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The Application of Hidden Markov Models to the Analysis of Real Convergence[**]

A b s t r a c t. This paper employs hidden Markov models and the Viterbi path to analyze the process of real convergence. Such an approach combines the analysis of cyclical and income-level convergence. Twelve macroeconomic variables in the sample of 28 EU countries observed in the 1995–2016 period are within the scope of the study. The results indicate, among others, the existence of real convergence of Poland toward the remaining EU countries in terms of the levels of GDP per capita at PPP and GDP growth rates, with a short-run period of divergence during the global crisis.

K e y w o r d s: catching-up; convergence; hidden Markov model; European Union; Viterbi path.

J E L Classification: C61, E32, O47, O52.

Introduction

Cyclical convergence and income-level convergence are usually tested separately in empirical studies. However, these are interrelated phenomena because business cycles are very closely linked with economic growth. That

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is why in order to obtain a full picture of output fluctuations and dynamics, it is necessary to apply a tool which would allow to test and assess simultaneously both cyclical and income-level convergence. Hidden Markov models (HMM) are used as such. The application of the method that combines the HMM and the Viterbi path in the analysis of real convergence is the main value added of the paper. The holistic approach involves the use of the Baum-Welch algorithm, Viterbi algorithm, and Monte Carlo simulations, which fills in the gap existing in the literature.

In the study, the procedure based on HMM is applied in order to assess the character and the rate of real convergence in Europe. While applied to the GDP time series, this approach encompasses both the cyclical and income-level convergence. The results allow us to verify the conformity of business cycles between various economies as well as the equalization of income levels between countries. Additionally, time series of some other macroeconomic variables are analyzed as a kind of robustness check of the proposed procedure as a tool to analyze the dynamics of different macroeconomic factors.

The analysis covers the 28 EU countries and the 1995–2016 period. The main results are plotted for Poland (its convergence toward the other 27 EU countries). The main results for Germany are also reported in the form of a robustness check. Germany is selected as the additional country due to three reasons. First, it is the biggest EU economy. Second, it is the main trade partner for Poland – a fact, which is even more important in the time periods around the big financial crisis (see Sledziewska and Witkowski, 2012). Third, Poland's macroeconomic performance depends deeply on the situation in Germany due to very close links between both countries caused e.g. by large flows of the factors of production (labor and capital).

The paper is organized as follows. After the introduction, the theoretical background of real convergence is described. The concept of the HMM and the Viterbi path are discussed in the next section. The macroeconomic variables included in the study and the method of estimation are then presented, which are followed by the empirical results. The last section concludes.

1. Real Convergence

Various ways of defining the process of convergence as well as many methods of testing a particular convergence hypothesis are proposed in the literature (see, e.g., Islam, 2003) In general, nominal and real convergence are the two types that can be outlined. The nominal convergence means the tendency of nominal variables, like price levels, interest rates, or exchange

rates to level up. The real convergence refers to variables given in real terms (mainly real output or income) and can be divided into cyclical convergence and income-level convergence. The cyclical convergence means the tendency of different economies toward the conformity of business cycles while the income-level convergence means the equalization of the GDP per capita levels.

This general classification has been extended in a number of ways, including the types and definitions of convergence, as well as the methods of verifying of a given concept of convergence. For example, the new concept of growth cycle has been developed to analyze cyclical fluctuations of many Western economies where GDP has not dropped for many years (see, e.g., Zarnowitz and Ozyildirim, 2006). Unlike the classical business cycle where recession means the absolute fall in GDP, if the growth cyclicality is observed, the GDP rises during both the contractionary and the expansionary phase. However, during the contractionary period the growth rate of GDP is less than the trend, while in the expansionary period the GDP growth rate exceeds the long term trend. Various definitions of cyclical fluctuations, as well as a variety of methods to extract the trend and to find peaks and troughs in business activity, implied that there are many quantitative methods to cope with output fluctuations and cyclical conformity. Furthermore, there is still much room to develop new methods and concepts, including those based on the HMM approach.

Similarly, there is no unique definition of income-level convergence. The existence of β convergence means that less developed countries (with lower GDP per capita) grow faster than the more developed ones. On the other hand, the σ convergence is observed when the differences of the income levels between countries (measured e.g. by the standard deviation of the logarithm of the GDP per capita levels) decrease over time (Barro and Sala-i-Martin, 2003).

The latest studies on convergence in the EU (including the papers on the catching-up process between the CEE countries and Western Europe) take into account the effects of the global crisis and the crisis in the eurozone (Kaitila, 2013; Dauderstadt, 2014; Nenovsky and Tochkov 2014; Simionescu 2014; Forgo and Jevcak, 2015). Many of these recent analyses prove that the convergence process decelerated after the crisis, indicating even some divergence tendencies. There are also studies that suggest the existence of convergence clubs in the EU (e.g. Borsi and Metiu, 2013; Monfort et al., 2013; Gligoric, 2014). The book by Jozwik (2017) presents the analysis of convergence at the national and regional levels, focusing on institutional changes due to systemic transformation, economic integration,

and the cohesion policy. Prochniak and Witkowski (2014) as well as Matkowski et al. (2016a) show the analysis of β and σ convergence between the CEE11 and EU15 in the 1993–2015 period. They conclude that the catching-up process was not continuous, showing some breaks and divergence episodes. The most intensive convergence was observed in the years 2000–2007, just before and after the EU's major enlargement. The other recent studies on convergence include, among others, papers by Batog (2013) and Grzelak and Kujaczynska (2013).

The review of the literature, focused on the studies published in the last years, shows the necessity to check carefully the time stability of the catching-up process along with structural breaks. Such an analysis is carried out in this study.

2. Hidden Markov Models (HMM)

A number of methods of analyzing convergence have been developed. Some of them have purely mathematical background, others involve the expert insight. In this paper, the HMM is used. Its concept is often identified with the name of Hamilton (1989), however it was present in the literature at least since the 60s of the previous century, long before the first articles by Hamilton. For the comprehensive description and characteristics of these models, one can refer to Cappe et al. (2005), while in this paper we discuss very briefly only those of the main definitions and ideas behind the HMM models, Markov chains (MC) and Viterbi paths, which are used extensively in the following sections of the article. We concentrate rather on the idea behind those terms, restricting to basic definitions and notation necessary to understand the proposed method of analysis of the real convergence.

Let $\{X_k, Y_k\}_{k\geq 0}$ be a discrete stochastic process satisfying the following conditions:

- the unobservable process $\{X_t\}_{t\geq 0}$ is a homogenous MC with a finite state space S,
- conditionally on the process $\{X_t\}_{t\geq 0}$ the observations $\{Y_t\}_{t\geq 0}$ are independent, and for each *t* the conditional distribution of Y_t depends only on X_t.

The $\{X_k, Y_k\}_{k\geq 0}$ which fulfills the above conditions is referred to as a Hidden Markov Model (HMM). In macroeconomic applications Y_t often has univariate or multivariate Gaussian distribution. In that we refer to a resulting HMM as to the normal HMM.

HMM are widely used in the areas, where the pattern recognition is explored. Therefore, the most common use of HMM is in speech or handwrit-

ing recognition, as well as in the cases when gesture or voice patterns need to be obtained. HMM is also a basic tool in bioinformatics, for example in the DNA sequencing process. In macroeconomics applying HMM is one of the methods of business cycles synchronization analysis and turning points identification. This approach, however, should be treated rather as a whole class of models, due to the possibility of choosing a form of an observable and unobservable component. For this reason, a huge variety of types of models were being under study over the years – see Hamilton (1994) or Koskinen and Oeller (2004) for a comprehensive review.

Unlike the classical Markov models, where all states are visible and the Markov model is defined only by the transition probabilities, in HMM the states are unobservable and need to be calculated based on another observable time series. Therefore, besides the transition probabilities, the parameters of the probability distribution related to each state are also present. The problem which needs to be solved is to find the unobservable path of states. The deterministic algorithm of finding the parameters of the HMM which is used was described by Baum et al. (1970) and is known under the name of the Baum-Welch algorithm. However, knowing the model parameters does not solve the stated problem as the state of an unobservable MC still remains to be estimated. There are few alternatives of performing this estimation. One of them are the smoothed probabilities given by Hamilton (1994). Second are the filtered probabilities, see Chauvet and Hamilton (2005) or Harding and Pagan (2002). In both those approaches, the most likely estimation of the state of the hidden MC at the given moment is chosen on the basis of those probabilities. So the states on the path of MC are estimated locally, which could be inefficient. However, an alternative, which makes use of the global decoding exists: instead of a single point of time, the whole period covered by the analysis is taken under consideration. The path of states being the result of this approach is called the Viterbi path. Elements of the Viterbi path are calculated with the use of the Viterbi algorithm, descried by Viterbi (1967).

The Viterbi algorithm together with the Baum-Welch algorithm provide the deterministic procedure of transforming the time series into the most probable path of states. Unfortunately, the results strongly depend on the initial values and can be far from optimal. Therefore, to increase the chance of finding the globally optimal solution, the computations are performed repeatedly with the same set of data and different initial values. This approach is usually referred to as Monte Carlo simulations. Depending on a number of factors (Bernardelli, 2013), computation can be quite time-consuming.

In this paper, due to the length of the input time series, only the HMMs with two-element state space are considered. Therefore the state space has the form of $S=\{0,1\}$ and the interpretation of states is as follows: 0 is associated with the periods of relatively good conditions and 1 is associated with a worse situation. We restrict ourselves to the analysis of normal HMM. Thus the observable component Y_t, which corresponds to economic time series under the analysis, must satisfy the conditions

$$Y_n|_{X_n=0} \sim N(\mu_0, \sigma_0) \text{ and } Y_n|_{X_n=1} \sim N(\mu_1, \sigma_1). \quad (1)$$

We additionally assume that $\mu_0 < \mu_1$, to have the same order of states in each considered case (state 1 is associated with a greater mean value), which in turn allows to compare Viterbi paths for different pairs of countries.

Theoretically, the state space could be extended to the case of more than two states. However, larger state space would obviously involve longer computation time and often cause problems with the numerical stability of results. The combination of the Viterbi path and HMM with three and four states are rather uncommon in the macroeconomic literature with a series of articles by Bernardelli and Dedys in 2012 (e.g. Bernardelli and Dedys, 2012) serving as an exemption. The authors explore the discussed method in order to describe the business cycle synchronization and identify turning points. However, the application of the HMM method and the Viterbi path to the analysis of real convergence is absent in the literature.

The discussed method has several advantages as compared with the standard β and σ convergence tests. First of all, we do not assume a priori both the number and the timing of turning points (structural breaks). Turning points are identified on the basis of the behavior of individual indicators and, based on this, convergence and divergence periods are identified. Second, convergence and divergence periods are identified for each pair of countries individually. Third, the concepts of income-level and cyclical convergence are integrated. As a result, even if the obtained empirical results in this case are intuitive and comply with expectations of most economists, this should be viewed as a confirmation of method's proper functioning.

3. Data and Method of Estimation

The empirical analysis includes 12 macroeconomic variables:

a) GDP per capita at purchasing power parity (PPP, constant 2011 international $) [gdppc_ppp],
b) growth rate of total real GDP (%) [g_gdp],
c) CPI inflation (%) [inf],

d) unemployment rate (%) [une],
e) household final consumption expenditure (constant 2010 US$) [cons_usd],
f) household final consumption expenditure growth rate (%) [cons_g],
g) government final consumption expenditure (constant 2010 US$) [gov_usd],
h) government final consumption expenditure growth rate (%) [gov_g],
i) foreign trade balance (current US$) [nx_usd],
j) foreign trade balance (% of GDP) [nx_gdp],
k) domestic credit provided by financial sector (% of GDP) [cred_byfin],
l) bank nonperforming loans (% of total gross loans) [nonp_loans].

The variables are taken from the International Monetary Fund and World Bank databases (IMF, 2017; World Bank, 2017).

The first two variables represent the level of GDP per capita at PPP and the real GDP growth rate. These are the two basic variables that are used in the studies on real convergence. The results for these two variables allow us to assess the cyclical and income-level convergence on the basis of the HMM analysis.

Given that one of the aims of this study is to check the appropriateness of the HMM algorithm in the analysis of the dynamics of various macroeconomic variables, the list has been extended by a few other time series. Firstly, both inflation and unemployment are included as important variables from the point of view of the wellbeing of the society and the standard of living. Variables which are components of the GDP (household consumption, government expenditure, and net exports) are also analyzed. Those are considered both in levels or as growth rates (except net exports which are taken as the level and percentage of GDP). Finally, the two variables that represent the stability and development of the financial sector are included. Those are very important in particular in the period which followed the global crisis and the crisis in the euro area when financial turbulences largely influenced the real economy.

The analysis covers the 28 EU countries observed in the 1995–2016 period. In each of the cases the series are annual. In the case of missing observations, the calculations include a shorter period or lower number of countries.

This study focuses on Poland. That is why the detailed results for Poland against the remaining EU countries are presented throughout the major part of the paper. It means that the results show – for each individual variable – the comparison of Poland with each of the 27 remaining EU countries (in the figures – as average values).

Table 1 shows the evolution of selected indicators for Poland. For the sake of conciseness, we do not present the remaining indicators for Poland and the other countries.

For comparison purposes and as a form of robustness check of the proposed method at the end of the paper selected results for Germany are included. Germany is the biggest economy in the EU. In 2016, its GDP constituted 21.2% of the EU28's GDP at current exchange rates and 20.0% of the EU28's GDP at purchasing power parity (European Commission, 2016). Hence, it is interesting to compare the Viterbi path for Poland's GDP with the analogous path for German GDP. The details of the applied method of the analysis are the same both in the case in Poland and Germany and are provided in detail in the remaining part of this section. Germany is compared with each of the other 27 EU countries.

Table 1. The evolution of selected indicators for Poland

	gdppc_ppp	g_gdp	inf	une	cred_byfin	nonp_loans
1995	11300	6,7	27,9	13,3	29,4	–
1996	11976	6,2	19,9	12,3	30,9	–
1997	12741	7,1	14,9	11,2	32,0	10,5
1998	13324	5,0	11,8	10,6	33,4	10,5
1999	13944	4,5	7,3	13,1	35,9	13,3
2000	14732	4,3	10,1	16,1	34,3	15,5
2001	14920	1,2	5,5	18,2	38,9	18,6
2002	15232	1,4	1,9	19,9	38,9	21,1
2003	15785	3,6	0,8	19,6	40,2	21,2
2004	16606	5,1	3,5	19,0	39,0	14,9
2005	17194	3,5	2,1	17,7	38,4	11,0
2006	18268	6,2	1,0	13,8	42,9	7,4
2007	19563	7,2	2,5	9,6	47,8	5,2
2008	20392	3,9	4,2	7,1	63,5	2,8
2009	20953	2,6	3,5	8,2	61,6	4,3
2010	21771	3,7	2,6	9,6	63,2	4,9
2011	22850	5,0	4,3	9,6	65,9	4,7
2012	23218	1,6	3,7	10,1	64,1	5,2
2013	23555	1,3	0,9	10,3	67,2	5,0
2014	24346	3,3	0,0	9,0	71,0	4,8
2015	25323	3,7	–0,9	7,5	73,2	4,3
2016	–	3,1	–0,6	6,3	–	4,4

Source: IMF, 2017; World Bank, 2017.

The procedure used in the empirical analysis explores the concept HMM and Viterbi path described in the previous section. In order to get the reliable results, Monte Carlo simulations are used. The initial values for the Baum-Welch algorithm are chosen randomly with the use of independent and iden-

tically distributed draws from the univariate distribution. The number of draws used for parameters estimation of the time series being under study was set to 1000. In order to choose the best model, three criteria are taken into account:

- Akaike's information criterion (AIC),
- Bayesian information criterion (BIC),
- the log likelihood value.

The procedure used in the analysis can be described in the following steps (Poland is used as an example for the clarity of description).

1. For each of the 12 variables and for each of the 27 EU countries (all except Poland), the time series of differences are constructed as

$$\tilde{v}_t^C = v_t^{PL} - v_t^C, \tag{2}$$

   where $t = 1995, 1996, \ldots, 2016$ and C refers to one of the EU countries.
2. The estimation of the HMM parameters is performed with the use of the Baum-Welch algorithm. The resulting estimates are used to find the Viterbi path. State 0 on that path identifies a year of greater similarity in terms of the variable under analysis, whereas state 1 indicates a divergence between countries (Poland and the country C).
3. For each year, averages of the states of Viterbi paths for all countries for the given variable are calculated. The value of 0 means perfect convergence while the year when the average equals 1 means the period of undisputable divergence between Poland and other EU countries.

This kind of approach allows to both analyse a pair of countries separately, as well as to consider the real convergence between the chosen country and the group of other countries jointly. Plots presented in the figures and summaries in the tables visualize both of those possibilities.

The discussed procedure is followed for the cases of Poland and Germany, however, it may be used to determine the convergence of the other countries as well. This study is the initial application of the HMM and the Viterbi path and, for the sake of conciseness, none of the advanced algorithms in calculating reference values were employed. In further studies, it is possible to extend the analysis by considering weighted averages or assessing convergence toward certain subgroups of countries (their clusters).

4. Empirical Analysis

The results of the empirical analysis for Poland are presented in Figures 1–12 and Tables 2–3. The figures show the averages of the states of the Viterbi paths calculated with the use of the estimated parameters of the

HMM for the differences between the annual values of a given variable for Poland and each of the other EU countries. Lower values (closer to 0) indicate the existence of real convergence (greater similarity in terms of a given variable) while higher values (closer to 1) – real divergence (bigger differences of a given variable).

The detailed results in the form of the states of the Viterbi path are given in Tables 2 and 3. For a given year, the HMM parameters were estimated for the differences in the values between Poland and each other EU country separately. The value of 1 is assigned when the calculated probabilities for being in certain state – on the basis of the Viterbi algorithm – are relatively large as compared with the other years in the whole period. On the other hand, when the difference is relatively low, a 0 value is assigned. Hence, value 1 indicates real divergence (bigger differences between countries) while value 0 can be interpreted as a real convergence (greater similarity). The functions plotted in Figures 1 and 2 are the arithmetic averages of the values provided in the respective columns of Tables 2 and 3.

Figure 1 shows the Viterbi path of the GDP per capita at PPP. The results indicate a clear-cut cyclical convergence before the beginning of the global crisis, that is from 1995 to 2008. In 2009, the differences between the GDP per capita at PPP in Poland and the other EU countries increased. This was caused by the economic and financial crisis. The global crisis led to the recession in all the EU countries, except Poland, which resulted in the significant change of the earlier convergence trends. This tendency is visible on the basis of the HMM method – the average values of the states of Viterbi paths rose in 2009 and 2010, indicating real divergence. Since 2011, GDP per capita levels between Poland and the other EU countries have converged in terms of the Viterbi path but the process has not been so intensive as in the first part of the analyzed period.

These results are in line with some other studies that confirm the existence of divergence tendencies in Europe in the last years (see, e.g., Mucha, 2012; Stanisic, 2012; Borsi and Metiu, 2013; Monfort et al., 2013). For example, the study by Matkowski et al. (2016b) showed – with the use of the σ convergence concept – that in 2009 and 2010, income differences among the 26 EU countries increased.

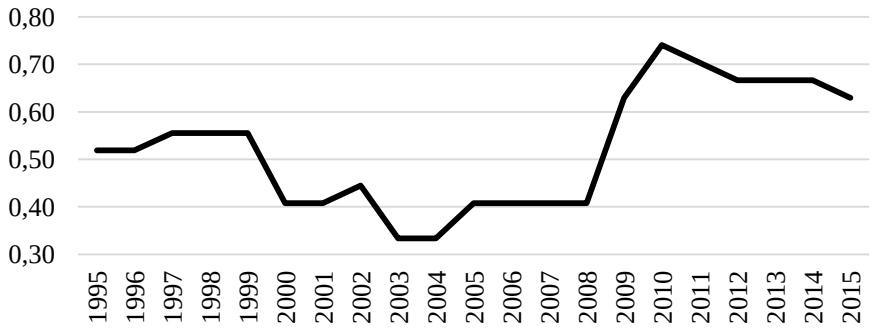

Figure 1. Cyclical convergence of GDP per capita at PPP [gdppc_ppp] between Poland and the other EU countries

Figure 2 illustrates the results for GDP growth rates. The curve plotted in Figure 2 shows a sharp increase in the values of averages of the states of the Viterbi paths in the year 2009. This means that a large rise in differences between GDP growth rates in Poland and the other EU countries was observed during the crisis. It was caused by the fact that in 2009 Poland was the only country that recorded the increase in GDP while all the other EU countries noted a recession. This atypical behavior which consisted in the difference between Poland and the remaining EU countries was confirmed by the results attained from the proposed procedure based on the HMM as reflected by the increase in the average values of the states of Viterbi paths for the pair of Poland and other EU countries for 2009.

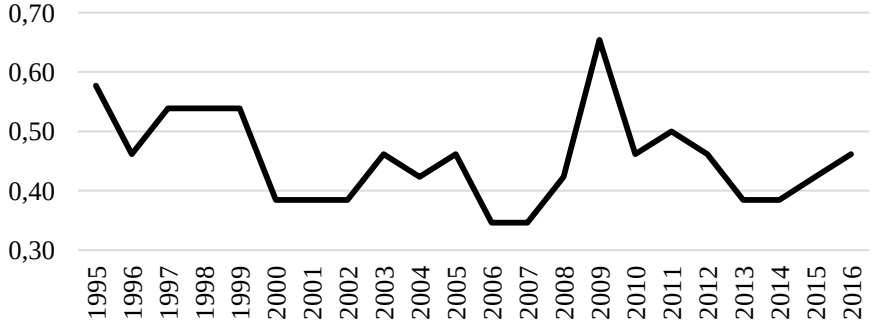

Figure 2. Cyclical convergence of GDP growth rates [g_gdp] between Poland and the other EU countries

The results for inflation rates are shown in Figure 3. The tendency of the Viterbi paths is declining throughout the whole 1995–2016 period. This out-

come points to a regular fall in differences in inflation rates between Poland and the other EU countries. A significant convergence of inflation is in line with the economic theory and official statistics – along with the further openness of the economy and integration with the EU, there took place nominal convergence in terms of price levels and inflation rates. Clearly the combination of the Baum-Welch and Viterbi algorithm, yielded economically justified results not only in terms of GDP convergence but also the inflation convergence.

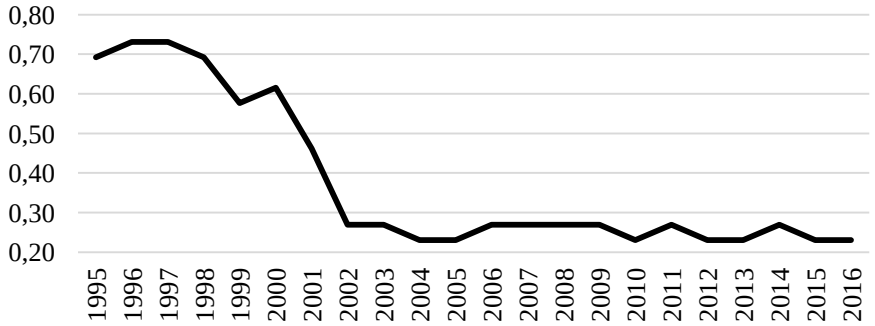

Figure 3. Cyclical convergence of inflation rates [inf] between Poland and the other EU countries

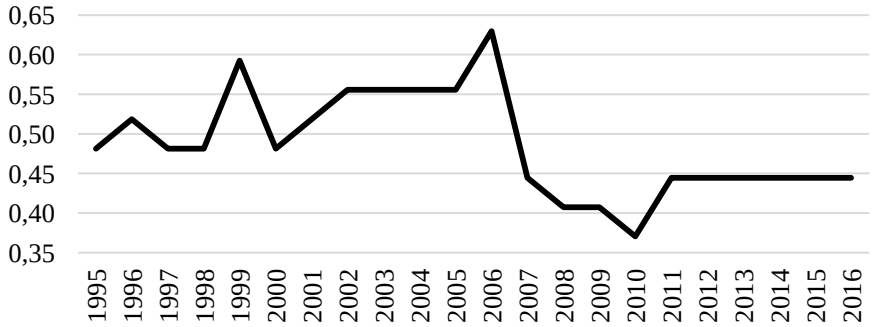

Figure 4. Cyclical convergence of unemployment rates [une] between Poland and the other EU countries

The results for unemployment rates (Figure 4) indicate the strengthening of the similarity of unemployment rates in the last years. This may be caused by the fact that the global crisis and the crisis in the euro area both were the factors leading to the increase of unemployment in many countries. Moreover, the official statistics for real economies often do not support the Okun's

law, meaning that economic growth in real countries needn't lead to the fall in unemployment. That is why the results for the convergence in unemployment rates are different compared with those for GDP.

Figures 5 and 6 show the results for consumption (in terms of levels and growth rates, respectively). Consumption constitutes the largest portion of GDP. As we can see, the Viterbi paths for both the level and growth rate of consumption are quite similar to the respective paths for GDP per capita levels and GDP growth rates. It reinforces the appropriateness of the HMM method as a tool to analyze real convergence. Given these outcomes, the results are unlikely to be a coincidence. Additionally, significant changes in consumption pattern may also confirm the appropriateness of the Keynesian consumption function where consumption depends mainly on current disposable income. The permanent income hypothesis, according to which short-run fluctuations in income do not influence the level of consumption, is unlikely to be supported by this study.

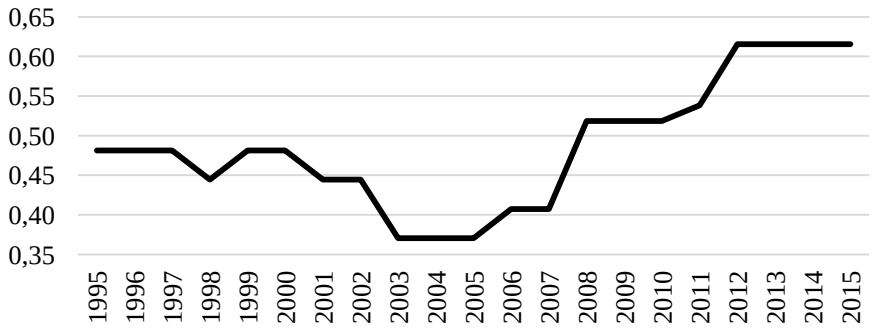

Figure 5. Cyclical convergence of household consumption [cons_usd] between Poland and the other EU countries

The results for convergence of government expenditures are shown in Figure 7 (the levels) and 8 (growth rates). Unlike private consumption that behaves very similarly to the total output, the averages of the states of Viterbi paths for government consumption are different. In terms of levels, there is a tendency toward decreasing cross-country differences. As regards the growth rates, the results reveal large fluctuations from one year to another. This is in line with the economic theory, including the Keynesian cross model, according to which the government purchases of goods and services are autonomous, that is independent of income. The level of government spending depends on the economic policy performed by a given country. The increasing convergence in terms of the level of government consumption may be also caused by the fact that, after the EU accession, Poland re-

ceived a lot of aid funds from the European Union (it was the main recipient of EU funds from the 2007–2013 budget).

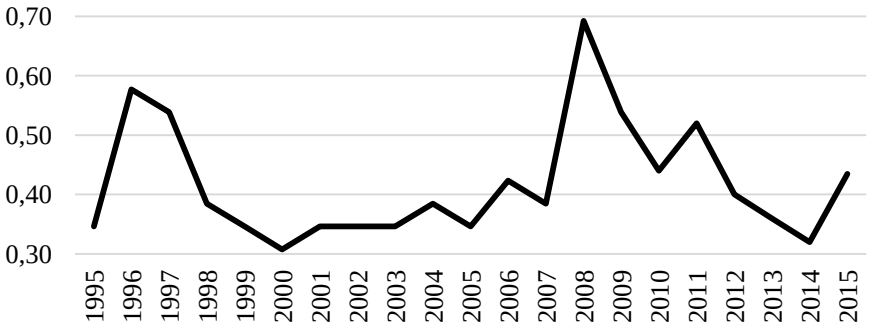

Figure 6. Cyclical convergence of household consumption growth rates [cons_g] between Poland and the other EU countries

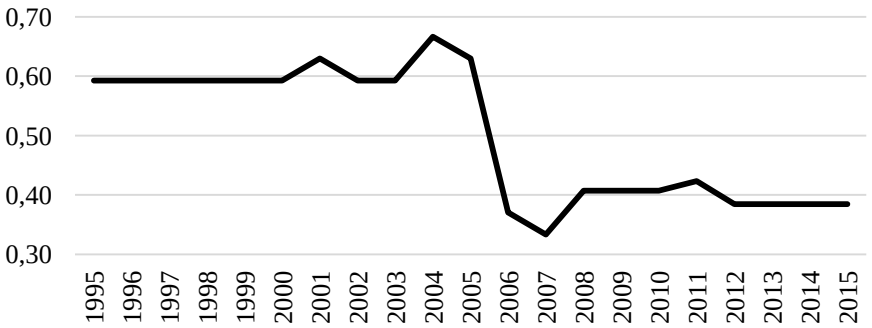

Figure 7. Cyclical convergence of government consumption [gov_usd] between Poland and the other EU countries

Data for net exports (Figures 9 and 10) indicate the increase in differences of the foreign trade balance between the countries. It is likely to be related to the different specialization of individual countries and a different reaction to various external shocks as well as different involvement in international flows of goods, services, assets, and labor. Greater differences in foreign trade balance may also be caused by the fact that the intraregional trade within the EU is substantial. In such a case, if a country increases its exports, another country's import must be increased. This hampers the convergence tendency of foreign trade balances.

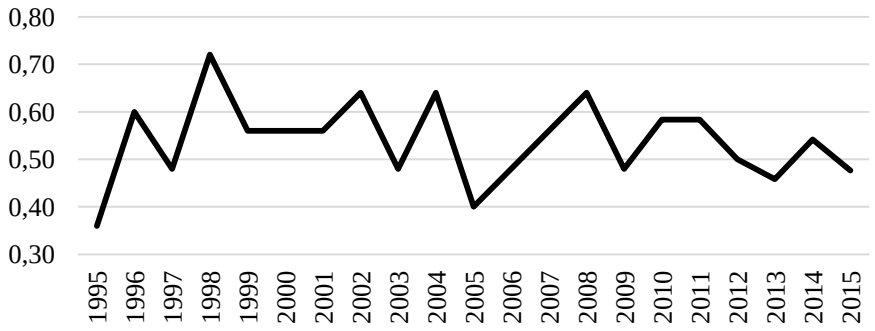

Figure 8. Cyclical convergence of government consumption growth rates [gov_g] between Poland and the other EU countries

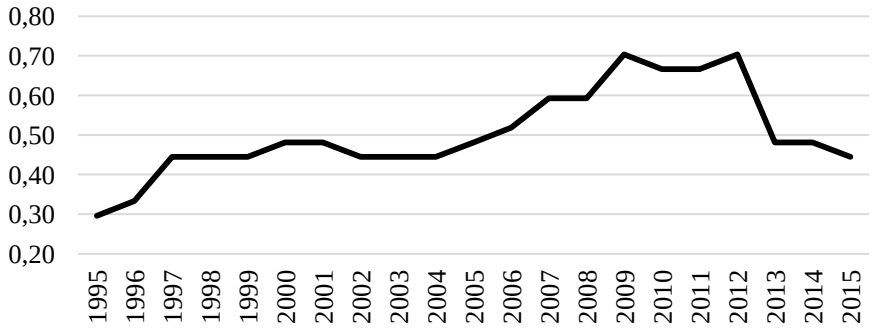

Figure 9. Cyclical convergence of foreign trade balance [nx_usd] between Poland and the other EU countries

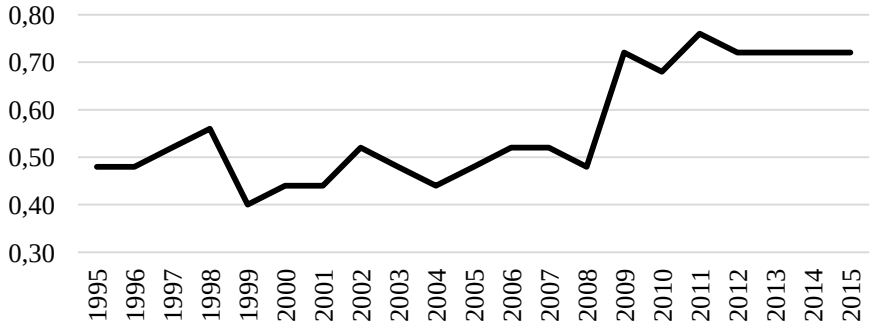

Figure 10. Cyclical convergence of foreign trade balance to GDP ratio [nx_gdp] between Poland and the other EU countries

The results for the financial sector development are mixed. On the one hand, the volume of domestic credit provided by the financial sector (% of GDP) shows the tendency toward divergence throughout the analyzed period (Figure 11). On the other hand, Figure 12 indicates that the volume of nonperforming loans (% of total loans) exhibits the tendency toward convergence. The latter outcome results from the fact that the extent of nonperforming loans depends on the situation in global markets. After the economic and financial crisis as well as the crisis in the euro area the majority of EU countries noticed a considerable rise in nonperforming loans. If the volume of nonperforming loans rises simultaneously in both countries, it means greater convergence.

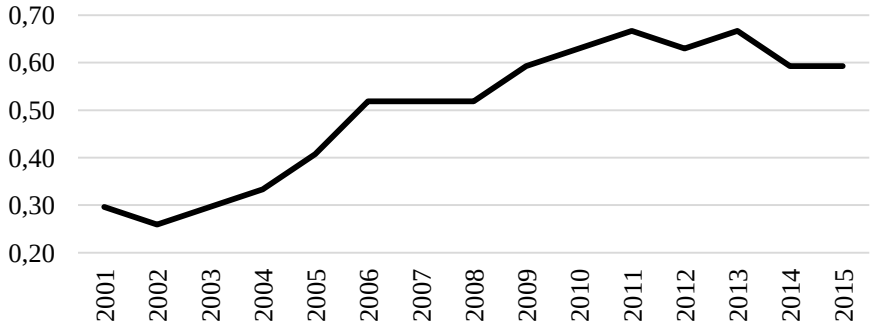

Figure 11. Cyclical convergence of domestic credit provided by financial sector [cred_byfin] between Poland and the other EU countries

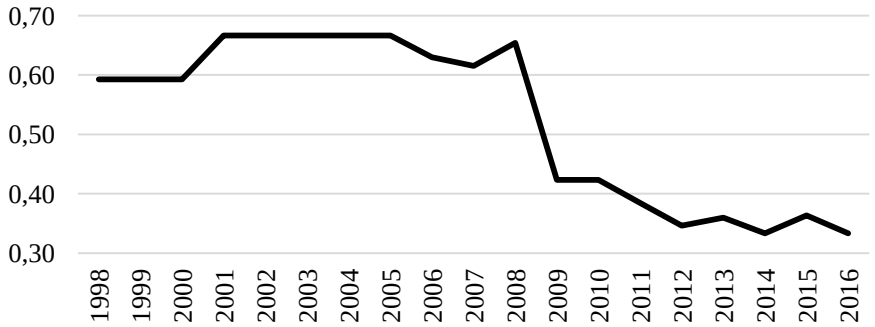

Figure 12. Cyclical convergence of bank nonperforming loans [nonp_loans] between Poland and the other EU countries

The results presented in Tables 2 and 3 indicate that there is no common regularity as regards catching-up toward the subgroups of countries (Central-

Eastern or Western Europe). In each subperiod (the 1990s, 2000s, and 2010s), there were countries both from Western Europe as well as Central-Eastern Europe which Poland converged to or Poland diverged from. It reflects the fact that there are many countries Poland cooperates with and the character of bilateral relations is different for different partners. Hence, it is possible to reveal convergence with some countries but divergence with another ones. This outcome is also in line with the hypothesis of club convergence – some studies suggest the necessity to divide world countries into clusters in the frame of convergence analysis (see, e.g., Battisti and Parmeter, 2013).

Table 2. Viterbi paths for GDP per capita at PPP [gdppc_ppp] between Poland and the individual EU countries

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Western Europe																					
AT	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
BE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
DK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
FI	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
FR	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
DE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
GR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
IE	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0
IT	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
LU	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NL	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
PT	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
ES	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
SE	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
UK	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Central-Eastern Europe																					
BG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
HR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
CZ	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
EE	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
HU	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
LV	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
LT	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
RO	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
SK	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
SI	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
CY	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
MT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1

Note: 0 indicates convergence in differences; 1 indicates a divergence in differences.

Table 3. Viterbi paths for GDP growth rates [g_gdp] between Poland and the individual EU countries

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Western Europe																						
AT	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
BE	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
DK	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
FR	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1
DE	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1
GR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
IE	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
IT	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LU	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
NL	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PT	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ES	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1
SE	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1
UK	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1
Central-Eastern Europe																						
BG	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
HR	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CZ	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
EE	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
HU	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0
LV	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
LT	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
RO	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
SK	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
SI	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
CY	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1
MT	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1

Note: 0 indicates convergence in differences; 1 indicates a divergence in differences. Finland was excluded from calculations due to having the time series that does not satisfy the conditions of the HMM procedure.

The Viterbi paths for GDP per capita levels and GDP growth rates for Germany have been estimated for both comparison and robustness check. Those are illustrated in Figures 13 and 14. In general, the results for Germany are quite similar to those for Poland (although some differences appear). This is economically justified as Germany is Poland's main trading partner and Polish economy vastly depends on the development of Germany. The theoretical structural model implies that the growth rate of the Polish economy should approximately follow the growth of output in Germany. The results are partly in line with this view. The Viterbi path for GDP per capita in Germany shows quite strong real convergence toward the 27 EU countries

at the beginning of the analyzed period and an evident real divergence afterwards (around the global crisis). The Viterbi path for GDP growth rates in Germany shows a peak in 2009 meaning that in this year the highest differences appeared. The latter outcome is the same as in Poland.

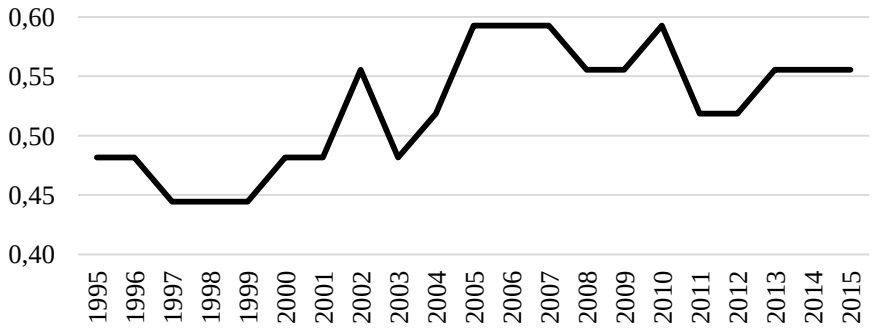

Figure 13. Cyclical convergence of GDP per capita at PPP [gdppc_ppp] between Germany and the other EU countries

Figure 14. Cyclical convergence of GDP growth rates [g_gdp] between Germany and the other EU countries

Conclusions

The analysis confirms that the procedure involving the use of the HMM, the Viterbi path and joining the paths using averages, is a good tool to analyze real convergence. It focuses on different aspects of catching-up as compared with the standard approaches and should be treated as complementary rather than substitutive. The majority of the results are economically justified.

In terms of the GDP per capita at PPP, the results indicate a clear-cut real convergence of Poland and the other EU countries before the beginning of the global crisis, that is from 1995 to 2008. In 2009, the differences between GDP per capita at PPP increased due to the economic and financial crisis. As regards GDP growth rates, there was a sharp increase in the values of averages of the states of the Viterbi paths in the year 2009, meaning that during the crisis there was observed a large rise in differences between GDP growth rates in Poland and the other EU countries, while prior to the global crisis and afterwards there was evident real convergence.

Considering other variables, the results indicate, among others, a clear-cut nominal convergence in inflation years between Poland and the other EU countries throughout the whole analyzed period.

HMM seems to be an effective method of analyzing the macroeconomic time series. Besides the turning point identification and synchronization of the business cycles, the proposed procedure should be considered as new, powerful method that could be extensively explored also in the real convergence studies.

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Zastosowanie ukrytych modeli Markowa w analizie realnej konwergencji

Z a r y s t r e ś c i. Artykuł przedstawia zastosowanie ukrytych modeli Markowa i ścieżki Viterbiego do badania realnej konwergencji (zbieżności). Takie podejście łączy analizę konwergencji cyklicznej i dochodowej. Badanie obejmuje 28 krajów UE, okres 1995–2016 oraz 12 zmiennych makroekonomicznych. Wyniki pokazują m.in. realną zbieżność Polski do pozostałych krajów UE w kategoriach poziomów PKB per capita wg PSN oraz stóp wzrostu PKB, z krótkim okresem dywergencji podczas kryzysu globalnego.

S ł o w a k l u c z o w e: konwergencja; ścieżka Viterbiego; ukryty model Markowa; Unia Europejska; zbieżność.