG20 in USD	0.0262	14.995	-19.463	2.2717	-0.2531	8.2124
WIG20 in EUR	0.0212	16.368	-17.481	2.3220	-0.1743	8.6857

Since the patterns of non-trading days in national stock markets differ, for the purpose of modeling dependencies the dates of observations for each pair of indices were checked and observations not corresponding to ones in the other index quotation series were removed. The time series under scrutiny are percentage logarithmic daily returns calculated by the formula:

$$r_t = 100(\ln P_t - \ln P_t)\,, \tag{19}$$

where P_t denotes the closing index value on day t.

The descriptive statistics of the analyzed return series are presented in Table 1. In Tables 2–5 we show in-sample estimates of the unconditional correlations.

Table 2. S&P500 and DAX. Estimates of the unconditional correlation of the returns

	S&P500	S&P500 in EUR
DAX	0.5590	0.5309
DAX in USD	0.5227	-----

Table 3. S&P500 and WIG20. Estimates of the unconditional correlation of the returns

	S&P500	S&P500 in EUR	S&P500 in PLN
WIG20	0.2682	-----	0.1212
WIG20 in USD	0.2824	-----	
WIG20 in EUR	-----	0.2749	

Table 4. S&P500 and WIG20. Estimates of the unconditional correlation of the returns

	DAX	DAX in USD	DAX in PLN
WIG20	0.4494	-----	0.3598
WIG20 in USD	-----	0.5042	
WIG20 in EUR	0.4887		

Table 5. DAX and NIKKEI225. Estimates of the unconditional correlation of the returns

	NIKKEI	NIKKEI in USD	NIKKEI in EUR
DAX	0.2993	-----	0.2225
DAX in USD	-----	0.2616	
DAX in JPY	0,2853		

3. Empirical Analysis of the Stock Market Linkages

The course of presented analysis is as follows. We investigate the dependencies between the returns for four pairs of indices: S&P500-DAX, S&P500-WIG20, DAX-WIG20, DAX-NIKKEI225. In each case we estimate the dynamic copula correlations by means of the DCC-*t*-copula model described in section 3. Each pair of indices is considered in a local currency, in the US dollar, and in the euro. For S&P500-WIG20 and DAX-WIG20 we additionally take into account denomination in the Polish zloty, and for DAX-NIKKEI225 – in Japanese yen. Moreover, we estimate jointly the dynamic copula correlations for triples of returns: S&P500-DAX-EUR/USD, S&P500-WIG20-USD/PLN, DAX-WIG20-EUR/PLN and DAX-NIKKEI225-EUR/JPY.

The advantage of copula models we apply here is that they allow to separate the dependence dynamics from the volatility dynamics. The feedback between these two features causes many problems in traditional analyses based on multivariate volatility models. In our approach the volatility dynamics is captured by means of GARCH models and then the dependence structure is modeled. It means that the DCC-*t*-copula models are estimated using a two-step maximum likelihood approach. The first step includes fitting a GARCH model to each return series (Laurent, 2009). The types of fitted models differ depending on currency used to denominate an index (Table 4–5). Next, the GARCH standardized residuals are transformed by means of their theoretical cumulative distribution functions to obtain the series of data uniformly distributed on [0,1]. In the second step the DCC-*t*-copula models are fitted to the transformed series. Thus, we follow the method of inference functions for margins (Joe and Xu, 1996).

The first observation coming from Tables 6–7 is that the conditional mean an volatility dynamics is sensitive to denomination. The return series under scrutiny are long and include some crisis periods so the fitted GARCH models are mostly asymmetric and with a skewed Student *t* as an error distribution.

The DCC-*t*-copula parameter estimates are presented in Tables 8–11. The results indicate that the dynamics of dependencies shows a high level of persistence in each considered case.

Table 6. S&P500 and NIKKEI225. Types of fitted ARMA-GARCH models

Return series	S&P500	S&P500 in EUR	S&P500 in PLN	NIKKEI225	NIKKEI225 in USD	NIKKEI225 in EUR
ARMA	(1,1)	(0,2)	(1,1)	(0,0)	(0,1)	(0,0)
GARCH	GJR-GARCH(1,2)	GJR-GARCH(1,2)	GARCH(1,1)	GJR(1,2)	GJR(1,2)	FIGARCH(1,1)
Error distribution	Skewed Student	Skewed Student	Skewed Student	Skewed Student	Skewed Student	Skewed Student

Table 7. DAX and WIG20. Types of fitted ARMA-GARCH models

Return series	DAX	DAX in USD	WIG20	WIG20 in USD	WIG20 in EUR
ARMA	(2,2)	(1,0)	(0,1)	(2,0)	(0,0)
GARCH	FIAPARCH(1,1)	GARCH(1,1)	FIAPARCH(1,1)	GJR-GARCH(1,2)	FIGARCH(1,1)
Error distribution	Skewed Student	Skewed Student	Student	Student	Student

Table 8. S&P500-DAX. Parameter estimates for the fitted DCC-*t*-copula model (standard errors in parentheses)

	S&P500 and DAX			
Parameter	in local currencies	in EUR	in USD	S&P500-DAX -USD/EUR
α	0.0146 (0.003	0.0204 (0.006)	0.0185 (0.003)	0.0175 (0.003)
β	0.9841 (0.004)	0.9750 (0.008)	0.9805 (0.004)	0.9789 (0.004)
ν	14.4538 (3.819)	13.1894 (3.582)	15.5688 (4.665)	12.5504 (1.843)

Table 9. S&P500-WIG20. Parameter estimates for the fitted DCC-*t*-copula model (standard errors in parentheses)

	S&P500 and WIG20				
Parameter	in local currencies	in EUR	in USD	in PLN	S&P500-WIG20 -USD/PLN
α	0.0075 (0.008)	0.0109 (0.004)	0.0102 (0.004)	0.0072 (0.002)	0.0125 (0.003)
β	0.9900 (0.016)	0.9836 (0.007)	0.9878 (0.005)	0.9871 (0.005)	0.9821 (0.005)
ν	14.6137 (3.710)	16.1879 (4.616)	14.9618 (3.911)	11.3950 (2.286)	16.6111 (3.004)

Table 10. DAX-WIG20. Parameter estimates for the fitted DCC-*t*-copula model (standard errors in parentheses)

	DAX and WIG20				
Parameter	in local currencies	in EUR	in USD	in PLN	DAX-WIG20 -EUR/PLN
α	0.0125 (0.005)	0.0181 (0.008)	0.0141 (0.003)	0.0109 (0.003)	0.0110 (0.002)
β	0.9875 (0.007)	0.9793 (0.011)	0.9852 (0.004)	0.9865 (0.003)	0.9863 (0.004)
ν	12.017 (2.468)	12.154 (2.555)	14.516 (3.640)	14.372 (3.435)	17.213 (3.199)

Table 11. DAX-NIKKEI225. Parameter estimates for the fitted DCC-*t*-copula model (standard errors in parentheses)

	DAX and NIKKEI225				
Parameter	in local currencies	in EUR	in USD	in JPY	DAX-NIKKEI -EUR/JPY
α	0.0150 (0.018)	0.0091 (0.004)	0.0080 (0.003)	0.0066 (0.003)	0.0114 (0.002)
β	0.9339 (0.142)	0.9889 (0.006)	0.9900 (0.004)	0.9864 (0.006)	0.9861 (0.003)
ν	15.029 (4.870)	21.162 (7.524)	23.087 (7.556)	17.4376 (6.412)	13.248 (2.118)

Figure 1 shows a comparison of the dynamic copula correlations for the pair S&P500-DAX obtained in all the considered cases. The dynamics of the correlations is quite strong. The strongest dependencies are observed in the years 2001-2004 and 2008-2009. The values of the correlations calculated for the indices denominated in local currencies, in the euro, and modeled jointly with the exchange rate EUR/USD are quite close each other. Only in the case of denomination in the US dollar the correlation estimates are clearly lower. The mean levels of the estimated dynamic copula correlations (Table 12) are significantly different and the highest mean is obtained in the case of the dependencies between the indices and the exchange rate EUR/PLN modeled jointly. The null hypothesis about equality of the means was tested using the Model Confidence

Set (MCS) procedure (Hansen et al., 2003, 2011; Hansen and Lunde, 2007) applied to the set of the dynamic copula correlations series.

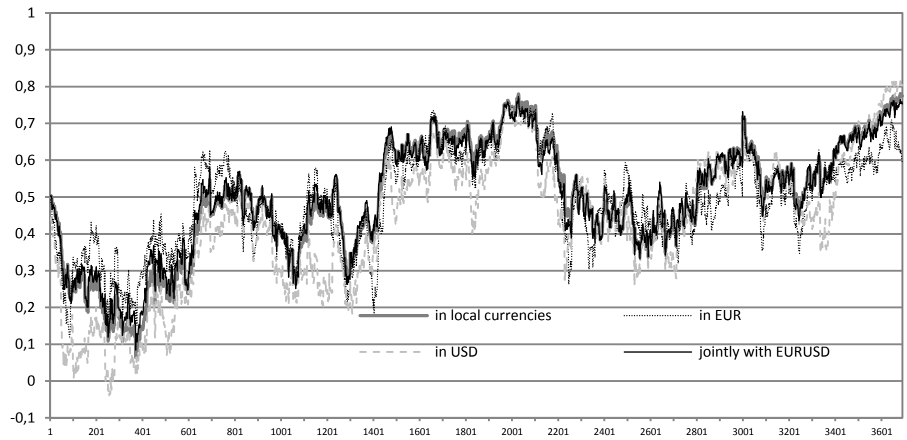

Figure 1. S&P500 and DAX. Dynamic copula correlations from DCC-*t*-copula model

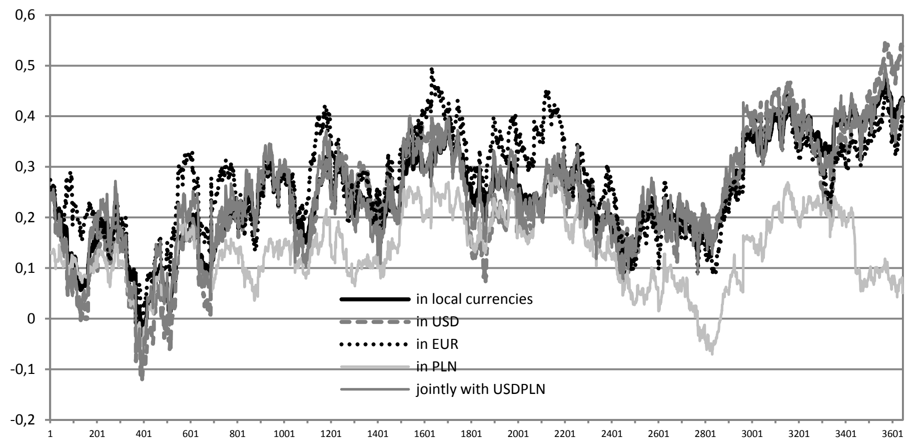

Figure 2. S&P500 and WIG20. Dynamic copula correlations from DCC-*t*-copula model

The estimates of dynamic copula correlations obtained for the pair S&P500-WIG20 are much lower but show similar pattern as in the previous case – the dynamics of the conditional copula correlations is strong but it does not depend significantly on the choice of currency. The only exception concerns the clearly weaker dependencies in the case of the indices denominated in the Polish zloty. The difference is more visible after Poland joining the EU. The testing procedure, the same as in the previous considered case, indicate that the mean levels of the estimated dynamic copula correlations (Table 12) are significantly different.

Table 12. Means of the dynamic copula correlation estimates for the pairs S&P500-DAX and S&P500-WIG20

S&P500 and	DAX	WIG20
in local currencies	0.4975	0.2434
in USD	0.4339	0.2345
in EUR	0.4903	0.2711
modeled jointly with the exchange rate	0.4992	0.2455
in PLN		0.1325

Table 13. Means of the dynamic copula correlation estimates for the pairs DAX-WIG20 and DAX-NIKKEI225

DAX and	WIG20	NIKKEI225
in local currencies	0.4185	0.2957
in USD	0.4432	0.2913
in EUR	0.4436	0.2601
modeled jointly with the exchange rate	0.4195	0.3015
in PLN	0.3386	------
In JPY	------	0.2571

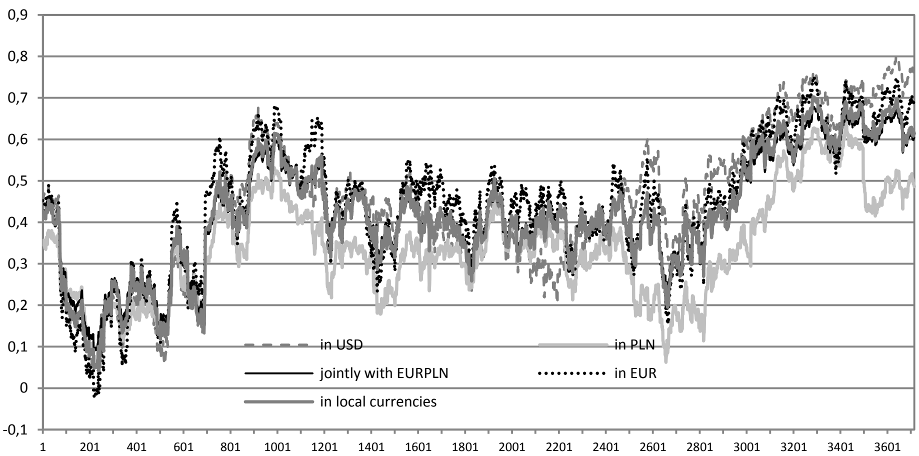

Figure 3. DAX and WIG20. Dynamic copula correlations from DCC-*t*-copula models

The estimates of dynamic copula correlations obtained for the pair DAX-WIG20 are presented in Figure 3. In general, the differences between the estimates are not very high and the dynamics in all cases is similar. However, once again, we can observe the impact of Poland's EU joining on the conditional correlations calculated for the indices denominated in the Polish zloty. Mean levels of the conditional correlation estimates are presented in Table 13. The testing MCS procedure indicates that the mean levels of the conditional correlations are statistically undistinguishable in the case of denomination in the EUR and in the USD and this mean level is the highest one.

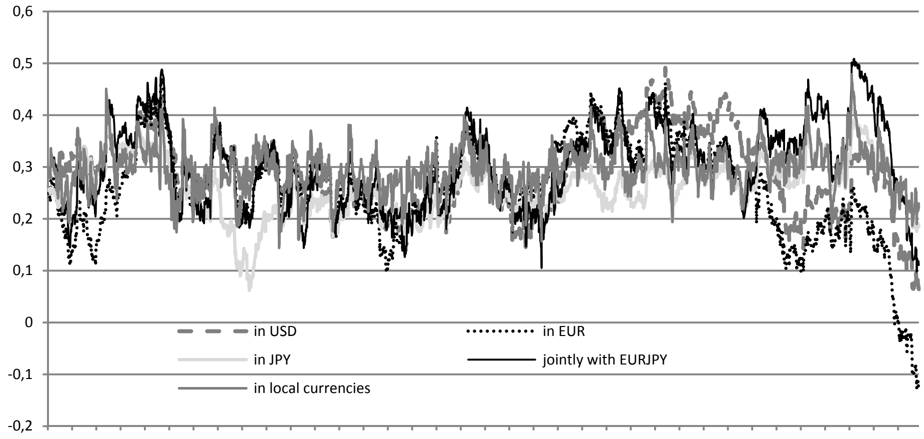

Figure 4. DAX and NIKKEI225. Dynamic copula correlations from DCC-*t*-copula models

The dynamics of the conditional correlations between the returns on the DAX and the NIKKEI 225 shows slightly different patterns (Figure 4). Denominating the indices in the yen results in the lowest values of the correlation estimates. The most interesting thing one can see in Figure 4 is that from the beginning of the financial crisis 2007-2009 the plots of the conditional correlations estimates start to disperse. Starting from that point, the dependencies measured for the indices denominated in the EUR are the weakest.

Conclusions

The aim of the presented research was to examine how the dynamics of linkages between stock markets changes when the exchange rate dynamics is introduced into the model. We considered dependencies between the S&P500 index and two European indices – the DAX and the WIG20, and for the pairs DAX-WIG20 and DAX-NIKKEI225. To analyze the stock indices linkages we used DCC-*t*-copula models. The advantage of the applied approach is that it allows to separate the dynamics of linkages from the volatility dynamics.

The presented results are slightly ambiguous but generally show that the impact of denomination or introducing the exchange rate directly into the model for dependencies is rather weak. However, as it was expected, some significant changes in the dynamics of the conditional dependence are observed for indices denominated in a common currency when events strongly influencing the considered exchange rates dynamics are present.

The question about the proper way of analyzing the dependencies remains still open. The problem seems to be less important in the case of indices denominated in major currencies, i.e. the USD or the EUR, and much more significant in the case of indices expressed in other (local) currencies.

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Wpływ dynamiki kursów walutowych na zależności na globalnym rynku akcji

Z a r y s t r e ś c i. Analiza powiązań pomiędzy narodowymi rynkami akcji jest zwykle oparta na modelach opisujących zależności pomiędzy stopami zwrotu z akcji lub indeksów. Przy tym w niektórych badaniach wykorzystuje się notowania w walutach lokalnych, a w innych – notowania denominowane w tej samej walucie (zwykle w dolarze amerykańskim). W artykule zajmujemy się badaniem, jak uwzględnienie dynamiki kursów walutowych w modelu powiązań dla giełdowych stóp zwrotu wpływa na opis zależności. Stosujemy i porównujemy dwa podejścia. Pierwsze polega na denominowaniu rozważanych notowań w tej samej walucie, a drugie sprowadza się do bezpośredniego wprowadzenia kursu walutowego do modelu struktury zależności. Prezentowana analiza jest oparta na szeregach stóp zwrotu z okresu 1995-2010. W celu opisu struktury zależności stosujemy dynamiczne modele kopuli. Podejście takie pozwala nam na oddzielenie dynamiki zależności od dynamiki zmienności notowań.

S ł o w a k l u c z o w e: rynek akcji, indeks giełdowy, powiązania, denominacja, kursy walutowe, kopula.