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Liquidity and Market Microstructure Noise: Evidence from the Pekao Data†

A b s t r a c t. The availability of ultra-high frequency data justifies the use of a continuous-time approach in stock prices modeling. However, this data contain, apart from the information about the price process, a microstructure noise causing a bias in the realized volatility. This noise is connected with all the reality of trade. In the paper we separate the microstructure noise from the price process and determine the noise to signal ratio for the estimates of the realized volatility in the case of the shares of the Polish company Pekao S.A. The results are used to discover the optimal sampling frequency for the realized volatility calculation. Moreover, we check the linkages between the noise and some liquidity measures.

K e y w o r d s: market microstructure, volatility, realized variance, liquidity, stock market, trading volume, high frequency data.

1. Introduction

Continuous-time econometric models are becoming now a standard tool for describing the financial market dynamics. They correspond well to the theoretical models of financial mathematics and can be quite easy estimated due to the availability of ultra-high frequency data. It seems natural that tick-by-tick data are the most useful in the context of continuous-time models. However, it is not the all truth. This kind of data contains apart from the useful information about the price process a noise which, for instance, causes a bias in the daily realized volatility estimates. The sources of the noise are connected with the reality of trade. Dealing with continuous-time models, we make many assumptions that are not satisfied in the real market. They concern time, price process, and market mechanism. The departures of the observed process from these assumptions

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are very often connected with the so-called market microstructure effects. The most known factors of market microstructure are liquidity, nonsynchronous trade, bid-ask spread, discrete-valued price, irregular time intervals between trades, and existence of diurnal pattern (Tsay, 2000). Usually these effects depend on legal regulations, market electronic systems, and traders knowledge and behavior.

Volatility is one of the most important parameter in risk management, derivative pricing and portfolio allocation. Nowadays, one of the most popular and promising estimator of daily volatility is the daily realized variance (Andersen, Bollerslev, 1998; Barndorff-Nielsen, Shephard, 2002). It is calculated as a sum of squared intraday returns and so it depends on the chosen frequency of observations. The frequency of intraday data should be high enough to capture as much as possible of available information and small enough to avoid including a noise into the realized variance estimates. It seems rather obvious that the problem of separating the noise from the „true price" process is of great importance for quality of the daily volatility estimates. In this connection, the most significant microstructure phenomenon is liquidity. The microstructure noise is usually weaker for very liquid shares.

The presented analysis applies the Aït-Sahalia and Yu (2009) approach to separate the microstructure noise from the price process in the case of shares of the Polish company Pekao S.A. Basing on the noise estimates, we determine the noise to signal ratio where the signal is the realized volatility. As a result of the analysis we obtain the optimal sampling frequency for the realized volatility calculation. Since liquidity is considered to be the crucial factor determining the noise level, we try to determine the dependencies between the noise to signal ratio and chosen liquidity measures. Moreover, we apply the signal to noise ratio to compare the strength of market microstructure effects observed in the analyzed Pekao data with that reported from more developed stock markets.

2. Realized Volatility and Market Microstructure Noise

We consider a daily log-price process $Y(t) = \ln(P(t))$ where t is measured in days. Then the logarithmic returns are given by formula $r(t,h) = Y(t) - Y(t-h)$, and the daily realized variance (volatility) (Andersen, Bollerslev, 1998; Barndorff-Nielsen, Shephard, 2002) is defined as

$$RV_t(h) = \sum_{j=1}^{1/h} (r(t-1+jh,h))^2, \tag{1}$$

where h denotes time between two consecutive observations.

The daily volatility σ_t^2 of a financial instrument is defined as the conditional variance of its daily return given the set of information Ω_{t-1} available on day $t-1$, i.e.

$$\sigma_t^2 = E((R_t - E(R_t \mid \Omega_{t-1}))^2 \mid \Omega_{t-1}). \tag{2}$$

Thus the volatility is an unobservable variable. The realized variance (1) is a possible estimator of it.

In the following discussion we assume that $Y(t)$ is described by the following stochastic differential equation

$$dY(t) = \mu(t)dt + \sigma(t)dW(t). \tag{3}$$

Here $W(t)$ denotes a Brownian motion, $\sigma(t)$ is an instantaneous volatility and $\mu(t)$ is a drift function. In such a framework an ideal ex post measure of the daily volatility σ_t^2 is the integrated variance

$$IV(t) = \int_{t-1}^{t} \sigma^2(u)du. \tag{4}$$

From the quadratic variation theory it follows that

$$RV_t(h) \to \int_{t-1}^{t} \sigma^2(u)du, \text{ if } h \to 0. \tag{5}$$

It means that in absence of market microstructure noise the realized variance is a consistent estimator of the integrated variance.

Following Aït-Sahalia and Yu (2009), we assume that the observed price X_t is a sum of the "true price" Y_t and the microstructure noise ε_t:

$$X_t = Y_t + \varepsilon_t, \tag{6}$$

and we are interested in determining the daily volatility σ_t^2 of the Y_t basing on discrete observations obtained in moments $0, \Delta, \ldots, n\Delta = T$.

The model given by (6) is deep-rooted in the market microstructure theory. Many authors consider the noise ε_t as a result of bid-ask spread (Roll, 1984; Huang Stoll, 1996), transaction costs (Huang, Stoll, 1996; Chan, Lakonishok, 1997), discrete price changes (Gottlieb, Kalay, 1985). Manganelli (2005) and Aït-Sahalia and Yu (2009) associate the noise with the low liquidity level. The framework of the presented investigation is based on the Hasbrouck (1993) model according to which the standard deviation of ε_t is a total measure of the market quality.

In the following empirical analysis our main goal is to separate the microstructure noise from the fundamental price and evaluate the share of noise in observed values of the daily realized variance. We can use this result to determine the frequency for intraday returns allowing to minimize bias in the realized variance estimates. Moreover, we try to discover the dependencies between liquidity and microstructure noise by modeling dependence of the later on a variety of liquidity measures.

From now on we assume that the conditional mean of the return process is equal to 0. It means that (3) reduces to

$$dY(t) = \sigma(t)dW(t). \tag{7}$$

Aït-Sahalia, Mykland and Zhang (2005) showed that in the parametric case this model is equivalent to that with constant σ. If $\varepsilon_t = 0$, i.e. if no microstructure noise is present, the observed log returns $r_i = X_{\tau_i} - X_{\tau_{i-1}}$ are i.i.d. $N(0, \sigma^2\Delta)$. The daily realized volatility is then the maximum likelihood estimator for σ^2 and

$$\sqrt{T}(RV(\Delta) - \sigma^2) \xrightarrow[n\to\infty]{} N(0, 2\sigma^4\Delta). \tag{8}$$

In such a case the best estimates of volatility are obtained for the smallest possible Δ (Aït-Sahalia, Yu, 2009; Aït-Sahalia, Mykland, Zhang, 2005).

The situation changes in presence of the microstructure noise. Assume now that the noise ε_t is i.i.d. with mean 0 and variance *a*. Thus the observed log-returns process is MA(1)

$$r_i = Y_{\tau_i} - Y_{\tau_{i-1}} = \sigma(W_{\tau_i} - W_{\tau_{i-1}}) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}} \equiv u_i + \eta u_i, \tag{9}$$

with $u_i \sim \text{i.i.d.}(0, \gamma^2)$, $\text{var}(r_i) = \sigma^2\Delta + 2a^2$ and $\text{cov}(r_i) = -a^2$.

The above dependencies form a theoretical framework for the empirical analysis presented in Section 4.

3. The Data

We consider the Polish bank Pekao S.A. stock returns. The period under scrutiny is from August 8, 2006 to February 13, 2009. The tick-by-tick data are provided by Stooq.pl.

Table 1. Number of observations in the considered frequencies

Type of observations	Number of observations
transactions	361 314
tick by tick	325 177
5 minute	53 520
10 minute	27 160
daily	629

The analysis was performed for 10, 5, 2 and 1 minute observations and for the duration returns which are calculated from transaction data. The time between the closing of the stock exchange and its opening next day is considered as equal to 0.

For the sake of place, we show here only the results for 5 and 10 minute returns. Table 1 contains the information about the number of observations in the considered frequencies. The plot showing the dynamics of daily returns is presented in Figure 1.

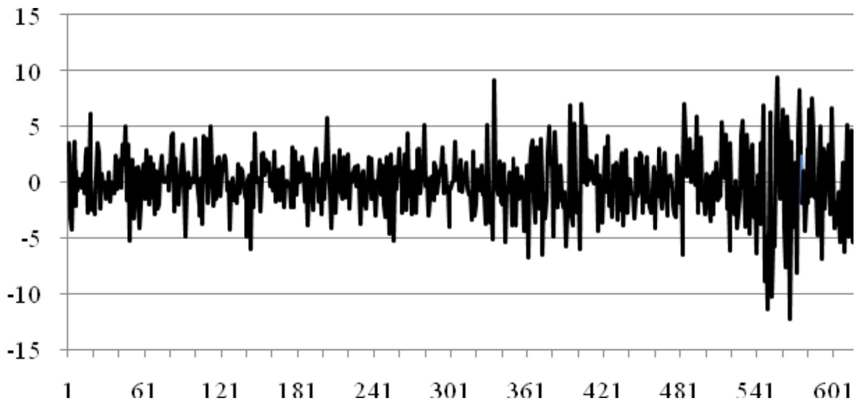

Figure 1. Daily returns of Pekao S.A. Period: August 8, 2006 to February 13, 2009

4. Empirical Results

The steps of the presented analysis are as follows. First we estimate the volatility of the fundamental price process X_t and the variance a_t of the microstructure noise ε_t for each considered day *t*. To evaluate the partition of the noise in the daily realized volatility estimates we calculate for each day the noise to signal ratio (NSR) from the following formula

$$\mathrm{NSR} = \frac{\mathrm{var(noise)}}{\mathrm{var(signal)}}. \tag{10}$$

Noise to signal ratio is a measure commonly used in science to quantify to what degree the observed signal has been corrupted by noise. In market microstructure theory it is often used as a market quality measure (Aït-Sahalia, Yu, 2009) because in some sense it allows to evaluate the level of the market friction.

The next part of the investigation deals with dependence of the market microstructure noise on liquidity. To determine the possible linkages we run two types of regressions. The first one is of the form

$$a_t = c_0 + c_1 x_{t-1} + \nu_t, \tag{11}$$

and captures the impact of liquidity (measured by a variable x_t) on the noise variance a_t. The second one,

$$\mathrm{NSR}_t = c_0 + c_1 x_{t-1} + \nu_t, \tag{12}$$

allows us to establish the connections between the noise to signal ratio and liquidity. The considered liquidity measures are logarithms of the daily volume (*V*), the daily mean transaction volume (*DMTV*), and the number of transactions during a day (*DTN*).

The estimates of the realized volatility based on 5 and 10 minute returns are presented in Figure 2. Figure 3 shows the plot of corresponding noise. Table 2 contains mean values and standard deviations of the daily volatility, variance of noise and noise to signal ratio. The lowest values of noise are obtained for 5 minute returns. The mean level of noise to signal ratio is about $1/3$ and this result is similar to that observed in developed stock markets (Aït-Sahalia, Yu, 2009).

Table 2. Mean and standard deviation of noise and realized volatility

	Frequency	5 min	10 min
a_t	mean	0.1123	0.1468
	standard deviation	0.0953	0.1408
σ_t	mean	2.4584	2.2690
	standard deviation	1.4047	1.3096
NSR	mean	0.3381	0.3314
	standard deviation	0.2784	0.2977

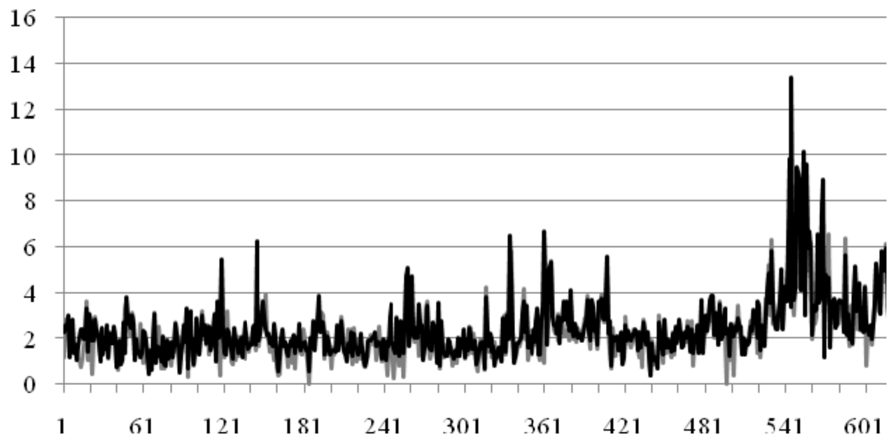

Figure 2. The realized volatility estimates based on 10 minute (grey line) and 5 minute (black line) returns

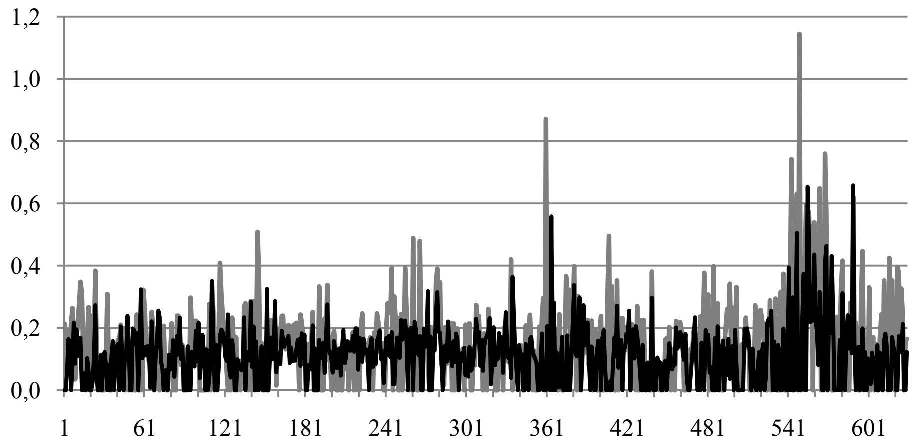

Figure 3. The market microstructure noise estimates for the realized volatility estimates based on 10 minute (grey line) and 5 minute (black line) returns

The results of analysis on the connections between the microstructure noise and liquidity are presented in Table 3.

Table 3. Parameter estimates for regressions (11) and (12)

	a_t			
Explanatory variable	5 min		10 min	
	c_1	R^2	c_1	R^2
Log(DNT)	0.012 (0.008)	0.01	0.042 (0.011)	0.03
Log(MDTV)	-0.005 (0.009)	0.004	0.004 (0.0129)	0.001
Log(V)	0.006 (0.006)	0.002	0.0278 (0.008)	0.02
	NSR			
Explanatory variable	5 min		10 min	
	c_1	R^2	c_1	R^2
Log(DNT)	-0.095 (0.019)	0.1	-0.066 (0.021)	0.02
Log(MVTD)	-0.037 (0.028)	0.03	0.017 (0.030)	0.0005
Log(V)	-0.072 (0.015)	0.03	-0.037 (0.016)	0.01

Surprisingly, the obtained estimates show rather weak connections between the both measures of the noise level and the considered liquidity measures. In the case of 10 minute returns there exists a positive and significant, though not very strong, dependence of the strength of noise and the number of transactions during a day, and the transaction volume. The expectations were that these dependencies should be negative (the higher liquidity, the lower noise). As concerns the noise to signal ratio, a significant negative dependence on the daily number of transaction is in agreement with our early conjecture, but the results concerning the remaining liquidity measures are unexpected. It seems that in the case of Pekao S.A. the measures based on trading volume are not good liquidity measures. Some explanation of this fact can be derived from the plots in Figures 4–5, which show a typical dynamics of returns in days with high and low level of the noise. During the days with high noise to signal ratio the tick-by-tick returns exhibit a very regular pattern caused probably by market makers activity. The high values of volume are presumably connected with this spurious trade. On the other hand, during the days with the noise to signal ratio close to zero the dynamics of the returns is irregular and strong, which is characteristic for the days with high activity of uninformed traders. So, the conclusion is that in the case of analyzed equities the microstructure noise is to a large extent connected with the market makers activity.

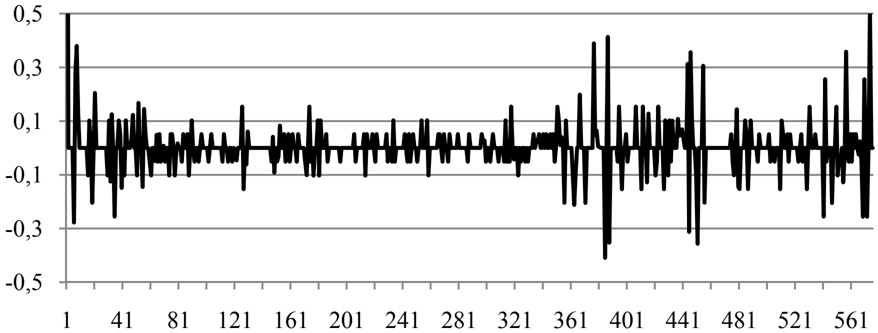

Figure 4. Tick-by-tick returns with the noise to signal ratio equal to 0.91 observed on May 13, 2008

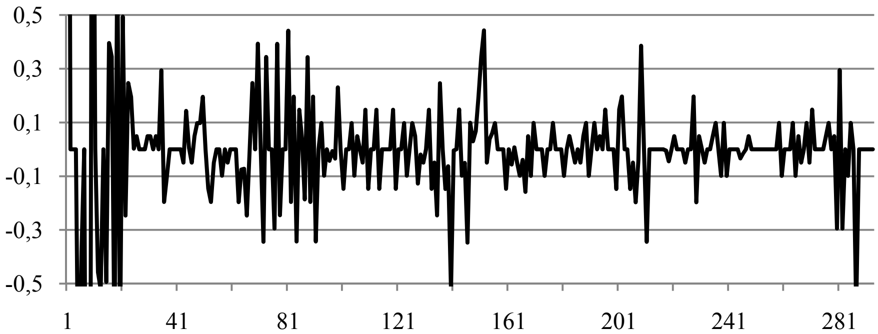

Figure 5. Tick-by-tick returns with the noise to signal ratio equal to 0.03 observed on September 7, 2006

5. Conclusions

Due to the availability of ultra-high frequency data, a continuous-time approach to modeling the stock markets dynamics is still becoming more popular. In fact, many of fruitful research areas in financial econometrics are based on this methodology and use the realized variance as an estimator of true volatility. The daily realized variance is calculated as a sum of squared intraday returns. However, the estimates of volatility obtained in such a way are usually biased due to the presence of the market microstructure noise in the observed data. The market microstructure effects include all the phenomena connected with the reality of the trade that usually contradict the continuous-time model assumptions.

In the paper we considered the quotations of the Polish stock company Pekao S.A. and attempted to separate the market microstructure noise from the observed daily realized variance process. Our main findings are as follows. The best volatility estimates are obtained for 5-minute returns. The market microstructure noise is to a large extent connected with market makers activity. The analyzed liquidity measures (volume, mean volume of transaction, number of transactions during a day) poorly explain the market microstructure noise. The mean level of the noise to signal ratio in the case of the Pekao data is comparable to that observed in developed markets. This result seems to support the opinion about good quality of market regulations and procedures on the Warsaw Stock Exchange.

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Płynność a szum mikrostruktury rynku na przykładzie notowań spółki Pekao

Z a r y s t r e ś c i. Dostępność danych giełdowych o bardzo wysokiej częstotliwości stanowi argument za stosowaniem do opisu dynamiki cen akcji modeli z czasem ciągłym. Jednak dane takie zawierają oprócz informacji na temat procesu ceny także szum mikrostruktury rynku, którego obecność powoduje obciążenie oszacowań zmienności. Szum ten jest związany z rzeczywistymi warunkami, w jakich odbywa się handel. W pracy dokonano oszacowania szumu mikrostruktury rynku w zmienności zrealizowanej cen akcji spółki Pekao SA oraz wyliczono stosunek sygnału do szumu. Wyniki badań wskazują, że optymalna częstotliwość wyliczania stóp zwrotu przy wyznaczaniu zmienności zrealizowanej to częstotliwość pięciominutowa, a obserwowany stosunek sygnału do szumu jest na poziomie zbliżonym do obserwowanego na rozwiniętych rynkach giełdowych. Ponadto, przeprowadzona została analiza powiązań pomiędzy wybranymi miarami płynności a poziomem szumu mikrostruktury rynku.

S ł o w a k l u c z o w e: mikrostruktura rynku, zmienność, wariancja zrealizowana, płynność, rynek giełdowy, wolumen obrotu, dane wysokiej częstotliwości.