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Verification of Information Flow Properties under Rational Observation

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Abstract: Information flow properties express the capability for an agent to infer information about secret behaviours of a partially observable system. In a languagetheoretic setting, where the system behaviour is described by a language, we define the class of rational information flow properties (RIFP), where observers are modeled by finite transducers, acting on languages in a given family \mathscr{L} . This leads to a general decidability criterion for the verification problem of RIFPs on \mathscr{L} , implying PSPACE-completeness for this problem on regular languages. We show that most trace-based information flow properties studied up to now are RIFPs, including those related to selective declassification and conditional anonymity. As a consequence, we retrieve several existing decidability results that were obtained by ad-hoc proofs.

Keywords: Information flow, Security predicates, Opacity, Declassification, Conditional anonymity, Rational transducers, Formal verification.

1 Introduction

Motivations. Generic models for information flow properties aim at expressing, in a uniform setting, the various capabilities of observers to infer information from partially observable systems. These models provide a description of the system behaviour, a parametric description of the observation by the environment and the secret parts of the system, and a security criterion. A security property is an instantiation of such a model, with the goal of avoiding a particular information flow. Generic models have been thoroughly investigated, for instance in [Man00, FG01, BKMR08]. They propose various classifications and comparisons of security properties, either for transition systems or directly for traces. In the case of transition systems [FG01, BKMR08], the branching structure permits to express security properties as

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equivalences like weak or strong (bi-)simulations. For trace-based models, properties are stated as relations between languages, also called security predicates in [Man00].

In addition to classification, an important question about security properties concern their verification: given a system *S* and a security property *P*, does *S* satisfy *P*? Since [FG01], much attention has been given to such questions for various classes of systems (or their sets of traces) and security properties [BKMR08, DHRS11, CDM12, DFK⁺12, BD12, MY14, CFK⁺14]. This is the problem we consider in this work, for a subclass of trace-based information flow properties.

Contributions. We first introduce the class of Rational Information Flow Properties (RIFP), in a language-theoretic setting. In this class, observations are modeled by rational transducers, called here rational observers. For a language L in some family of languages \mathcal{L} , an RIFP is then defined as an inclusion relation $L_1 \subseteq L_2$, where L_1 and L_2 are obtained from L by inductively applying rational observers, unions and intersections. This mechanism produces the set of properties $RIF(\mathcal{L})$, and a generic decidability result can be stated for the verification problem of these properties. In the particular case of the family $\Re eg$ of regular languages, generated by finite automata (also called labelled transition systems), we obtain a PSPACE-complete verification problem for the class $RIF(\mathcal{R}eg)$. We then proceed to show that this result subsumes most existing decidability results for security properties on regular languages, thus establishing the pertinency of our model. This involves expressing properties in our formalism by designing suitable rational observers. We first consider the particular case where observations are functions and we show that opacity properties with regular secrets are RIFPs. To illustrate the expressiveness of RIFPs, we introduce a subclass of functional rational observers that we call rational Orwellian observers and show that several properties including *intransitive non-interference* and selective intransitive non-interference for a language $L \in \mathscr{L}$ are in $RIF(\mathscr{L})$. We also reduce their verification to the verification of opacity w.r.t. Orwellian observers. These observers are more powerful than those considered so far in literature as they model not only observers constrained to a fixed *a priori* interpretation of unobservable events (static observers) or even to observers able to base this interpretation on observation of previous events (dynamic observers), but also able to re-interpret past unobservable events on the base of subsequent observation. We finally consider general observers and we show that all Mantel's Basic Security Predicates (BSPs) are RIFPs. Finally, we illustrate the applicability of our framework by providing the first formal specification for conditional anonymity guaranteeing anonymity of agents unless revocation (for instance, the identity of an agent discovered to be dishonest can be revealed).

Outline. The rest of the paper is organized as follows. Rational Information flow properties are defined in Section 2, with the associated decidability results. RIFPs w.r.t. rational observation functions are investigated in Section 3: rational opacity properties as RIFP are presented in Subsection 3.1, Orwellian observers in Subsection 3.2 and their application to *intransitive non-interference* and *selective intransitive non-interference* in Subsection 3.3. RIFPs w.r.t. general rational observation relations are investigated in Section 4: BSPs as RIFPs are presented in Subsection 4.1 and an application of general rational observation relation to conditional anonymity is presented in Subsection 4.2. In Section 5, we discuss related work and we conclude in Section 6.



2 Rational Information flow properties

We briefly recall the notions of finite automata and finite transducers before defining rational information flow properties.

2.1 Automata and transducers

The set of natural numbers is denoted by \mathbb{N} and the set of *words* over a finite alphabet *A* is denoted by A^* , with ε for the empty word and $A^+ = A^* \setminus \{\varepsilon\}$. The length of a word *w* is written |w| and for any $a \in A$, $|w|_a$ is the number of occurrences of *a* in *w*. A *language* is a subset of A^* .

Finite Labelled Transition Systems. A finite labelled transition system (LTS or automaton for short), over a finite set *Lab* of labels, is a tuple $\mathscr{A} = \langle Q, I, \Delta, F \rangle$, where *Q* is a finite set of states, $I \subseteq Q$ is the subset of initial states, $\Delta \subseteq Q \times Lab \times Q$ is a finite transition relation and $F \subseteq Q$ is a set of final states. Note that *Lab* can be an alphabet but also a (subset of a) monoid.

Given two states $q, q' \in Q$, a *path* from q to q' with *label* u, written as $q \xrightarrow{u} q'$, is a sequence of transitions $q \xrightarrow{a_1} q_1, q_1 \xrightarrow{a_2} q_2, \cdots, q_{n-1} \xrightarrow{a_n} q'$, with $a_i \in Lab$ and $q_i \in Q$, for $1 \leq i \leq n-1$, such that $u = a_1 \cdots a_n$. The path is *accepting* if $q \in I$ and $q' \in F$, and the language of \mathscr{A} , denoted by $L(\mathscr{A})$, is the set of labels of accepting paths. A regular language over an alphabet A is a subset of A^* accepted by a finite LTS over the set of labels A.

Finite Transducers. A finite transducer (or transducer for short) is a finite LTS \mathscr{T} with set of labels $Lab \subseteq A^* \times B^*$ for two alphabets *A* and *B*. A label $(u, v) \in A^* \times B^*$ is also written as u|v. The subset $L(\mathscr{T})$ of $A^* \times B^*$ is a *rational relation* [Sak09] from A^* to B^* . The transducer \mathscr{T} is said to realize the relation $L(\mathscr{T})$ (see Figure 1 for basic examples of transducers).

Given a rational relation *R*, we write $R(u) = \{v \in B^* \mid (u,v) \in R\}$ for the image of $u \in A^*$, $R^{-1}(v) = \{u \in A^* \mid (u,v) \in R\}$ for the inverse image of $v \in B^*$, possibly extended to subsets of A^* or B^* respectively, and dom $(R) = \{u \in A^* \mid \exists v \in B^*, (u,v) \in R\}$ for the domain of *R*. The relation *R* is complete if dom $(R) = A^*$, it is a function if for each $u \in \text{dom}(R)$, R(u) contains a single element $v \in B^*$.

For a subset *P* of *A*^{*}, the identity relation $\{(u, u) | u \in P\}$ on $A^* \times A^*$ is denoted by Id_{*P*}. The composition of rational relations R_1 on $A^* \times B^*$ and R_2 on $B^* \times C^*$, denoted by R_1R_2 (from left to right) or by $R_2 \circ R_1$ (from right to left), is the rational relation on $A^* \times C^*$ defined by $\{(u, w) | \exists v (u, v) \in R_1 \land (v, w) \in R_2\}$ ([EM65]). The family of regular languages is closed under rational relations [Ber79].

2.2 Rational observers

Information flow properties are related to what an agent can learn from a given system. In a language-based setting, the behavior of the system is described by a language *L* over some alphabet *A*, and some function \mathcal{O} associates with each $w \in L$ its observation $\mathcal{O}(w)$ visible by the agent. We generalize the notion of observation by defining \mathcal{O} as a relation on $A^* \times B^*$ for some alphabet *B*, but we restrict \mathcal{O} to be a rational relation.



Definition 1 (Rational observer) A rational observer is a rational relation \mathcal{O} on $A^* \times B^*$, for two alphabets *A* and *B*. The observation of a word $w \in A^*$ is the set $\mathcal{O}(w) = \{w' \in B^* \mid (w, w') \in \mathcal{O}\}$ and for any language $L \subseteq \text{dom}(\mathcal{O})$, the observation of *L* is $\mathcal{O}(L) = \bigcup_{w \in L} \mathcal{O}(w)$.

As pointed out in [DHRS11], a large amount of information flow properties of a language L are expressed as relations of the form $op_1(L) \subseteq op_2(L)$, for some language theoretic operations op_1 and op_2 . Actually, we show below that op_1 and op_2 are often rational relations corresponding to some specific observations of L. Also, we define the class of rational information flow properties as those using rational observers, and positive boolean operations:

Definition 2 (Rational information flow property) A rational information flow property (RIFP) for a language *L* is any relation of the form $L_1 \subseteq L_2$, where L_1 and L_2 are languages given by the grammar:

$$L_1, L_2 ::= L \mid \mathscr{O}(L_1) \mid L_1 \cup L_2 \mid L_1 \cap L_2$$

where \mathcal{O} is a rational observer.

Hence, from Def. 1, we recover information flow properties of *L* of the form $\mathscr{O}_1(L) \subseteq \mathscr{O}_2(L)$ for two rational observers, as a particular case. However it has to be noted that Def. 1 does not reduce to these inclusions since rational relations are not closed under intersection [Ber79]. Given a family of languages \mathscr{L} , we define $RIF(\mathscr{L})$ as the set of RIFPs for languages in \mathscr{L} . We immediately have the following general result:

Proposition 1 Let \mathscr{L} be a family of languages closed under union, intersection, and rational transductions, such that the relation \subseteq is decidable in \mathscr{L} . Then any property in $RIF(\mathscr{L})$ is decidable.

In particular, the class $\Re eg$ of regular languages satisfies the conditions above and it has a PSPACE-complete inclusion problem. We then have:

Corollary 1 The problem of deciding a property in $RIF(\Re eg)$ is PSPACE-complete.

Proof. It follows from the remark above that the problem is in PSPACE. For PSPACE-hardness, recall that for a language *K*, the relation \mathcal{O}_K defined by $\mathcal{O}_K(w) = \{w\} \cap K$ is a rational observer if (and only if) *K* is a regular language [Sak09]. Let L_1 and L_2 be two regular languages, and let $\mathcal{O}_{L_1}, \mathcal{O}_{L_2}$ be the two corresponding relations, then for $L = A^*$, we have $L_1 \subseteq L_2$ if and only if $\mathcal{O}_{L_1}(L) \subseteq \mathcal{O}_{L_2}(L)$.

This corollary subsumes many existing decidability results for IF properties. The rest of the paper is devoted to establish reductions of some of these to the $RIF(\Re eg)$ verification problem.

3 RIF properties with rational functions

In this section, we consider the generic model of opacity introduced in [BKMR08] for transition systems. Opacity is parametrized with observation functions, that are classified in [BKMR08]



as *static*, *dynamic* or *Orwellian* to reflect the computational power of the observer. In a static observation, actions are always interpreted in the same way. It is defined as a morphism and hence, it is a rational function. A particular case of static observer is the projection π_B from A^* into B^* for a subalphabet *B* of *A*, so that $\pi_B(a) = a$ if $a \in B$ and $\pi_B(a) = \varepsilon$ otherwise. In a dynamic observation function, interpretation of the current action depends on the sequence of actions observed so far and hence, it is also a rational function.

Example 1 In Figure 1 (where all states are final states), the left hand side depicts a transducer realizing the projection from $\{a,b\}^*$ onto $\{b\}^*$ while the right hand side depicts a transducer realizing the following dynamic observation function (translated from [CDM12]): The first occurrence of the first action is observed, then nothing is observed until the first occurrence of the second action (*b* if the trace begins with *a* and *a* otherwise) and everything is observed in clear as soon as this second action occurs that is, $\mathcal{O}(aa^*bu) = abu$ and $\mathcal{O}(bb^*au) = bau$ for any $u \in \{a,b\}^*$.

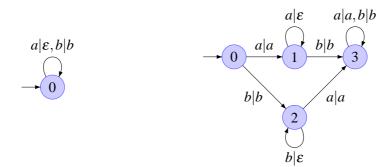


Figure 1: Examples of transducers realizing basic observation functions

In Orwellian observation functions, the current observation depends not only on the prefix of actions observed so far but also on the complete trace. It reflects the capability of the observer to use subsequent knowledge to re-interpret past actions. In the rest of this section we will study opacity w.r.t. rational Orwellian observers.

3.1 Opacity w.r.t. rational functions

In its original setting, opacity is related to a language $L \subseteq A^*$ modelling the behaviour of a system, a function \mathcal{O} from A^* to B^* and in addition, a predicate φ given as a subset of L, describing a secret. Two words w and w' of L are observationally equivalent for \mathcal{O} if $\mathcal{O}(w) = \mathcal{O}(w')$. The *observation class* of w in L is the set $[w]_{\mathcal{O}}^L = \{w' \in L \mid \mathcal{O}(w) = \mathcal{O}(w')\} = L \cap \mathcal{O}^{-1}(\mathcal{O}(w))$.

The secret φ is opaque in *L* for \mathscr{O} if for any word in φ , there is another word in $L \setminus \varphi$ such that *w* and *w'* are observationally equivalent. Hence, φ is opaque if and only if $\mathscr{O}(\varphi) \subseteq \mathscr{O}(L \setminus \varphi)$, which we take as definition when \mathscr{O} is a rational function:

Definition 3 (Rational Opacity) Given a language $L \subseteq A^*$, a language $\varphi \subseteq L$ and a rational function \mathcal{O} , φ is rationally opaque in *L* for \mathcal{O} if $\mathcal{O}(\varphi) \subseteq \mathcal{O}(L \setminus \varphi)$.

The information flow deduced by an observer when the system is not opaque is captured by



the notion of secret disclosure: A word $w \in L$ discloses the secret S w.r.t. \mathscr{O} if $[w]_{\mathscr{O}}^{L} \subseteq \varphi$. We have:

Proposition 2 *Rational opacity properties on languages in some family* \mathscr{L} *for regular secrets belong to* $RIF(\mathscr{L})$.

Proof. As already seen in the proof of Corollary 1, intersection with a regular set *K* is a rational observation \mathscr{O}_K . Since the secret φ is regular, opacity of φ in *L* for \mathscr{O} is equivalent to $\mathscr{O}(\mathscr{O}_{\varphi}(L)) \subseteq \mathscr{O}(\mathscr{O}_{\neg \varphi}(L))$.

Non-interference and weak and strong anonymity have been shown to reduce to opacity w.r.t. suitable observers (see [BKMR08]). In [CDM12], PSPACE-hardness is established for opacity of regular secrets for regular languages w.r.t. static and dynamic observers.

3.2 Rational Orwellian observers

In the sequel, we denote the disjoint union by \oplus . In our context, Orwellian observation functions from [BKMR08] are realized by rational Orwellian observers:

Definition 4 (Rational Orwellian Observer) A *rational Orwellian observer* is a rational function, given as a disjoint union of functions: $\mathcal{O} = \bigcup_{1 \le i \le n} \mathcal{O}_i$, where the domains $\{\operatorname{dom}(\mathcal{O}_i), 1 \le i \le n\}$ form a partition of A^* . The partial functions \mathcal{O}_i are called *views*.

Note that \mathcal{O} is a function because the domains of the views are disjoint. We simply call these functions Orwellian observers for short, since there is no ambiguity in our context. The terminology *Orwellian* comes from the ability of the observer to somehow see in the future, as illustrated in the following example.

Example 2 (A simple example) The function $\mathcal{O} = \mathcal{O}_a \oplus \mathcal{O}_b \oplus \mathcal{O}_{\varepsilon}$ is an Orwellian observer on $\{a, b\}$ realized by the transducer depicted in Figure 2. The function is defined by $\mathcal{O}(\varepsilon) = \varepsilon$ and:

$$\mathscr{O}(w) = \begin{cases} \pi_{\{b\}}(w) \text{ if the last letter of } w \text{ is a} \\ \pi_{\{a\}}(w) \text{ if the last letter of } w \text{ is b.} \end{cases}$$

Hence, the observer interpretation of the current event depends on the last event of the trace. If it is *a* then \mathcal{O} interprets the trace as its projection over $\{b\}$ and the other way around, if it is *b* then it interprets the trace as its projection over $\{a\}$.

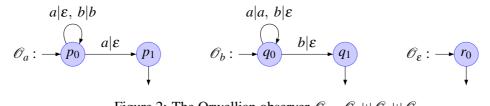


Figure 2: The Orwellian observer $\mathscr{O} = \mathscr{O}_a \uplus \mathscr{O}_b \uplus \mathscr{O}_{\varepsilon}$.



Despite its observational power, this observer is not able to deduce whether the first event in the trace in $L = (a+b)(a^*+b^*)(a+b)$ is an *a*. Indeed, let $\varphi = a(a^*+b^*)(a+b)$ be the secret, corresponding to the set of traces in *L* with *a* as the first event. Then φ is opaque w.r.t. \mathcal{O} in *L*. To see this, if a secret trace *w* is observed, examine what \mathcal{O} can deduce from this observation.

- If w ends with an a then $\mathscr{O}(w) = b^n$ for some $n \ge 0$ but $b^n a \notin \varphi$ is also observed by b^n .
- If w ends with a b then $\mathscr{O}(w) = a^n$ for some $n \ge 0$ but $ba^n b \notin \varphi$ is also observed by a^n .

Example 3 (Static and dynamic observers) Static and dynamic observations are of course special cases of Orwellian observers, where \mathcal{O} consists of a single complete view. Note that static and dynamic observations preserve prefixes while it is not necessarily the case for Orwellian observations (see examples 2 and 4).

Example 4 (Intransitive non-interference) Let $A = V \uplus C \uplus D$ be a partition of the alphabet into visible actions in *V*, confidential actions in *C* and declassification actions in *D*. When a declassification action occurs in a word, the prefix is observed in clear. The corresponding observation function is called in [MY14] the projection on *V* unless *D*, and defined as a mapping $\pi_{V,D}: A^* \to A^*$ such that $\pi_{V,D}(\varepsilon) = \varepsilon$ and

$$\pi_{V,D}(ua) = \begin{cases} ua & \text{if } a \in D, \\ \pi_{V,D}(u)a & \text{if } a \in V, \\ \pi_{V,D}(u) & \text{otherwise.} \end{cases}$$

A language *L* satisfies intransitive non-interference (INI) if $\pi_{V,D}(L) \subseteq L$. Again:

Proposition 3 The function $\pi_{V,D}$ is an Orwellian observer, hence INI for languages in \mathscr{L} belongs to $RIF(\mathscr{L})$.

Proof. The function $\pi_{V,D}$ is a sum of two views: $\pi_{V,D} = \mathcal{O}_{\varepsilon} \uplus \mathcal{O}_{D}$, realized by the transducers depicted in Figure 3.

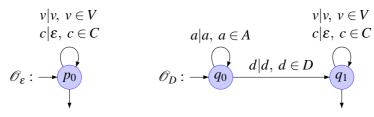


Figure 3: The Orwellian observer $\pi_{V,D} = \mathscr{O}_{\varepsilon} \uplus \mathscr{O}_{D}$.

It has been shown in [MY14] that a language *L* satisfies intransitive non-interference (INI) if and only if $\varphi_{INI} = \{w \in L \mid \pi_{V,D}(w) \neq w\}$ is opaque in *L* w.r.t. the observer $\pi_{V,D}$.

This can be generalized as follows, showing that many non-interference like properties reduce to opacity w.r.t. Orwellian observers.

Proposition 4 Let \mathcal{O} be a rational idempotent function (i.e. $\mathcal{O}^2 = \mathcal{O}$). Then $\mathcal{O}(L) \subseteq L$ if and only if $\varphi_{\mathcal{O}} = \{w \in L \mid \mathcal{O}(w) \neq w\}$ is opaque in L for \mathcal{O} .

Proof. First assume that $\mathscr{O}(L) \subseteq L$ and let $w \in \varphi_{\mathscr{O}}$. Then $\mathscr{O}(w) \neq w$. For $w' = \mathscr{O}(w)$, we have: $w' \in L$ and $\mathscr{O}(w') = \mathscr{O}^2(w) = \mathscr{O}(w) = w'$, hence $w' \notin \varphi_{\mathscr{O}}$. Opacity of $\varphi_{\mathscr{O}}$ follows.

Conversely, assume that $\varphi_{\mathcal{O}}$ is opaque and let *w* be an element of *L*. If $w \in \varphi_{\mathcal{O}}$, then there exists $w' \in L \setminus \varphi_{\mathcal{O}}$ such that $\mathscr{O}(w) = \mathscr{O}(w')$. Since $w' \notin \varphi_{\mathcal{O}}$, $\mathscr{O}(w') = w'$, hence $w' = \mathscr{O}(w) \in L$. Otherwise, $w \notin \varphi_{\mathcal{O}}$ implies $\mathscr{O}(w) = w \in L$. In all cases, $\mathscr{O}(w) \in L$ and $\mathscr{O}(L) \subseteq L$.

Finally, we can state the following:

Proposition 5 Given an Orwellian observer \mathcal{O} , deciding opacity of regular secrets w.r.t. \mathcal{O} for regular languages is PSPACE-complete.

Proof. Corollary 1 implies that the problem is in PSPACE. For the PSPACE-hardness, it suffices to observe that dynamic or static observers are particular Orwellian observers for which the problem is already PSPACE-hard. \Box

In the next paragraph, we show that the observation function defined for selective declassification is an Orwellian observer.

3.3 Selective declassification

Intransitive non-interference with selective declassification (INISD) generalizes INI by allowing to each downgrading action to declassify only a subset of confidential actions. It has recently been proposed in [BD12] for a class of Petri net languages (that does not include rational languages). To formalize INISD, the alphabet is partitioned into $A = V \uplus C \uplus D$ as in example 4. In addition, with each declassification action $d \in D$ is associated a specific set $C(d) \subseteq C$ of confidential events, with the following meaning: An occurrence of d in a word w declassifies all previous occurrences of actions from C(d), hence these actions are observable while other confidential events in C are not.

Let $\Sigma(D) = \{ \sigma \in D^* \mid |w|_d \le 1 \text{ for all } d \in D \}$ be the set of repetition-free sequences of downgrading actions in *D*. With any $\sigma = d_1 d_2 \dots d_n \in \Sigma(D)$, we associate the sets:

$$A_{\sigma} = V \uplus C \uplus \{d_1, \dots, d_n\}$$

$$W_{\sigma} = A_{\sigma}^* \cdot d_1 \cdot (A_{\sigma} \setminus \{d_1\})^* \cdot d_2 \cdot \dots \cdot d_n \cdot (A_{\sigma} \setminus \{d_1, \dots, d_n\})^*$$

$$V_{\sigma,i} = V \cup \{d_j, i+1 \le j \le n\} \cup \bigcup_{j=i+1}^n C(d_j), \text{ for every } i \in \{0, \dots, n\}$$

with the convention $V_{\sigma,n} = V$, and the projections $\pi_{\sigma,i} : A^* \to V_{\sigma,i}^*$ for every $i \in \{0, \ldots, n\}$.

For a given $\sigma = d_1 \dots d_n \in \Sigma(D)$, the set W_{σ} contains the words w in A^* where the set of all downgrading actions is precisely $\{d_1, \dots, d_n\}$ and such that the last occurrence of d_i precedes the last occurrence of d_{i+1} for any $1 \le i \le n-1$. Note that the family of all these sets $\{W_{\sigma}, \sigma \in \Sigma\}$ form a partition of A^* . Besides, the projection $\pi_{\sigma,i}$ observes in clear any confidential event in $\bigcup_{i=i+1}^n C(d_i)$, in addition to the visible events in V and the declassifying events from σ .



Now the property called INISD in [BD12] can be stated in our general context for a language *L* as follows: For any $\sigma \in \Sigma(D)$ and for any word $w = w_0 d_1 w_1 \dots d_n w_n$ in $L \cap W_{\sigma}$, there exists a word $w' = w'_0 d_1 w'_1 \dots d_n w'_n$ in $L \cap W_{\sigma}$ such that for every $i \in \{0, \dots, n\}$, $w'_i \in V^*_{\sigma,i}$ and $\pi_{\sigma,i}(w_i) = \pi_{\sigma,i}(w'_i)$. We have:

Proposition 6 The INISD property for languages in \mathscr{L} belongs to $RIF(\mathscr{L})$.

Proof. We build an (idempotent) Orwellian observer \mathcal{O}_{SD} such that a language *L* satisfies INISD if and only if $\mathcal{O}_{SD}(L) \subseteq L$. Let $\mathcal{O}_{SD} = \biguplus_{\sigma \in \Sigma(D)} \mathcal{O}_{\sigma}$, where the view $\mathcal{O}_{\varepsilon}$ and a generic view \mathcal{O}_{σ} for some non empty $\sigma = d_1 \dots d_n \in \Sigma(D)$ are depicted in Figure 4.

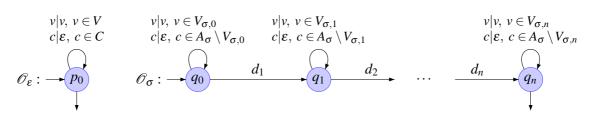


Figure 4: Views of the observation \mathcal{O}_{SD}

Let $w = w_0 d_1 w_1 \dots d_n w_n$ be a word in $L \cap W_{\sigma}$, the observation of *w* is

$$\mathscr{O}_{\sigma}(w) = \pi_{\sigma,0}(w_0) d_1 \pi_{\sigma,1}(w_1) \dots d_n \pi_{\sigma,1}(w_n)$$

Then *L* satisfies INISD if and only if $\mathscr{O}_{\sigma}(L \cap W_{\sigma}) \subseteq L \cap W_{\sigma}$ for any $\sigma \in \Sigma(D)$. Since the family $\{W_{\sigma}, \sigma \in \Sigma\}$ is a partition of A^* , the family $\{L \cap W_{\sigma}, \sigma \in \Sigma\}$ is a partition of *L* and the result follows. Each view \mathscr{O}_{σ} is idempotent and the partitionning also ensures that \mathscr{O}_{SD} itself is idempotent. As a consequence, proposition 4 applies here.

Remark 1 Also note that a secret φ is opaque for a language L w.r.t. \mathscr{O}_{SD} if and only if for all $\sigma \in \Sigma(D)$, $\varphi \cap W_{\sigma}$ is opaque for $L \cap W_{\sigma}$ w.r.t. \mathscr{O}_{σ} . Indeed, the result again holds because the family $\{L \cap W_{\sigma}, \sigma \in \Sigma\}$ is a partition of L: for all $\sigma \in \Sigma(D)$, $\mathscr{O}_{\sigma}(W_{\sigma}) \subseteq W_{\sigma}$, we have that φ is opaque for L w.r.t. \mathscr{O}_{SD} if and only if for all $\sigma \in \Sigma(D)$,

$$\mathscr{O}_{\sigma}(\varphi \cap W_{\sigma}) \subseteq \mathscr{O}_{\sigma}((L \setminus \varphi) \cap W_{\sigma}) = \mathscr{O}_{\sigma}((L \cap W_{\sigma}) \setminus (\varphi \cap W_{\sigma}))$$

Like before, for regular languages, decidability of INISD as well as opacity under \mathcal{O}_{SD} , are consequences of corollary 1 and proposition 6 above. This property is studied in [BD12] for the prefix languages of (unbounded) labelled Petri nets. This family is closed under intersection, inverse morphisms and alphabetical morphisms, hence it is also closed under rational transductions (by Nivat's theorem [Ber79]), but it has an undecidable inclusion problem. A very nice proof is given in [BD12] for the decidability of the INISD property: it relies on the decidability of the inclusion problem for the particular case of free nets (where all transitions have distinct labels, different from ε).

The following example (inspired from [BD12]) tries to explain the essence of selective declassification.



Verification of Information Flow Properties under Rational Observation

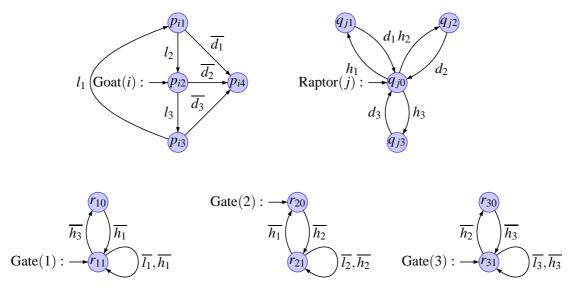


Figure 5: The dining Raptors

Example 5 (The Dining Raptors) A circuit followed by a herd of goats is divided in three sections. Each section is guarded by a gate. When gate *i* is open, goats can move clockwise from section *i* to section $i+1 \pmod{3}$. The center of the circuit is occupied by a den of raptors. When gate $i+1 \pmod{3}$ is open, a raptor can leave the den and hide around gate *i* after opening it and closing gate $i+1 \pmod{3}$ to increase chance of success. When a raptor is embushed near a section and there is a goat in this section, the raptor can catch prey and come back to the den.

This scenario is modelled with the transition system

$$DR(n,m) = \prod_{i=1}^{n} Goat(i) \times \prod_{j=1}^{m} Raptor(j) \times \prod_{k=1}^{3} Gate(k)$$

obtained by synchronizing the components depicted in Figure 5 on the complementary actions. Goats' move from gate *i* to gate *i*+1 (mod 3) is modelled with visible action l_1 , raptors' embush action at section *i*, with the confidential action h_i and the raptors' catch action in section *i*, by the declassification action d_i . Opacity of φ_{DR} w.r.t. \mathcal{O}_{SD} in L(DR(m,n)) where

$$\varphi_{DR} = \{ u \in L(DR(m,n)) \mid \mathscr{O}_{SD}(u) \neq u \}$$

comes down to absence of information the goats can get from environment about the moment they will be caught until this happens. Hence there is no strategy that they can oppose to the raptors. In the case where initially goats are in section 2 and gates 1 and 3 are opened, as shown in Figure 5, L(DR(n,m)) is not opaque w.r.t. φ_{DR} since $l_3l_1h_2l_2$ reveals the secret $(h_2l_3l_1l_2, l_3h_2l_1l_2)$ and $l_3l_1h_2l_2$ are the only traces observed as $l_3l_1l_2$) and this, for any number of raptors and goats. This example may be of course modified in various ways as follows. If all three gates are open, goat 1 never realizes she dies since $l_3l_1h_1d_1$ does not reveals the secret but following this, as gate 2 is now close, goat 2 after $l_3l_1l_2$ will know that a raptor is embushed at gate 2 since $l_3l_1h_1d_1l_3l_1h_2l_2$ reveals the secret. If only gate 3 is open, $l_3h_2d_2h_1l_1$ reveals to the herd,



that one of them is now trapped in section 2. Finally, if we dismantle all three gates, the only synchronizing actions are now the declassification ones and φ_{DR} becomes opaque w.r.t. \mathcal{O}_{SD} .

4 RIF properties with full rational relations

In this section, we first revisit Basic Security Predicates (BSP) presented in [Man00, Man01] and used as building blocks of the Mantel's generic security model. In the second part, we investigate anonymity properties.

4.1 Basic Security Predicates

For BSPs, the alphabet A is partitioned into $A = V \uplus C \uplus N$, where V is the set of visible events, C is the set of confidential events and N is a set of internal events. Informally, a BSP for a given language L over A, is an implication stating that for any word w in L satisfying some restriction condition, there exists a word w' also in L which is observationnally equivalent to w and which fulfills some closure condition describing the way w' is obtained from w by adding or removing some confidential events. The conditions are sometimes parametrized by an additional set $X \subseteq A$ of so-called admissible events. We prove:

Proposition 7 Any BSP over languages in some family \mathscr{L} belongs to $RIF(\mathscr{L})$.

Due to lack of space, the proof is omitted (but will be found in a long version). It mainly consists in exhibiting rational observers together with an inclusion relation such that a language L satisfies a given BSP if and only if this relation holds.

In [DHRS11], the decidability results for all 14 BSPs on regular languages are obtained by ad-hoc proofs establishing that regularity is preserved by the various op_1 , op_2 operations. These include auxiliary functions on languages (like *mark*, *unmark*, etc.) that are unnecessary in our setting. Actually, we show how decidability of BSPs is an immediate consequence of corollary 1 and proposition 7 above. The more difficult case of pushdown systems (generating prefix-closed context-free languages) is also investigated in [DHRS11]: Although context-free languages are closed under rational transductions, they are not closed under intersection and the inclusion problem is undecidable for context-free languages [Ber79]. Finally, several undecidability results are presented in [DHRS11]. In particular, they exhibit an information flow property called Weak Non Inference (WNI) shown to be undecidable even for regular languages. Hence, WNI cannot be expressed neither as a conjunction of BSPs, and as matter of fact, neither as an RIFP. Also, in order to get decidable cases, authors had to restrict the languages and/or the class of properties like reducing the size of the alphabet ($card(V) \leq 1$ and $card(C) \leq 1$).

4.2 Conditional anonymity

Conditional or escrowed anonymity is concerned with the revocation of the guarantee, under well-defined conditions, to which an agent agrees, that his identification w.r.t. a particular (non-secret) action will remain secret and in such case, conditional anonymity guarantees the un-linkability of revoked users in order to guarantee anonymity to "legitimate" agents [DS08]. As



suggested in [BKMR08], Orwellian observation can be used to model *conditional anonymity* but [BKMR08] contains neither a definition of such a property, nor any investigation of its decidability. We close the gap in this paper.

The alphabet is partitioned into $A = V \uplus P \uplus R$ where *P* is the set of actions performed by anonymous participants, *V* is the set of visible actions and *R* is the set of anonymity revocation actions, such that for each participant corresponds a dedicated revocation action *r* allowing to reveal the subset P(r) of all its anonymous actions. Hence the sets P(r) are mutually disjoint.

In [SS96], definitions of *weak and strong anonymity* are given in the setting of the process algebra CSP. A language is *strongly anonymous* (SA) if it is stable under any "perturbation" of anonymous actions where an anonymous action in P can be replaced by any other element of P. It is *weakly anonymous* (WA) if it is stable under any permutation on the set of anonymous actions. For a finite set Z, we denote by S_Z the set of all permutations on Z. We first have:

Proposition 8 Weak and strong anonymity on languages in \mathcal{L} belong to $RIF(\mathcal{L})$.

Proof. For these two properties, the subalphabet R of revocation actions is empty. We express the properties in our language-based setting, similarly as in [BKMR08].

A language *L* is strongly anonymous w.r.t. *P* if $\mathscr{O}_{SA}^{P}(L) \subseteq L$ where \mathscr{O}_{SA}^{P} is the mapping defined on $A = V \uplus P$ by: $\mathscr{O}_{SA}^{P}(a) = P$ if $a \in P$ and $\mathscr{O}_{SA}^{P}(a) = \{a\}$ otherwise. Such mappings (called rational substitutions in [Ber79]) are well known to be rational relations, hence the result follows.

A language *L* is weakly anonymous w.r.t. *P* if $\mathscr{O}_{WA}^P(L) \subseteq L$ where $\mathscr{O}_{WA}^P = \biguplus_{\alpha \in S_P} \mathscr{O}_{\alpha}$ and \mathscr{O}_{α} is the morphism which applies the permutation α on letters of *P*:

 $\mathscr{O}_{\alpha}(a) = \alpha(a)$ if $a \in P$ and $\mathscr{O}_{\alpha}(a) = a$ otherwise.

With any $\sigma \subseteq R$, we associate:

- $W_{\sigma} = \{ w \in A^* \mid \pi_R(w) \in \sigma^* \}$, the set of words w in A^* where the set of revocation actions appearing in w is σ ,
- $P_{\sigma} = P \setminus \bigcup_{r \in \sigma} P(r)$, the set of actions of legitimate agents.

We denote by 2^R the powerset of R and remark that here also, the sets W_{σ} for $\sigma \in 2^R$ form a partition of A^* . In order to provide at any moment strong (weak) anonymization to legitimate agents, we define conditional anonymity as follows:

Definition 5 With the notations above, a language *L* on $V \uplus P \uplus R$ is:

- conditionally weakly anonymous (CWA) if for any $\sigma \subseteq R$, $L \cap W_{\sigma}$ is WA w.r.t. P_{σ} ,
- *conditionally strongly anonymous* (CSA) if for any $\sigma \subseteq R$, $L \cap W_{\sigma}$ is SA w.r.t. P_{σ} .

Now we have:

Proposition 9 Weak and strong conditional anonymity on languages in \mathcal{L} belong to $RIF(\mathcal{L})$.

Proof. We build rational observers, with a view-like component for each possible subset σ of revoked users, corresponding to \mathcal{O}_{SA} (resp. \mathcal{O}_{WA}) localized to W_{σ} , *i.e.* revocation actions are those in σ , anonymous actions are restricted to P_{σ} and visible actions are extended to $V \uplus \biguplus_{r \in \sigma} P(r)$:



$$\mathscr{O}_{CSA} = \biguplus_{\sigma \in 2^R} \mathscr{O}_{SA}^{P_{\sigma}} \text{ and } \mathscr{O}_{CWA} = \biguplus_{\sigma \in 2^R} \mathscr{O}_{WA}^{P_{\sigma}}$$

Then *L* is conditionally strongly anonymous (CSA) if and only if $\mathscr{O}_{CSA}(L) \subseteq L$ and *L* is conditionally weakly anonymous (CWA) if and only if $\mathscr{O}_{CWA}(L) \subseteq L$, which yields the result. \Box

5 Related works.

Along the lines, important connections between RIFPs and information flow properties have been established, hence in this section, we will focus on extending the picture.

Algorithms for verifying opacity in Discrete Event Systems w.r.t. projections are presented together with applications in [BBB⁺07, TK09, SH11, Lin11]. In [BBB⁺07], the authors consider a concurrent version of opacity and show that it is decidable for regular systems and secrets. In [TK09], the authors define what they called *secrecy* and provide algorithms for verifying this property. A system property satisfies *secrecy* if the property and its negation are state-based opaque. In [Lin11] the author provides an algorithm for verifying state-based opacity (called *strong opacity*) and shows how opacity can be instantiated to important security properties in computer systems and communication protocols, namely anonymity and secrecy. In [SH11], the authors define the notion of *K*-step opacity where the system remains state-based opaque in any step up to depth-*k* observations that is, any observation disclosing the secret has a length greater than *k*. Two methods are proposed for verifying *K*-step opacity. All these verification problems can be uniformly reduced to the RIFP verification problem.

In [FG01], the authors provide decision procedures for a large class of trace-based security properties that can all be reduced to the RIFP verification problem for regular languages. In [MZ07], decision procedures are given for trace-based properties like non-deducibility, generalized non-interference and forward correctability. The PSPACE-completeness results for these procedures can be reduced to our results.

Concerning intransitive information flow (IIF), *non-interference* (NI) and *intransitive non-interference* (INI) for deterministic Mealy machines have been defined in [Rus92]. In [Pin95], an algorithm is provided for INI. A formulation of INI in the context of non-deterministic LTSs is given in [Mul00], in the form of a property called *admissible interference* (AI), which is verified by reduction to a stronger version of NI. This property, called *strong non-deterministic non-interference* (SNNI) in [FG01], is applied to N finite transition systems where N is the number of downgrading transitions of the original system. This problem was also reduced to the opacity verification problem w.r.t. Orwellian projections in [MY14]. In [BPR04], various notions of trace-based INI declassification properties are considered and compared. In contrast, our generic model is instantiable to a much larger class of IIF properties.

In [vdM07], the author has argued that Rushby's definition of security for intransitive policies suffers from some flaw, and proposed some stronger variations. The considered flaw relies to the fact that, if $u \in W_{d_1}$ and $v \in W_{d_2}$, that is u (resp. v) declassifies only $h_1 \in H(d_1)$ (resp. $h_2 \in H(d_2)$), then the shuffle of u and v resulting of their concurrent interaction will reveal the order in which h_1 and h_2 have been executed. The proof techniques used in this paper for deciding the RIFP



verification problem relies on their end-to-end execution semantics and hence does not address this problem.

6 Conclusion

We introduce a language-theoretic model for trace-based information flow properties, the RIFPs where observers are modelled by rational transducers. Given a family \mathscr{L} of languages, our model provides a generic decidability result to the $RIF(\mathscr{L})$ verification problem: Given $L \in \mathscr{L}$ and a security property P in RIF(\mathscr{L}), does L satisfy P? When \mathscr{L} is the class $\mathscr{R}eg$ of regular languages, the problem is shown PSPACE-complete. This result subsumes most decidability results for finite systems. In order to prove that, we have shown that opacity properties and Mantel's BSPs, two major generic models for trace-based IF properties, are RIFPs. We illustrated the expressiveness of our model by showing that the verification problem of INI and INISD can be reduced to the verification problem of opacity w.r.t. a subclass of rational observers called rational Orwellian observers. Finally we illustrated the applicability of our framework by providing the first formal specification of *conditional anonymity*.

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