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An Evaluation of a Third-Party Logistics Provider: The Application of the Rough Dombi-Hamy Mean Operator

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Abstract: Third-party logistics (3PL) has involved a significant response among researchers and practitioners in the recent decade. In the global competitive scenario, multinational companies (MNCs) not only improve the quality of the service and increase efficiency, but they also decrease costs by means of 3PL. However, the assessment and selection of 3PL is a very critical decision to make, comprising intricacy due to the existence of various imprecisely based criteria. Also, uncertainty is an unavoidable part of information in the decision-making process and its importance in the selection process is relatively high and needs to be carefully considered. Consequently, incomplete and inadequate data or information may occur among other various selection criteria, which can be termed as a multi-criteria decisionmaking (MCDM) problem. Rough numbers are very flexible to model this type of uncertainty occurring in MCDM problems. In this paper, the Hamy Mean (HM) operator and Dombi operations are expanded by rough numbers (RNs) so as to propose the Rough Number Dombi-Hamy Mean (RNDHM) operator. Then, the Multi-Attribute Decision-Making (MADM) model is designed with the RNDHM operator. Finally, the RNDHM is employed to achieve the final ranking of the 3PL providers.

Key words: *MADM*; rough numbers; Rough Dombi-Hamy Mean Operator; third-party logistics.

1. Introduction

The number of transport requests for the relocation of this specific type of goods is inevitably increasing. The basic difference in relation to the transport of the classical types of goods is that, in the transport of dangerous materials, participants are faced with additional safety requirements. This type of transport is specific because of the

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risks that present themselves during the implementation of this logistics process (Tanackov et al., 2018), which may endanger people's life and health, the environment, and material resources due to the dangerous properties of the transported matter and potential reactions (Di Fazio et al., 2016). That is the reason why, at both the national and international levels, the hazardous substance domain is one of the most regulated areas in terms of the set directives tailored for each type of transport.

The companies that produce or use hazardous materials are frequently unable to provide full transport services as they have the specific and varied capacities that must be used in the provision of complete transport services, which is why they often turn to 3PL services. Also, numerous extra security requirements specific to dangerous materials must be met in organizing transport. When systematically viewed in the economic context, there are companies which find it more economically viable to outsource transport to third-party logistics providers (3PL), rather than possess and use their own fleets. When assessing the logistics sector, a number of companies will abolish or reduce their own fleet and hire 3PL service providers to handle transport, with the goal of reducing total costs and increasing profits (Azadi & Saen, 2013). In certain cases in practice, this solution has proven to be correct (Lien & Day, 2017), which is especially interesting as being relevant to the dangerous goods transport field, where the cost is not the only criterion. The evaluation of 3PL providers is a critical step for any manufacturer seeking to select a suitable 3PL provider as a business partner (Jung, 2017). Today, the evaluation of 3PL providers is most often performed by using multi-criteria decision-making methods.

To deal with uncertainties and determine the values of qualitative attributes, the majority of authors use fuzzy sets (Zadeh, 1965) or various extensions of fuzzy theory, such as: interval-valued fuzzy sets (Sizong and Tao, 2016; Zywica, 2016), intuitionistic fuzzy sets (Atanassov, 1986), interval intuitionistic fuzzy sets (Nguyen, 2016), hesitant fuzzy sets (Ngan, 2017), as well as other extensions. Fuzzy sets are a very powerful and commonly used tool for dealing with imprecision. However, when selecting an appropriate membership function for fuzzy sets, subjectivity may affect the final decision, for which reason particular care needs to be taken with that respect (Pamucar et al., 2017; Pamucar et al., 2018). In addition to fuzzy theory, rough set theory, originally introduced by Pawlak (1991), is another suitable tool for treating imprecision. In recent years, rough set theory has been successfully implemented in various fields of human activities. Its application can be said to be adequate and usually irreplaceable when handling uncertainty and inaccuracy analyses. Knowing the advantages of rough set theory (Pawlak 1991), the application of rough sets and rough numbers is fully justified in today's modern practice in the decision-making process when it includes imprecision in data.

Although RNs have been effectively used in some fields, all the existing approaches are unsuitable to depict the interrelationships among any number of RNs assigned by a variable vector. The Hamy mean (HM) operator (Li et al., 2018) is a famous operator able to show interrelationships among any number of arguments assigned by a variable vector. Therefore, the HM operator can assign a robust and flexible mechanism to solve an information fusion in MADM problems. Thus, the HM operator is proposed in this paper in order to overcome these limits. How to aggregate RNs by applying traditional HM operators while simultaneously using the Dombi T-norms and co-norms (Sremac et al., 2018) is an interesting issue. So, the purpose of this paper is to propose the HM operator in order to solve a MADM problem of the evaluation of managers in the human resource department with RNs. In order to do that, the rest of the paper is organized as follows: in Section 2, IRNs are introduced; in Section 3, the HM operator with RNs is developed based on the Dombi operations; in Section 4, an

example of the evaluation of managers in a human resource department with RNs is presented, and finally in Section 5, comments are given at the end of the paper.

2. Rough Numbers and Operations

Rough numbers (RNs), consisting of the upper, lower and boundary intervals, respectively, determine the intervals of multiple experts' evaluations without requiring any additional information, only relying on original data (Stevic et al., 2018). Hence, the obtained experts' preferences objectively represent and improve the decision-making process. The definition of RNs according to Song et al. (2013) is given below.

Let U be a universe containing all the objects and let X be a random object from U. Then, a set of k classes is assumed to exist, which represents the DM's preferences, $R = (J_1, J_2, ..., J_k)$, with the condition $J_1 < J_2 < ..., < J_k$. Then, for every $X \in U$, $J_q \in R$, $1 \le q \le k$, the lower approximation $\underline{Apr}(J_q)$, the upper approximation $\overline{Apr}(J_a)$ and the boundary interval $Bnd(J_q)$ are determined as follows:

$$\underline{Apr}(J_q) = \bigcup \left\{ X \in U \,/\, R(X) \le J_q \right\} \tag{1}$$

$$\overline{Apr}(J_q) = \bigcup \left\{ X \in U \,/\, R(X) \ge J_q \right\}$$
⁽²⁾

$$Bnd(J_q) = \bigcup \left\{ X \in U / R(X) \neq J_q \right\} = \left\{ X \in U / R(X) > J_q \right\} \cup \left\{ X \in U / R(X) < J_q \right\}$$
(3)

The object can be represented by a rough number with the lower limit $\underline{Lim}(J_q)$ and the upper limit $\overline{Lim}(J_q)$ in Eqs. (4)-(5).

$$\underline{Lim}(J_q) = \frac{1}{M_L} \sum R(X) | X \in \underline{Apr}(J_q)$$
(4)

$$\overline{Lim}(J_q) = \frac{1}{M_U} \sum R(X) \left| X \in \overline{Apr}(J_q) \right|$$
(5)

where M_L and M_U represent the sum of the objects given in the lower and upper object approximations of J_q , respectively. For the object J_q , the rough boundary interval ($IRBnd(J_q)$) is the interval between the lower and upper limits (Pamucar et al., 2018). The rough boundary interval is a measure of uncertainty. A bigger $IRBnd(J_q)$ value shows that there are variations in experts' preferences, while smaller values show that experts' opinions do not considerably differ. All the objects between the lower limit $\underline{Lim}(J_q)$ and the upper limit $\overline{Lim}(J_q)$ of the rough number $RN(J_q)$ are included in $IRBnd(J_q)$. Since RNs belong to a group of interval numbers, arithmetic operations applied to interval numbers are also appropriate for RNs.

3. The Dombi Operations of RNs and the Dombi-Hamy Mean Operator With Rough Numbers

Definition 1. Let *p* and *q* be any two real numbers. Then, the Dombi T-norm and T-conorm between *p* and *q* are defined as follows (Dombi, 1982):

$$O_{D}(p,q) = \frac{1}{1 + \left\{ \left(\frac{1-p}{p}\right)^{\rho} + \left(\frac{1-q}{q}\right)^{\rho} \right\}^{1/\rho}}$$
(6)
$$O_{D}^{c}(p,q) = 1 - \frac{1}{1 + \left\{ \left(\frac{p}{1-p}\right)^{\rho} + \left(\frac{q}{1-q}\right)^{\rho} \right\}^{1/\rho}}$$
(7)

where $\rho > 0$ and $(p,q) \in [0,1]$.

In accordance with the Dombi *T*-norm and the *T*-conorm, Dombi operations are defined by rough numbers.

$$\begin{array}{ll} Definition & 2. \\ RN(\varphi_2) = [\underline{Lim}(\varphi_2), \overline{Lim}(\varphi_2)] & \text{are two rough numbers, } \rho, \gamma > 0 & \text{and let} \\ f\left(RN(\varphi_i)\right) = \left[f\left(\underline{Lim}(\varphi_i)\right), f\left(\overline{Lim}(\varphi_i)\right)\right] = \left[\frac{\underline{Lim}(\varphi_i)}{\sum_{i=1}^{n}\underline{Lim}(\varphi_i)}, \frac{\overline{Lim}(\varphi_i)}{\sum_{i=1}^{n}\overline{Lim}(\varphi_i)}\right] & \text{be a rough} \end{array}$$

function, then some operational laws of rough numbers based on the Dombi *T*-norm and *T*-conorm can be defined as follows:

(1) Addition "+"

$$RN(\varphi_{1}) + RN(\varphi_{2}) = \begin{bmatrix} \sum_{j=1}^{2} \underline{Lim}(\varphi_{j}) - \frac{\sum_{j=1}^{2} \underline{Lim}(\varphi_{j})}{1 + \left\{ \left(\frac{f(\underline{Lim}(\varphi_{1}))}{1 - f(\underline{Lim}(\varphi_{1}))} \right)^{\rho} + \left(\frac{f(\underline{Lim}(\varphi_{2}))}{1 - f(\underline{Lim}(\varphi_{2}))} \right)^{\rho} \right\}^{\nu_{\rho}}, \\ \sum_{j=1}^{2} \overline{Lim}(\varphi_{j}) - \frac{\sum_{j=1}^{2} \overline{Lim}(\varphi_{j})}{1 + \left\{ \left(\frac{f(\overline{Lim}(\varphi_{1}))}{1 - f(\overline{Lim}(\varphi_{1}))} \right)^{\rho} + \left(\frac{f(\overline{Lim}(\varphi_{2}))}{1 - f(\overline{Lim}(\varphi_{2}))} \right)^{\rho} \right\}^{\nu_{\rho}}} \end{bmatrix}$$
(8)

(2) Multiplication "×"

$$RN(\varphi_{1}) \times RN(\varphi_{2}) = \begin{bmatrix} \frac{\sum_{j=1}^{2} \underline{Lim}(\varphi_{j})}{1 + \left\{ \left(\frac{1-f(\underline{Lim}(\varphi_{1}))}{f(\underline{Lim}(\varphi_{1}))} \right)^{\rho} + \left(\frac{1-f(\underline{Lim}(\varphi_{2}))}{f(\underline{Lim}(\varphi_{2}))} \right)^{\rho} \right\}^{1/\rho}}, \\ \frac{\sum_{j=1}^{2} \overline{Lim}(\varphi_{j})}{1 + \left\{ \left(\frac{1-f(\overline{Lim}(\varphi_{1}))}{f(\overline{Lim}(\varphi_{1}))} \right)^{\rho} + \left(\frac{1-f(\overline{Lim}(\varphi_{2}))}{f(\overline{Lim}(\varphi_{2}))} \right)^{\rho} \right\}^{1/\rho}} \end{bmatrix}$$
(9)

(3) Scalar multiplication, where $\gamma > 0$

$$\gamma RN(\varphi_{1}) = \begin{bmatrix} \underline{Lim}(\varphi_{1}) - \frac{\underline{Lim}(\varphi_{1})}{1 + \left\{ \gamma \left(\frac{\underline{Lim}(\varphi_{1})}{1 - \underline{Lim}(\varphi_{1})} \right)^{\rho} \right\}^{1/\rho}}, \\ \underline{Lim}(\varphi_{1}) - \frac{\overline{Lim}(\varphi_{1})}{1 + \left\{ \gamma \left(\frac{\overline{Lim}(\varphi_{1})}{1 - \overline{Lim}(\varphi_{1})} \right)^{\rho} \right\}^{1/\rho}} \end{bmatrix}$$
(10)

(4) Power, where $\gamma > 0$

$$\left\{RN(\varphi_{1})\right\}^{\gamma} = \left[\frac{\underline{Lim}(\varphi_{1})}{1 + \left\{\gamma\left(\frac{1-\underline{Lim}(\varphi_{1})}{\underline{Lim}(\varphi_{1})}\right)^{\rho}\right\}^{\nu_{\rho}}}, \frac{\overline{Lim}(\varphi_{1})}{1 + \left\{\gamma\left(\frac{1-\overline{Lim}(\varphi_{1})}{\overline{Lim}(\varphi_{1})}\right)^{\rho}\right\}^{\nu_{\rho}}}\right]$$
(11)

The Hamy mean (HM) (Hara et al., 1998) is used to aggregate values, simultaneously including mutual correlations among multiple arguments, and is defined in the following way:

Definition 3 (Hara et al., 1998). Assume that x_i (i = 1, 2, ..., n) represents a set of nonnegative real numbers and the parameter k = 1, 2, ..., n, then the HM is defined as:

$$HM^{(k)}(x_1, x_2, ..., x_n) = \frac{1}{\binom{n}{k}} \sum_{1 \le i_1 < ... < i_k \le n} \left(\prod_{j=1}^k x_{i_j} \right)^{1/k}$$
(12)

where $(i_1, i_2, ..., i_k)$ includes all the *k*-tuple combinations of (1, 2, ..., n) and $\binom{n}{k}$ represents a binomial coefficient calculated as:

 $\binom{n}{n}$ $\binom{n}{n}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{13}$$

Based on the RN operators (8)-(11), the RN Dombi-Hamy mean (RNDHM) operator is proposed.

Theorem 1. Let $RN(\varphi_j) = [\underline{Lim}(\varphi_j), \overline{Lim}(\varphi_j)]; (j = 1, 2, ..., n)$, the collection of RNs in *R*, then the RNDHM operator is defined as follows:

where
$$f(RN(\varphi_i)) = \begin{cases} f(\underline{Lim}(\varphi_i)) = \underline{Lim}(\varphi_i) / \sum_{i=1}^{n} \underline{Lim}(\varphi_i); \\ f(\overline{Lim}(\varphi_i)) = \overline{Lim}(\varphi_i) / \sum_{i=1}^{n} \overline{Lim}(\varphi_i). \end{cases}$$
 represents a rough

function.

The proof. Based on the Dombi operations of the RN, the following is obtained:

$$\mathbf{a} \quad \prod_{j=1}^{k} RN(\varphi_{i_{j}}) = \left[\frac{\underline{Lim}(\varphi_{i_{j}})}{1 + \left\{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)}\right)^{\rho}\right\}^{\nu\rho}}, \frac{\overline{Lim}(\varphi_{i_{j}})}{1 + \left\{\sum_{j=1}^{k} \left(\frac{1 - f\left(\overline{Lim}(\varphi_{i_{j}})\right)}{f\left(\overline{Lim}(\varphi_{i_{j}})\right)}\right)^{\rho}\right\}^{\nu\rho}}\right]$$

Thus,

$$\left(\prod_{j=1}^{k} RN(\varphi_{i_{j}})\right)^{1/k} = \left[\frac{\underline{Lim}(\varphi_{i_{j}})}{1 + \left\{\frac{1}{k}\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\overline{Lim}(\varphi_{i_{j}})}{1 + \left\{\frac{1}{k}\sum_{j=1}^{k} \left(\frac{1 - f\left(\overline{Lim}(\varphi_{i_{j}})\right)}{f\left(\overline{Lim}(\varphi_{i_{j}})\right)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

b) Thereafter,

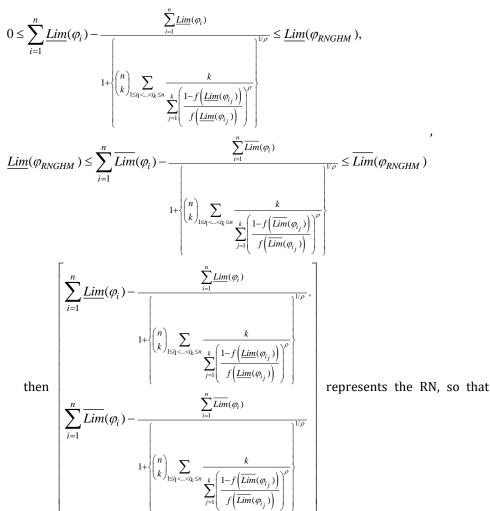
$$\sum_{1 \leq i_1 < \ldots < i_k \leq n} \left(\prod_{j=1}^k RN(\varphi_{i_j}) \right)^{1/k} = \begin{bmatrix} \underline{Lim}(\varphi_{i_j}) - \frac{\underline{Lim}(\varphi_{i_j})}{1 + \left\{ \sum_{1 \leq i_1 < \ldots < i_k \leq n} \frac{k}{\sum_{j=l}^k \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_j})\right)}{f\left(\underline{Lim}(\varphi_{i_j})\right)} \right)^{\rho} \right\}^{1/\rho}} \\ \overline{Lim}(\varphi_{i_j}) - \frac{\overline{Lim}(\varphi_{i_j})}{1 + \left\{ \sum_{1 \leq i_l < \ldots < i_k \leq n} \frac{k}{\sum_{j=l}^k \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_j})\right)}{f\left(\underline{Lim}(\varphi_{i_j})\right)} \right)^{\rho} \right\}^{1/\rho}} \end{bmatrix}$$

c) Therefore,

$$RNDHM^{(k)} \left\{ RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n) \right\}^{k,\rho} = \frac{1}{\binom{n}{k}} \sum_{1 \le i_1 < ... < i_k \le n} \left(\prod_{j=1}^k RN(\varphi_{i_j}) \right)^{1/k}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} \underline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \sum_{1 \leq i_{1} \leq \ldots \leq i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)} \right)^{\rho}} \right\}^{1/\rho}, \\ \sum_{i=1}^{n} \overline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \overline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \sum_{1 \leq i_{1} < \ldots < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\overline{Lim}(\varphi_{i_{j}})\right)}{f\left(\overline{Lim}(\varphi_{i_{j}})\right)} \right)^{\rho}} \right\}^{1/\rho}} \end{bmatrix}$$

Besides, since



Theorem 1 has been proven and Equation (14) is correct.

The RNDHM operator also contains the following properties: *Idempotency, Boundedness* and *Monotonicity*.

Theorem 2. (Idempotency). If $RN(\varphi_j) = RN(\varphi) = [\underline{Lim}(\varphi), \overline{Lim}(\varphi)]$ for all (j = 1, 2, ..., n) then $RNHM \{RN(\varphi), RN(\varphi), ..., RN(\varphi)\} = RN(\varphi) = [\underline{Lim}(\varphi), \overline{Lim}(\varphi)]$.

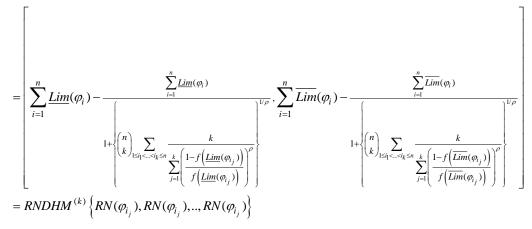
The proof. Since $RN(\varphi_j) = [\underline{Lim}(\varphi_j), \overline{Lim}(\varphi_j)] = RN(\varphi)$; (j = 1, 2, ..., n), then the following calculations are obtained by using *Theorem 1*:

$$\begin{split} & \text{RNDHM}^{(k)} \left\{ \text{RN}(\varphi), \text{RN}(\varphi), \dots, \text{RN}(\varphi) \right\} = \left[\underline{Lim}(\varphi), \overline{Lim}(\varphi) \right] = \\ & = \left[\sum_{i=1}^{n} \underline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \sum_{1 \leq i_{1} \leq \dots \leq i_{k} \leq n} \frac{k}{\sum_{j=1}^{n} \left\{ \frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)} \right\}^{p}} \right\}^{1/p}} \right] \\ & = \left[\sum_{i=1}^{n} \overline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \overline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \sum_{1 \leq i_{1} \leq \dots \leq i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\overline{Lim}(\varphi_{i_{j}})\right)}{f\left(\overline{Lim}(\varphi_{i_{j}})\right)} \right)^{p}} \right\}^{1/p}} \right] \\ & = \left[\sum_{i=1}^{n} \underline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \left(\underline{Lim}(\varphi_{i})\right)} \right\}^{1/p}}, \sum_{i=1}^{n} \overline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \overline{Lim}(\varphi_{i})}{1 + \left\{ \binom{n}{k} \left(\overline{Lim}(\varphi_{i_{j}})\right)} \right\}^{1/p}} \right] \\ & = \left[\underline{Lim}(\varphi), \overline{Lim}(\varphi) \right] = RN(\varphi) \end{split}$$

Theorem 3. (*Commutativity*). Let the RN rough set $(RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n))$ be any permutation of $(RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n))$. Then, there is $RNDHM \{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\} = RNDHM \{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\}$. *The proof.* Based on *Definition 2*, the conclusion is obvious:

$$RNDHM^{(k)}\left\{RN(\phi_{1}^{'}), RN(\phi_{2}^{'}), ..., RN(\phi_{n}^{'})\right\}$$

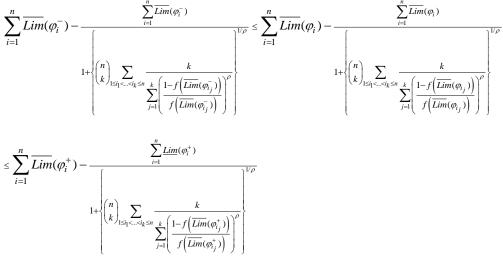
$$= \left[\sum_{i=1}^{n} \underline{Lim}(\phi_{i}^{'}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\phi_{i}^{'})}{1 + \left\{\binom{n}{k} \sum_{1 \leq i_{1} < ... < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\phi_{i_{j}}^{'})\right)}{f\left(\underline{Lim}(\phi_{i_{j}}^{'})\right)}\right)^{\rho}}\right\}^{V,\rho}}, \sum_{i=1}^{n} \overline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \overline{Lim}(\phi_{i}^{'})}{1 + \left\{\binom{n}{k} \sum_{1 \leq i_{1} < ... < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\phi_{i_{j}}^{'})\right)}{f\left(\underline{Lim}(\phi_{i_{j}}^{'})\right)}\right)^{\rho}}\right\}^{V,\rho}}\right]$$



Theorem 4. (Boundedness). Let $RN(\varphi_j) = [\underline{Lim}(\varphi_j), \overline{Lim}(\varphi_j)]; (j = 1, 2, ..., n)$, tte collection of RNs in R, let $RN(\varphi_j^-) = \left[\min\left\{\underline{Lim}(\varphi_j)\right\}, \min\left\{\overline{Lim}(\varphi_j)\right\}\right]$ and $RN(\varphi_j^+) = \left[\max\left\{\underline{Lim}(\varphi_j)\right\}, \max\left\{\overline{Lim}(\varphi_j)\right\}\right], \max\left\{\overline{Lim}(\varphi_j)\right\}\right],$ then $RN(\varphi_j^-) \le RNHM\left\{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\right\} \le RN(\varphi_j^+).$

 $RN(\varphi_{j}^{-}) = \min \left\{ RN(\varphi_{1}), RN(\varphi_{2}), ..., RN(\varphi_{n}) \right\} = \left[\underline{Lim}(\varphi_{j}^{-}), \overline{Lim}(\varphi_{j}^{-}) \right]$ $RN(\varphi_{j}^{+}) = \max \left\{ RN(\varphi_{1}), RN(\varphi_{2}), ..., RN(\varphi_{n}) \right\} = \left[\underline{Lim}(\varphi_{j}^{+}), \overline{Lim}(\varphi_{j}^{+}) \right].$ Then, $\underline{Lim}(\varphi_{j}^{-}) = \min_{j} \left\{ \underline{Lim}(\varphi_{j}) \right\}, \quad \overline{Lim}(\varphi_{j}^{-}) = \min_{j} \left\{ \overline{Lim}(\varphi_{j}) \right\}, \quad \underline{Lim}(\varphi_{j}^{+}) = \max_{j} \left\{ \underline{Lim}(\varphi_{j}) \right\} \text{ and }$ $\overline{Lim}(\varphi_{j}^{+}) = \max_{i} \left\{ \overline{Lim}(\varphi_{j}) \right\}.$ Based on that, it follows:

$$\begin{split} &\sum_{i=1}^{n} \underline{Lim}(\varphi_{i}^{-}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i}^{-})}{1 + \left\{ \begin{pmatrix} n \\ k \end{pmatrix}_{1 \leq i_{1} < \dots < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)} \right)^{\rho} \right\}^{V,\rho} \leq \sum_{i=1}^{n} \underline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i})}{1 + \left\{ \begin{pmatrix} n \\ k \end{pmatrix}_{1 \leq i_{1} < \dots < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)} \right)^{\rho} \right\}^{V,\rho}} \\ &\leq \sum_{i=1}^{n} \underline{Lim}(\varphi_{i}) - \frac{\sum_{i=1}^{n} \underline{Lim}(\varphi_{i}^{-})}{1 + \left\{ \begin{pmatrix} n \\ k \end{pmatrix}_{1 \leq i_{1} < \dots < i_{k} \leq n} \frac{k}{\sum_{j=1}^{k} \left(\frac{1 - f\left(\underline{Lim}(\varphi_{i_{j}})\right)}{f\left(\underline{Lim}(\varphi_{i_{j}})\right)} \right)^{\rho} \right\}^{V,\rho}} \end{split}$$



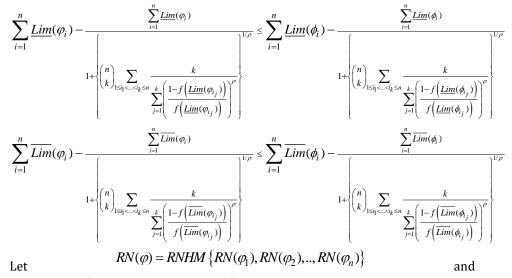
According to the foregoing inequalities, it can be concluded that $RN(\varphi_j^-) \leq RNHM \{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\} \leq RN(\varphi_j^+)$.

Theorem 5. (Monotonicity). Let $RN(\varphi_i) = [\underline{Lim}(\varphi_i), \overline{Lim}(\varphi_i)]$,

 $RN(\phi_j) = [\underline{Lim}(\phi_j), \overline{Lim}(\phi_j)]; (j = 1, 2, ..., n), \text{ be the two collections of RNs, if}$ $\underline{Lim}(\phi_j) \leq \underline{Lim}(\phi_j), \ \overline{Lim}(\phi_j) \leq \overline{Lim}(\phi_j) \text{ for all } j \text{ , then}$

 $RNHM\left\{RN(\phi_1), RN(\phi_2), ..., RN(\phi_n)\right\} \le RNHM\left\{RN(\phi_1), RN(\phi_2), ..., RN(\phi_n)\right\}.$

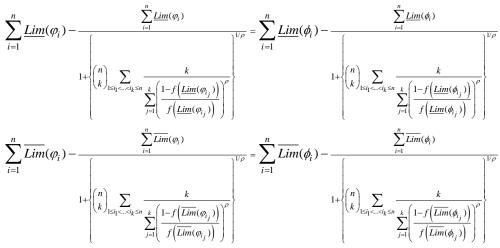
The proof. Since $0 \le \underline{Lim}(\phi_j) \le \underline{Lim}(\phi_j)$, $0 \le \overline{Lim}(\phi_j) \le \overline{Lim}(\phi_j)$, then, based on *Theorem 1*, the following is obtained:



 $RN(\phi) = RNHM \{RN(\phi_1), RN(\phi_2), ..., RN(\phi_n)\}$ be two RNs, then:

(1) If $\underline{Lim}(\varphi) \leq \underline{Lim}(\phi)$ and $\overline{Lim}(\varphi) \leq \overline{Lim}(\phi)$, then the following is obtained: $RNHM\left\{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\right\} \leq RNHM\left\{RN(\phi_1), RN(\phi_2), ..., RN(\phi_n)\right\}_{:}$

(2) If $\underline{Lim}(\phi) = \underline{Lim}(\phi)$ and $\underline{Lim}(\phi) = \underline{Lim}(\phi)$, then it can be concluded that there are the following equalities:



Finally, it can be concluded that there is the following inequality $RNHM\left\{RN(\varphi_1), RN(\varphi_2), ..., RN(\varphi_n)\right\} \le RNHM\left\{RN(\phi_1), RN(\phi_2), ..., RN(\phi_n)\right\}.$

4. A Numerical Example: An Evaluation of a Third-Party Logistics Provider

The practical application of the RNDHM aggregator is a logical conclusive phase of the subject-matter research. The multi-criteria model consists of a total of the five logistics providers who represent the alternatives having been evaluated based on the four criteria presented in detail in Table 1. The research study involved five experts, who evaluated the significance of the criteria, i.e. the alternatives according to the given criteria. The experts used a 1-9 scale for the evaluation of 3PL.

No	Criteria	Description of the criteria
1.	Financial stability	The logistics provider delivers a financial report on positive business practices and a certificate of the settlement of tax liabilities, i.e. a certificate issued by the tax authorities, which confirms no extant debts in connection with the payment of taxes, fees, etc.
2.	The professionalization of drivers	The logistics provider employs the drivers who have a certificate of approval for dangerous goods transport. The ongoing education of drivers is organized through seminars, additional and specialist training programs, safe and eco-friendly driving programs, etc.
3.	The cost of transport	Covers all costs incurred by the carrier during the performance of services.
4.	The application of risk mitigation measures	The logistics provider has one or more designated security advisors for the transport of dangerous

Table 1. The criteria based on which the logistics provider was evaluated

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No	Criteria	Description of the criteria
		goods. At points of loading/unloading, the property
		of the contracting authority or third parties is treated
		with special care. The carrier acquaints his workers
		with the rules of conduct, movement, safety, fire
		protection, and all other measures and procedures.

The five logistics providers *Ai* (*i*=1, 2, 3, 4, 5) were evaluated: C_1 C_2 C_3 C_4

	C1	C2	<i>C</i> 3	C4	
A_1 A_2 $X^{(e)} = A_3$	4,5,6,8	8,8,9,8	8, 8, 9, 7	8,8,9,9	
A_2	5,5,5,5	7,8,6,9	8, 8, 8, 7	9,9,6,9	
$X^{(e)} = A_3$	7,5,8,4	9,9,9,7	7,7,8,9	6,6,7,6	; $1 \le e \le m$
A_4	6,5,8,3	7,5,6,8 4,4,5,8	5, 5, 5, 6	9,9,8,7	
A_5	5,5,6,4	4,4,5,8	9,9,9,8	6, 6, 8, 6	

where *m* (in our case *m*=4) is the number of experts. By applying the rules for the transformation of crisp numbers into rough numbers, the elements of the matrix $X^{(e)}$ are transformed into RNs and presented in the matrix $X^{(e)}_{_{RN}}$ (Table 2).

Alt./Crit.	C1	C2
A1	[4.0,5.75];[4.5,6.33];[5,7];[5.75,8]	[8,8.25];[8,8.25];[8.25,9];[8,8.25]
A2	[5.0,5.0]; [5.0,5.0]; [5.0,5.0]; [5.0,5.0]	[6.5,8.0];[7,8.5];[6.0,7.5];[7.5,9.0]
A3	[5.33,7.5];[4.5,6.67];[6,8];[4.0,6.0]	[8.5,9.0];[8.5,9.0];[8.5,9.0];[7.0,8.5]
A4	[4.67,7];[4,6.33];[5.5,8];[3,5.5]	[6.0,7.5];[5.0,6.5];[5.5,7.0];[6.5,8.0]
A5	[4.67,5.33];[4.67,5.33];[5,6];[4,5]	[4,5.25];[4,5.25];[4.33,6.5];[5.25,8]
	22	
Alt./Crit.	C3	C4
Alt./Crit.	<u> </u>	<u> </u>
	60	01
A1	[8,8.25];[8,8.25];[8.25,9];[8,8.25]	[8,8.5];[8,8.5];[8.5,9];[8.5,9]
A1 A2	[8,8.25];[8,8.25];[8.25,9];[8,8.25] [7.75,8];[7.75,8];[7.75,8];[7,7.75]	[8,8.5];[8,8.5];[8.5,9];[8.5,9] [8.25,9];[8.25,9];[6,8.25];[8.25,9]
A1 A2 A3	[8,8.25];[8,8.25];[8.25,9];[8,8.25] [7.75,8];[7.75,8];[7.75,8];[7,7.75] [7,7.75];[7,7.75];[7.33,8.5];[7.75,9]	[8,8.5];[8,8.5];[8.5,9];[8.5,9] [8.25,9];[8.25,9];[6,8.25];[8.25,9] [6,6.25];[6,6.25];[6.25,7];[6,6.25]

Table 2. The expert rough decision matrix ($X_{RN}^{(e)}$)

The aggregated initial rough decision matrix (X_{RN}) is obtained by using the RNDHM aggregator. Thus, based on Eq. (14), the element $RN(x_{11})$ in the matrix $X_{RN}^{(e)} = [RN(x_{ij})]_{5\times4}$ is obtained as follows:

$$RNDHMA\left\{RN(x_{11}^{(1)}), RN(x_{11}^{(2)}), RN(x_{11}^{(3)}), RN(x_{11}^{(4)})\right\} = \left\{\left[4, 5.75\right]; \left[4.5, 6.33\right]; \left[5, 7\right]; \left[5.75, 8\right]\right\}$$

$$\begin{bmatrix} \underline{Lim}(x_{11}) = 19.25 - \frac{19.25}{\left(1 + \left(\frac{2}{\binom{4}{2}} \times \left(\frac{1}{3.81^{1} + 3.28^{1}} + \frac{1}{3.81^{1} + 2.85^{1}} + \frac{1}{3.81^{1} + 2.35^{1}} + \frac{1}{3.28^{1} + 2.85^{1}} + \frac{1}{3.28^{1} + 2.35^{1}} + \frac{1}{3.28^{1} + 2.35^{1}} + \frac{1}{2.85^{1} + 2.35^{1}}\right) \right)^{1/1}} = 4.76$$

$$\boxed{Lim}(x_{11}) = 27.08 - \frac{27.08}{\left(1 + \left(\frac{2}{\binom{4}{2}} \times \left(\frac{1}{3.71^{1} + 3.28^{1}} + \frac{1}{3.71^{1} + 2.87^{1}} + \frac{1}{3.71^{1} + 2.39^{1}} + \frac{1}{3.28^{1} + 2.87^{1}} + \frac{1}{3.28^{1} + 2.39^{1}} +$$

where the values $f(RN(x_{11}^{(e)}))$; $1 \le e \le 4$ represent the rough functions. In a similar way, the other aggregated elements of the matrix X_{RN} are obtained.

$$K1 K2 K3 K4 A_1 \begin{bmatrix} 4.76; 6.71 & 8.06; 8.43 & 8.06; 8.43 & 8.25; 8.75 \\ A_2 & 5.00; 5.00 & 6.72; 8.23 & 7.55; 7.94 & 7.62; 8.81 \\ \begin{bmatrix} 4.89; 7.00 & 8.10; 8.87 & 7.26; 8.23 & 6.06; 6.43 \\ A_4 & 4.18; 6.64 & 5.72; 7.23 & 5.06; 5.43 & 7.73; 8.72 \\ A_5 & 4.57; 5.4 & 4.36; 6.14 & 8.56; 8.94 & 6.12; 6.84 \end{bmatrix}$$

In the following steps, the final values of the criteria functions of the alternatives were obtained and ranked by applying additive normalization and weighting the normalized matrix using the following equation $Q(A_i) = \sum_{i=1}^{n} x_{ij} \cdot w_j$, where x_{ij} represents the elements of the normalized matrix and $w_j = (0.2, 0.22, 0.28, 0.30)^T$ represents the criteria weights. Thus, the alternative score functions: $Q(A_i) = [0.2; 0.25], \quad Q(A_2) = [0.18; 0.23], \quad Q(A_3) = [0.18; 0.24], \quad Q(A_4) = [0.15; 0.22] \text{ and } Q(A_5) = [0.16; 0.21]$ were obtained. It is desirable for an alternative to have the highest possible value of its criteria function, so $A_1 > A_2 > A_3 > A_5 > A_4$.

5. Conclusion

Dangerous goods transport and its potential consequences arouse the attention of the public because of the detrimental effect of these materials on the environment, as well as people, and possible accidents, too. In order to meet the complex demands of today's market, the hazardous substance increased consumption trend has appeared, thereby increasing the volume of the production and transport of these goods. Transport is a mandatory logistic activity for supplying users with these materials, regardless of whether they are in a raw, semi-processed, or fully-processed form.

In this paper, MADM problems with RNs were subjected to investigation, which was only followed by the utilization of the HM operator and the Dombi operations in order to design an HM operator with RNs, i.e. the RNDHM operator. After that, the RNDHM was used so as to propose a model for MADM problems with RNs. Finally, a real example of the evaluation of 3PL was used in order to show the developed approach.

The procedure for selecting 3PL carriers is particularly specific in the field of dangerous substances due to high risks and additional security requirements. This reflects in the criteria defined for the evaluation of the logistics providers. One of the benefits of using 3PL carriers' services is that they have a wider range of operations and are able to satisfy the clients who have a high frequency of transport needs on a weekly basis. Specialized carriers better understand the market and customers' needs. They also have well-designed strategies and business models to continue improving their offers, regional coverage, and specialization in all industry sectors.

A direction for a further research study implies supporting the environmental sustainability of 3PL dangerous material providers, particularly in the areas marked as the five topical areas: influencing factors, green actions, an impact on performance, information and communication technology (ICT) tools supporting green actions, energy efficiency in road freight transport, and shippers' perspective and

collaboration. In subsequent studies, the extension and application of RNs needs to be studied in many other uncertain environments and other applications.

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