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Forecasting EUR/PLN Exchange Rate: the Role of Purchasing Power Parity Hypothesis in ESTVEC Models[**]

A b s t r a c t. The purpose of this paper is to verify empirical consequences of imposing various forms of purchasing power parity (PPP) within a class of smooth transition vector error correction models (ESTVECM) for analysis of EUR/PLN exchange rage. Empirical importance of exponential smooth transition functions is confronted with the linear error-correction mechanism. A class of competing models for recursive samples are compared by the likelihood ratio test, information criteria, and out of sample forecast accuracy measures.

K e y w o r d s: PPP; ESTVECM; cointegration; exchange rate forecasting

J E L Classification: C32; F31; F37

1. Introduction

Purchasing power parity (PPP) is one of the oldest theories regarding exchange rate. Its empirical importance has been investigated intensively for decades. Rogoff (1996) and Officer (1982) point out that PPP theory had

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been initially articulated by the scholars of Salamanca school in Spain in the sixteenth century. Modern analyses and formulations of PPP theory could be dated on early 20's of the 20th century (e.g. Cassel, 1918). Nowadays PPP constitutes theoretical foundations of many long-run exchange rate models, as e.g. sticky-prices monetary models (Dornbusch, 1976), flexible-price monetary model (e.g. Johnson, 1978) or asset pricing models (Lucas, 1982). Consequently, the PPP hypothesis is also embedded in larger macroeconomic models, as e.g. in DSGE models for small open economy or two economies (e.g. Ca'Zorzi et al., 2017; Senbeta, 2011). Furthermore, PPP could be useful in forecasting exchange rate, in particular in the long-run (e.g. Ca'Zorzi et al., 2016).

Despite of wide applications of PPP theory, empirical evidence is far from being conclusive (e.g. Arize et al., 2015; Kelm, 2013, p. 68). While most of the existing literature do not confirm or even reject PPP hypothesis in the short-run (Arize et al., 2015), for the long-run results may vary. For example Chang et al. (2010), Wang (2000) or Pappel (1997) reject long-run PPP, while Arize et al. (2004; 2015), Cheung et al. (2004), Lothian & Taylor (1996) found support for PPP in terms of cointegrating relationships or real exchange rate for most of their samples. However, conclusion from most of the existing research should be treated with caution due to its methodological drawbacks (for discussion see e.g., Kelm, 2013, pp. 58–67). Among PPP studies, positive results are reported quite frequently, when the smooth transition (STR) approach is utilized, either to test stationarity of the real exchange rate (e.g. Kapetanios et al., 2003; Sollis, 2009; McMillan, 2009; Kelm, 2013) or to verify cointegration relationship (e.g. Gefang, 2008).

The aim of this study is to utilize STR cointegration framework to investigate PPP hypothesis. The research is conducted for EUR/PLN exchange rate as an interesting example of emerging market currency. The methodology of this research is motivated by the work of Gefang (2008), though the paper includes a number of extensions. Firstly, we allow for STR mechanism separately in selected components of the VECM model. This gives opportunity to find optimal specification, preferably linking parsimony of parametrization and good explanatory power. Furthermore, in this research strong-form PPP is considered, while Gefang (2008) investigated only weak-form PPP. Moreover, we conduct in-sample and out-of-sample analysis.

The research sample covers the period between January 1999 and February 2016, while out of sample forecasts are tested until February 2017. On the one hand, it takes into account only period with one exchange rate regime, while on the other hand it has heterogeneous features as in this period

Poland accessed to European Union, and global financial crisis as well as Euro Area sovereign debt crisis took place.

The article is organized as follows. Firstly, we provide a brief summary of PPP theory, methods of its verification and existing empirical literature. In section 3 we introduce the models used here, while section 4 contains empirical comparison of different models in terms of model fit and out of sample forecast performance.

2. Purchasing Power Parity – General Concept and Empirical Importance

The PPP theory in all variants is rooted in the "*law of the one price*" (LOP). It states that for any good *i:*

$$P_{it} = S_t^{eq} P_{it}^* \tag{1}$$

where P_{it} is the domestic-currency price of good *i* at time *t*, P_{it}^* is the foreign currency price of good *i* at time *t* and S^{eq} is the equilibrium exchange rate at time *t*, defined as home price of foreign currency. The relationship (1) could be expressed also for price indices:

$$P_t = \sum_{i=1}^{n} \omega_i P_{it} = S_t^{eq} \sum_{i=1}^{n} \omega_i P_{it}^* = S_t^{eq} P_t^*, \tag{2}$$

where ω_i is the weight of price of good *i* in domestic and foreign price indices and *n* denotes the number of goods in domestic and foreign price indices. Deviations from LOP for one item does not mean deviation from LOP for the whole price index, as aggregations and weighing may compensate impact of this breaches; see Wdowiński (2010). Thus the *strong* (or *strict*) form of *PPP*[1] states:

$$S_t^{eq} = P_t / P_t^* \tag{3}$$

or, in the log-linear form:

$$\ln\left(S_t^{eq}\right) = \ln(P_t) - \ln\,(P_{it}^*). \tag{4}$$

Hence, the real exchange rate *Q* can be written as:

$$\ln(Q_t) = \ln\left(S_t^{eq}\right) - \ln(P_t) + \ln(P_{it}^*) = \bar{\bar{q}}, \tag{5}$$

where $\bar{\bar{q}}$ is a constant value.

Empirical testing of existence of the strict PPP and law of the one price is based on assumptions that market works perfectly and any deviations from

[1] Alternatively, PPP models could be distinguished between satisfying and non-satisfying the long-term homogeneity restriction.

the aforementioned relationships (4) or (5) are not persistent. Thus two empirical strategies could be applied. In the first one, testing of unit root for the logarithm of the exchange rate is utilized. Stationarity of the real exchange rate is interpreted as empirical support of existence of the strong-form of PPP. The second approach bases on estimation of the following long run equation:

$$\ln\left(S_t^{eq}\right) = \delta_1 \ln\,(P_t) + \delta_2 ln(P_t^*) + u_t, \tag{6}$$

where u_t is a stationary error term and restriction $\delta_1 = -\delta_2 = -1$ is subject to analysis.

Economic assumptions that guarantee the existence of the strong-form of PPP are rather restrictive. For instance, on has to assume costless spatial arbitrage (e.g. no quotas, import tariffs, transportation costs), no measurement errors, or information costs (Arize et al., 2015), which implicitly is equivalent to existence of rational expectations. Further conditions mentioned in the literature are: non-existence of *pricing-to market* (PTM) strategies, lack of nominal rigidities and lack of entry-barriers to international markets (Kelm, 2013, p. 28). Consequently, three general approaches have been developed in the literature to relax the assumption.

In the first approach, the restriction: $\delta_1 = -\delta_2 = -1$ in (6) is relaxed and only the symmetry restriction $-\delta_1 = \delta_2$ is analysed. It reflects presence of transportation costs, other trade barriers, measurement problems (e.g. Taylor, 1988; Arize et al., 2015).

Second approach is connected with conclusion from Dumas (1992), Uppal (1993), Sercu et al. (1995), O'Connel & Wei (1997) and more recently Pavlidis et al. (2011) emphasizes the role of transaction costs:

$$\ln\,(S_t^{eq}) = \kappa + \ln(P_t) - \ln\,(P_t^*). \tag{7}$$

Therefore two regimes can be identified. The first (so-called *inner regime*) is defined by condition $\ln(Q)\,\epsilon\,(-\kappa_i\,, \kappa_j)$, while the second regime (called *outer regime*) is obtained in the case $\ln(Q)\epsilon(-\infty, -\kappa_i) \cup (\kappa_j, +\infty)$. Within the *inner regime* the real exchange rate can be described as $I(1)$ process. Within the *outer regime* real exchange rate should return toward *inner regime*. Empirical testing should take into accounts existence of three regimes: the inner regime and two – positive and negative outer regimes. Consequently different variants of the threshold autoregressive (TAR) or smooth transition autoregressive (STAR) models are applied as a natural generalization of the linear scheme. Similarly as in more restrictive cases above, two strategies utilizing TAR or STAR approach has been developed. The first and the most popular one, allows for testing the unit root hypothesis of logarithm of the exchange rate against TAR/STAR alternatives (e.g. Kapetanios et al., 2003; Sollis, 2009; McMillan, 2009; Månsson & Sjölander 2014). The second approach, which is also applied in this study, is connected with verification of the PPP hypothesis in the multivariate framework, within Threshold or Smooth Transition Vector Error Correction frameworks (TVECM/STVECM), both *strong-form* PPP (e.g. Wu & Chen, 2008; Nakagawa, 2010), as well as *weak-form* PPP (e.g. Gefang, 2008).

Third approach is connected with a reformulation of (6) resulting in:

$$\mathrm{Ln}\left(S_t^{eq}\right) = \delta_0 + \delta_1 \ln\,(P_t) + \delta_2 \ln(P_t^*) + \varphi'_{1(k)} l_{(k)t} + \\ + \varphi'_{2(k)} m_{(k)t} + \varphi'_{3(k)} s_{(k)t} \; + u_t \qquad (8)$$

where $l_{(k)t}$, $m_{(k)t}$, $s_{(k)t}$ – are vectors of long, medium and short run determinants of nominal exchange rate (except PPP), $\varphi_{i(k)} = [\varphi_{i1}, \dots, \varphi_{iK}]'$ – vector of equilibrium coefficients, $i = 1,2,3$, $k = 1, \dots, K$ (see Kelm & Bęza-Bojanowska, 2005; Kelm, 2010). Restriction $\varphi_{1(k)} = \varphi_{2(k)} = \varphi_{3(k)} = 0$ supports PPP hypothesis; see Kelm (2013, p. 27).

The empirical literature where PPP hypothesis is analysed is vast. In this paper only key papers are mentioned. Firstly the enormous short-term volatility of the real exchange rate with the extremely slow rate at which shocks appear to damp out still misses satisfactory and sound explanation (PPP puzzle, see Rogoff,1996) and that the nominal rigidities and market frictions are still unable to explain why real exchange rates (RER) deviate from the PPP level and high estimates of RERs' half-lives (3–5 years), see Kelm (2017). Secondly, allowing for smooth transition mechanism, seems to solve at least partially "PPP puzzle" (e.g. Schnatz, 2007, Norman, 2010). On the other hand, both theoretical and empirical soundness of testing procedures in many cases could be doubtful (e.g. Kelm, 2013, p. 90–92). For example Månsson & Sjölander, (2014), and Emirmahmutoglu & Omay, (2014) emphasized weak power of KSS (Kapetanios, Shin & Snell) and AKSS (augmented KSS) tests. Thirdly, verification of the PPP within the cointegration framework might be difficult if the number of cointegrating vectors differs from one.

Empirical results of PPP testing for the Polish Zloty depend on the sample used in the research and on the choice of the price indices. In particular if the sample starts before 1999, the *strong-form* PPP is not supported, due to characteristics of the exchange rate regimes in Poland before 1999 (as de facto Polish Zloty became free-floating). More specifically, both fixed exchange rate regime (before October 1991) and crawling peg (before May 1995) represented the nominal anchor feature (Kokoszczyński, 2001). In terms of price indices, one should note that production price index (PPI) in manufacturing seems to be the best proxy for *tradables* prices (see Kelm, 2013, p. 141).

Hence in prevailing part of the literature, *strong-form* PPP for the Polish Zloty is not supported, since the research sample contains data from mid 1990s (e.g. Rubaszek & Serwa; 2009; Wdowiński 2010; Chang & Tzeng, 2011) or even include the whole transition period (Arize et al., 2015). Furthermore, in many studies (Arize et al., 2015), at least for some specifications (Rubaszek & Serwa, 2009; Wdowiński; 2010) several forms of *weak-form* PPP (as proportionality restrictions) were rejected. Recently for samples including only free floating exchange rate regime period the *weak-form* PPP is confirmed or not rejected quite frequently. Surprisingly, the *strong-form* PPP is supported only exceptionally. Kelm (2013, p.188) does not reject symmetry restriction in one of specification of VECM with $I(2)$ variables and rejects null hypothesis about unit rot against ESTAR process for the real exchange rate. However both cases are controversial – as in VECM with $I(2)$ variables p-value for symmetry restriction is only 0.118 and interpretation of model is not straightforward as in VECM with only $I(1)$ variables. Furthermore, results of unit root test against ESTAR are driven by abnormal observations, mainly from the year 2008 (Kelm, 2013, p.172–173).

3. Econometric Framework Utilizing Nonlinear Cointegration

The econometric framework applied here is designed to fulfill assumptions of existence of transaction costs. In the multivariate framework, Threshold VECM (TVECM) or Exponentially Smooth Transition VECM (ESTVECM) are relevant. In these cases the dynamics of the adjustment changes across regimes (*inner* and *outer*), while in the simple linear VECM adjustment is described by a linear function of the magnitude of the deviations from the long run equilibrium. The driving forces of the regime changes are governed by the observed deviations from the equilibrium through the transition function. In a TVECM, the regime changes are assumed to be discrete, whereas in an ESTVECM, the regimes change smoothly.

The final ESTVECM specification used in the research is consistent with Gefang (2008), with some modifications – allowing for testing the strong-form PPP and more flexible specification.

Let $Y_t = [\ln(S_t), \ln(P_t), \ln(P_t^*)]$, where S, P, and P^* are respectively EUR/PLN exchange rate, domestic price index and foreign price index. Assuming that cointegration relationship is common among regimes, the EST-VECM is described for $t = 1, \dots, T$ as follows:

$$\Delta \mathrm{Y_t} = \mathrm{Y_{t-1}}\beta\alpha + \mathrm{D_t}\xi + \sum_{h=1}^{p} \Delta \mathrm{Y_{t-h}}\Gamma_h + \mathrm{F(z_t)}(\mathrm{Y_{t-1}}\beta\alpha^Z + \\ + \mathrm{Dt}\xi\mathrm{Z} + \mathrm{h}{=}1\mathrm{p}\Delta \mathrm{Yt}{-}\mathrm{h}\Gamma\mathrm{h\ z}) + \varepsilon\mathrm{t} \quad (9)$$

where $\Delta\mathrm{Y_t} = \mathrm{Y_t} - \mathrm{Y_{t-1}}$, the error time ε_t is Gaussian white noise process with $E(\varepsilon_t) = 0$ and:

$$E(\varepsilon_t, \varepsilon_s) = \begin{cases} \Sigma \text{ for } s = t \\ \mathbf{0} \text{ for } s \neq t \end{cases}.$$

Finally, the deterministic term D_t contains intercept only. The dimensions of Γ_h and Γ_h^z are $[3 \times 3]$, the dimension of β is $[3 \times r]$,while the dimensions of α and α^Z are $[1 \times r]$, with r denoting the cointegration rank. In this research it is assumed that $r = 1$. If strong-form PPP holds, we have: $\beta' = [1 \quad -1 \quad 1]'$.

In (9) changes of regimes are driven by past deviations from the equilibrium relationships and dynamics of changes of regimes are captured by the exponential smooth transition function proposed by Teräsvirta (1994):

$$\mathrm{F(z_t)} = 1 - \exp\left(-\gamma(\mathrm{z_t} - \mathrm{c})^2\right), \quad (10)$$

where the transition variable $\mathrm{z_t} = \mathrm{Y_{t-d}}\beta$ is the cointegrating combination between ln(S), ln(P) and ln(P*) at time *t–d*[2], *c* denotes the equilibrium level of cointegration relationship and γ is the smoothness parameter. Higher γ induces faster transition. This function has symmetric *U* shape, which illustrative example for the case with *c* = 0 is presented on Figure 1.

Formulas (9) and (10) allow for a set of models, varying the order of the autoregressive process, lag length of the transition variable and presence of nonlinearity in loading coefficients, autoregressive process and deterministic terms. In this research we assume that $h = 1$, $d = 1$, while different variants of non-linearity in particular element of ESTVECM are allowed in (9). These variants of nonlinearities could be described by zero restrictions in α^Z, ξ^Z, Γ_h^z. The set of competing specifications are presented in Table 1.

[2] When strict PPP holds it could be interpreted as real exchange rate at time *t-d*.

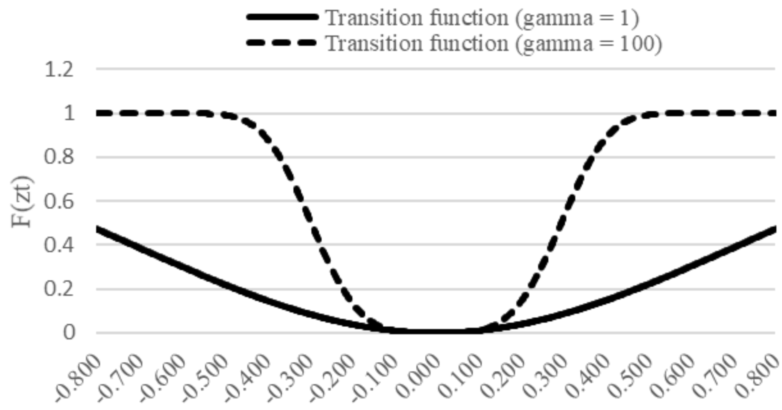

Figure 1. Illustrative example for the exponential smooth transition function with different values of γ

Table 1. The set of competing specifications

Models	ESTVEC Full	ESTVEC α	ESTVEC α,ξ	ESTVEC α,Γ	ESTVEC Γ,ξ	ESTVEC Γ	ESTVEC ξ
Parameters under zero restrictions	none	ξ^Z, Γ_1^Z	Γ_1^Z	ξ^Z	α^Z	α^Z, ξ^Z	α^Z, Γ_1^Z

4. Empirical Results

We used monthly data of the nominal exchange rate EUR/PLN (monthly average) and PPI manufacturing indices in Poland and Euro area. The full sample covers the period from January 1999 to February 2016 and hence the total number of observations, including starting values, is 206. We also performed recursive estimation and prediction on the basis of expanding windows, starting from the smallest sample covering the period from January 1999 to February 2006 and ending up on the sample from January 1999 to February 2016.

We estimated parameters of all competing models on the basis of Maximum Likelihood estimator. The strong form PPP is tested with the use of LR statistics. The model under the null hypothesis is obtained by the following restriction:

$$\beta' = [1 \quad -1 \quad 1]'.$$

We compared the forecasting performance of models on the basis of MAE and RMSE summaries. Also Diebold and Mariano (1995) test (DM)

was applied to check significance of differences of generated series of forecasts from the analogous series obtained on the basis of the Random Walk (RW) strategy. The DM procedure was applied together with small sample corrections (HLN) proposed by Harvey, Leybourne and Newbold (1997).
Table 2 contains model comparison results for VEC specifications and several generalisations towards smooth transition mechanism. We present logarithmic values of the likelihood calculated at ML estimates (LL) and AIC, BIC and HIC scores, respectively. Estimation is conducted for unrestricted cases and alternatively with PPP restriction imposed. The likelihood inference clearly indicates superiority of the smooth transition mechanism against simple VEC construct. The greatest data support, measured by LL value, received Full ESTVEC model and some limited parameterisations with nonlinearities in Γ and ξ separately and jointly Γ, ξ. However these models seem to be too heavily parameterised and are penalised substantially by information criteria scores. Among unrestricted specifications, smooth transition mechanism in parameters ξ seems to be an optimal compromise resulting with good data fit and parsimony.

Analyses conducted within a class of models with PPP restriction imposed show again superiority of nonlinear mechanisms. The full ESTVECM model receives again the highest LL value, but it is rejected by information scores as unparsimonious. Among restricted cases the smooth transition construct in parameters α is supported by the data and receives the best information score. Also the case with the smooth transition mechanism in parameters Γ receives attention.

In the next step we analysed statistical significance of parameters of ESTR function and restriction that guarantees PPP effect. On the basis of expanding data window described above we performed a sequence of appropriate LR tests. On Figure 2 and 3 we present fractions of analysed subsamples where ESTR coefficients were significant (Figure 2) and where strong form of PPP restriction was not rejected (Figure 3).

Decisive and more importantly time invariant inference about significance of underlying nonlinearities is impossible for heavy parameterised models. Full ESTVEC model and these limitations with smooth transition parameters imposed on pairs of groups of parameters – α and Γ, Γ and ξ, α and ξ perform worse and there are subsamples that do not support statistical significance of ESTR coefficients. The model ESTVEC-ξ, performs the best receiving in all analysed subsamples statistically significant ESTR construct at 0.1 and 0.05 significance level.

Rejection of the strong form of PPP are analysed in subsamples on Figure 3. In this element of analyses simple linear VECM is also considered.

We report surprisingly good performance of VECM specification and also relatively worse results given models with smooth transition component.

Table 2. Log-likelihood and information criteria obtained for all competing specifications in case of the whole sample

	Model	LL	AIC	BIC	HIC
Unrestricted specifications	VEC	2267.2	–4500.3	–4443.9	–4477.5
	ESTVEC Full	2282.3	–4496.6	–4383.8	–4450.9
	ESTVEC α	2274.8	–4505.6	–4432.6	–4476.1
	ESTVEC α,ξ	2277.8	–4505.6	–4422.7	–4472.1
	ESTVEC α,Γ	2280.0	–4498.0	–4395.1	–4456.4
	ESTVEC Γ,ξ	2282.6	–4503.2	–4400.3	–4461.6
	ESTVEC Γ	2283.4	–4510.9	–4418.0	–4473.3
	ESTVEC ξ	2280.6	–4517.1	–4444.1	–4487.6
Models with strong-form PPP restriction imposed	VEC	2265.1	–4500.3	–4450.5	–4480.1
	ESTVEC Full	2280.5	–4497.1	–4390.9	–4454.1
	ESTVEC α	2272.6	–4505.1	–4438.8	–4478.3
	ESTVEC α,ξ	2273.3	–4500.6	–4424.3	–4469.8
	ESTVEC α,Γ	2277.8	–4497.7	–4401.5	–4458.8
	ESTVEC Γ,ξ	2278.9	–4499.7	–4403.5	–4460.8
	ESTVEC Γ	2277.9	–4503.7	–4417.5	–4468.8
	ESTVEC ξ	2267.4	–4494.7	–4428.4	–4467.9

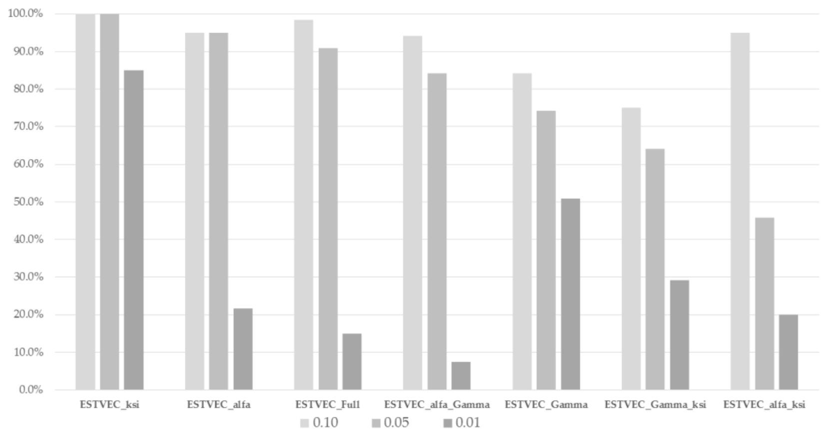

Figure 2. Fraction of subsamples, where ESTR coefficients are significant on given significance level, according to LR test

We also compare competing specifications with respect to the forecasting power. For the whole set of observations we generated 1, 3, 6 and 12 month-ahead forecasts. In Table 3 we put RMSE values normalised to RW strategy. We also present *p*-values (in italics) of the HLN-DM test against RW for quadratic loss function. Simple VECM specification exhibits much better forecasting performance in the short term. For one month-ahead case none of analysed smooth transition models generate better forecasts. Also in case of this horizon all competing specifications are much better than Random Walk and generate lower RMSE. However only simple VECM and ESTVECM- ξ generate forecasts significantly different than RW case at the significance level of 0.1. In case of unrestricted models specification ESTVEC with smooth transition mechanism in parameters ξ outperform other models for long term forecasting. For the case of 12 months ahead forecasts simple VECM is worse than RW and differences in point forecasts are statistically insignificant. For a set of restricted models we report relatively good forecasting performance in smooth transition class except the full case and cases with ST mechanism in α, jointly α,Γ and ξ. VECM model produces much better forecasts than ST class and outperforms RW case.

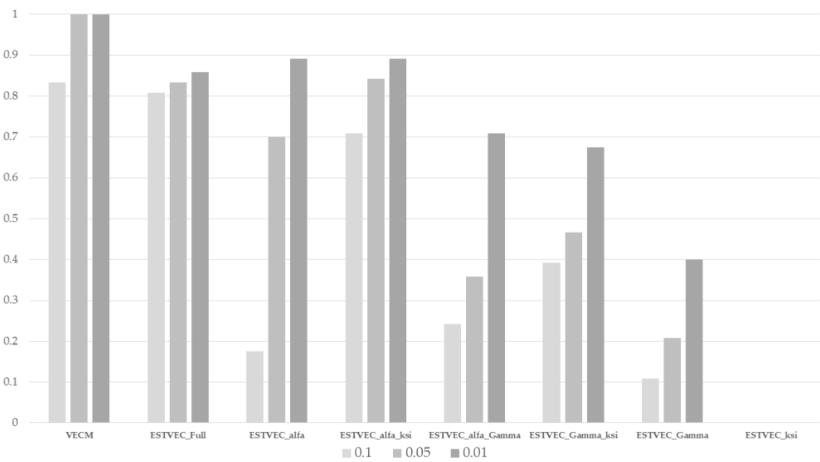

Figure 3. Fraction of subsamples, where strong-form PPP restrictions are not rejected given significance level, according to LR test

Analysing statistical significance of differences in forecasting performance (against RW) on the basis of HLN-DM test, in the set of unrestricted models, we can only be sure that data support poor forecasting performance in case of some ESTVEC specifications. Simple VECM model generates better forecasts that RW, but with differences to RW being statistically insignificant in case of all analysed horizons. In case of restricted models only the case of one month horizon exhibit significant differences from RW strategy in all cases. The strongest data support in favour of significance of differences is attached to simple VECM model.

Table 3. RMSE for the whole sample and *p*-values (in italics) for HLN-DM test against RW for quadratic loss function

Relative RMSE (RW=1) – log(FX) unrestricted models								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	**0.917**	0.958	**0.933**	0.948	0.962	0.992	0.961	0.983
	0.1	*0.3*	***0.1***	*0.2*	*0.3*	*0.5*	*0.3*	*0.4*
3m.	0.955	1.049	0.965	0.996	1.055	1.068	1.102	1.065
	0.3	*0.7*	*0.4*	*0.5*	*0.7*	*0.7*	*0.8*	*0.8*
6m.	0.997	1.142	0.937	0.965	1.117	1.080	161.531	1.153
	0.5	*1.0*	*0.2*	*0.3*	*0.9*	*0.2*	*0.2*	*1.0*
12m.	1.014	1.265	**0.889**	**0.921**	1.201	76.448	>1e+10	1.321
	0.7	*0.9*	***0.06***	***0.1***	*0.9*	*0.8*	*0.8*	*1.0*
Relative RMSE (RW=1) – log(FX) restricted models								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	**0.900**	**0.918**	**0.916**	**0.900**	**0.925**	**0.931**	**0.928**	**0.933**
	0.07	***0.1***	***0.1***	***0.07***	***0.1***	***0.1***	***0.1***	***0.1***
3m.	0.923	0.969	0.950	0.923	0.956	0.946	0.962	0.969
	0.3	*0.4*	*0.3*	*0.3*	*0.4*	*0.3*	*0.4*	*0.4*
6m.	0.949	0.997	0.958	0.949	0.946	0.932	1.036	0.969
	0.3	*0.5*	*0.3*	*0.3*	*0.3*	*0.3*	*1.0*	*0.4*
12m.	0.936	1.049	0.944	0.936	0.989	0.913	176.208	1.015
	0.3	*0.9*	*0.3*	*0.3*	*0.5*	*0.3*	*0.8*	*0.5*

The forecasting performance of both (restricted and unrestricted) class of models for subsamples is presented in Tables 4 and 5. Table 4 presents RMSE relative to RW for a group of unrestricted specifications, while Table 5 shows results for models with strong PPP restriction imposed. The sequence of subsamples were split into two sets, the first one covers the subsamples ending ad 2011:03 to 2016:02 and the second one contains these ending at 2011:03 to 2016:02. Just like in case of the whole sample, we report in all Tables *p*-values of the HLN-DM test against RW for the quadratic loss function. Again, in case of a set of unrestricted models ESTVECM with smooth transition function in ξ provides much better forecasts for longer horizon in both series of subsamples. In short term forecasting this model also beats simple VECM in second set of subsamples (2011:03 – 2016:02). Simple VECM performs relatively better than RW only in case of first series of subsamples.

Analysing results presented in Table 5 we report relatively good forecasting power of ESTVECM specification in the long term. Also simple VECM provide the best forecasts in case of one month ahead and three month-ahead horizon.

Table 4. RMSE for the sequential forecasting comparison out of sample periods for unrestricted models and *p*-values (in italics) for HLN-DM test against RW for quadratic loss function

Relative RMSE (RW=1) – log(FX) –for subsamples: 2006:03–2011:02								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	**0.904**	0.948	0.926	0.945	0.955	1.005	0.959	0.983
	0.1	*0.3*	*0.2*	*0.2*	*0.3*	*0.5*	*0.3*	*0.4*
3m.	0.917	1.022	0.937	0.972	1.022	1.069	1.120	1.037
	0.3	*0.6*	*0.3*	*0.4*	*0.6*	*0.7*	*0.8*	*0.6*
6m.	0.988	1.126	0.929	0.954	1.087	1.091	174.947	1.137
	0.4	*1.0*	*0.2*	*0.3*	*1.0*	*0.8*	*0.8*	*1.0*
12m.	1.010	1.249	**0.886**	**0.910**	1.172	79.682	>1e+10	1.314
	0.6	*1.0*	***0.04***	***0.1***	*1.0*	*0.9*	*0.9*	*1.0*
Relative RMSE (RW=1) – log(FX) –for subsamples: 2011:03–2016:02								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	0.955	0.979	0.946	0.954	0.977	0.953	0.965	0.982
	0.3	*0.4*	*0.2*	*0.2*	*0.4*	*0.2*	*0.3*	*0.4*
3m.	1.085	1.136	1.060	1.079	1.165	1.066	1.031	1.163
	0.8	*0.9*	*0.8*	*0.9*	*0.9*	*0.8*	*0.6*	*1.0*
6m.	1.056	1.225	0.971	1.022	1.271	1.009	1.188	1.241
	0.8	*1.0*	*0.4*	*0.6*	*1.0*	*0.5*	*1.0*	*0.9*
12m.	1.056	1.434	0.892	1.014	1.487	1.013	>5e+5	1.385
	0.7	*0.9*	*0.2*	*0.5*	*0.8*	*0.5*	*0.8*	*0.8*

However, only a few analysed cases generate forecasts significantly different compared to the RW-based ones. Relatively good-performing simple VECM model for one month ahead horizon as well as ESTVEC-ξ and ESTVEC-Γ for 12 months ahead differs significantly from the RW case for unrestricted models and the first set of subsamples; see Table 4. In this case the best forecasting performance, obtained for 12 month horizon in ESTVEC-ξ model receives also the strongest data evidence against RW forecasts as tested by HLN-DM quadratic score.

We report not so strong statistical significance in case of restricted models; see Table 5. Only in case of short term forecasts *p*-values are not greater than 0.1 making weak data evidence in favour of significant differences of generated forecasts from RW case.

Table 5. RMSE for the sequential forecasting comparison out of sample periods for models with strong-form PPP restriction and p-values (in italics) for HLN-DM test against RW for quadratic loss function

Relative RMSE (RW=1) – log(FX) –for subsamples: 2006:03–2011:02								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	**0.886**	**0.905**	**0.907**	**0.886**	0.909	0.927	0.919	0.919
	0.1	***0.1***	***0.1***	***0.1***	*0.2*	*0.2*	*0.2*	*0.2*
3m.	0.882	0.928	0.909	0.882	0.900	0.912	0.921	0.921
	0.2	*0.3*	*0.3*	*0.2*	*0.3*	*0.3*	*0.3*	*0.3*
6m.	0.928	0.973	0.924	0.928	0.896	0.892	1.032	0.925
	0.2	*0.4*	*0.3*	*0.2*	*0.2*	*0.2*	*0.9*	*0.3*
12m.	0.905	1.038	0.894	0.905	0.955	0.859	183.662	0.981
	0.2	*0.8*	*0.2*	*0.2*	*0.3*	*0.2*	*0.9*	*0.4*
Relative RMSE (RW=1) – log(FX) –for subsamples: 2011:03–2016:02								
horizon	VECM	ESTVEC α	ESTVEC ξ	ESTVEC Γ	ESTVEC α,ξ	ESTVEC Γ,ξ	ESTVEC α,Γ,ξ	ESTVEC Full
1m.	0.938	0.954	0.943	0.938	0.963	0.939	0.955	0.969
	0.2	*0.3*	*0.2*	*0.2*	*0.3*	*0.2*	*0.3*	*0.3*
3m.	1.062	1.108	1.087	1.062	1.134	1.062	1.102	1.128
	0.8	*0.9*	*0.9*	*0.8*	*1.0*	*0.9*	*0.9*	*1.0*
6m.	1.067	1.130	1.136	1.067	1.186	1.134	1.062	1.188
	0.8	*1.0*	*1.0*	*0.8*	*1.0*	*1.0*	*0.9*	*1.0*
12m.	1.246	1.175	1.399	1.246	1.300	1.386	1.131	1.337
	0.8	*1.0*	*0.9*	*0.8*	*1.0*	*1.0*	*0.9*	*1.0*

Conclusions

In this paper we investigated PPP hypothesis for the Polish Zloty in a multivariate dynamic econometric model. We confront the explanatory power of relatively broad class of models utilizing certain nonlinear cointegration concepts. We depart from a simple VECM framework into the STVECM class and verify empirical importance of generalizations (and hence the strength of statistical evidence in favor of PPP hypothesis).

The in-sample results of model comparison by information criteria (based on full dataset) deliver no clear-cut conclusions, though in general the strongest support is allocated to two specifications: a linear VECM model with PPP restriction and an unrestricted ESTVEC ξ specification.

For a more detailed analysis we consider an out-of-sample expanding-window recursive forecasting experiment with two verification windows, one for the period 2006–2011 and one for 2011–2016. The results are the following: in short forecast horizons (1–3 months ahead), the imposition of

PPP improves the forecasting performance across all the specifications and verification windows under consideration. The winning specification is a linear VEC model (with PPP). However, for longer forecasting horizons (6–12 months-ahead), the results are more complicated. In general, ESTVECM specifications involving smooth transition mechanism for the ξ parameters provide the best performance. There are though important differences between the two verification periods. In the first period, the ESTVECM-ξ model with PPP imposed is an overall winner for the 12-months ahead forecasts, although some other models involving smooth transition for ξ (with or without PPP) also offer similar performance. In the second period, the imposition of PPP restrictions results in a drop in long-horizon forecasting performance of the models mentioned above. In general, all the winning specifications mentioned above outperform simple RW-type forecasts.

Hence, our main results are the following. On the one hand, for the short-term prediction, a linear VECM with PPP seems to be a preferable tool. On the other hand, as longer horizons are involved, the need for nonlinear dynamics (as in ESTVECM-ξ) becomes more evident. As to the validity of PPP, the empirical support (based on long-horizon performance) is however time-inhomogeneous – it seems that more recent observations provide evidence against the PPP restrictions of the form considered here.

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Weryfikacja hipotezy parytetu sił nabywczej dla kursu walutowego EUR/PLN w ramach wektorowych modeli korekty błędu z funkcją wygładzonego przejścia (ESTVECM)

Z a r y s t r e ś c i. Celem artykułu jest ocena empirycznych konsekwencji narzucenia hipotezy PPP w formie mocnej (ang. *strong-form*) dla kursu EUR/PLN przy wykorzystaniu wybranych modeli kointegracji nieliniowej, to jest modeli ESTVEC. Zasadność wykładniczej funkcji przejścia dla mechanizmu korekty błędu jest testowana w odniesieniu do liniowego modelu VEC. Konkurencyjne modele są porównywane zarówno pod względem dopasowania wewnątrz próby, jak i zdolności predyktywnych. Wyniki wspierają mechanizm wygładzonego przejścia w składniku deterministycznym. Żaden z modeli ESTVECM nie generuje systematycznie lepszych prognoz niż liniowy model VECM

S ł o w a k l u c z o w e Parytet siły nabywczej; ESTVECM; kointegracja; Prognozowanie kursu walutowego.