Revista Colombiana de Estadística
July 2015, Volume 38, Issue 2, pp. 321 to 334
DOI: http://dx.doi.org/10.15446/rce.v38n2.51663

Parameter Estimation of Power Function Distribution with TL-moments

Estimación de parámetros de distribuciones de funciones de potencia con momentos TL

Mirza Naveed-Shahzad[1,a], Zahid Asghar[2,b], Farrukh Shehzad[3,c], Mubeen Shahzadi[1,d]

[1]Department of Statistics, University of Gujrat, Gujrat, Pakistan
[2]Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan
[3]Department of Statistics, COMSATS Institute of Information Technology, Lahore, Pakistan

Abstract

Accurate estimation of parameters of a probability distribution is of immense importance in statistics. Biased and imprecise estimation of parameters can lead to erroneous results. Our focus is to estimate the parameter of Power function distribution accurately because this density is now widely used for modelling various types of data. In this study, L-moments, TL-moments, LL-moments and LH-moments of Power function distribution are derived. In addition, the coefficient of variation, skewness and kurtosis are obtained by method of moments, L-moments and TL-moments. Parameters of the density are estimated using linear moments and compared with method of moments and MLE on the basis of bias, root mean square error and coefficients through simulation study. L-moments proved to be superior for the parameter estimation and this conclusion is equally true for different parametric values and sample size.

***Key words*:** Moments, Monte Carlo Simulation, Order Statistics, Parameter estimation, Power function distribution.

[a]Lecturer. E-mail: nvd.shzd@uog.edu.com
[b]Assistant Professor. E-mail: zahid.g@gmail.com
[c]Assistant Professor. E-mail: fshehzad.stat@gmail.com
[d]School Teacher. E-mail: mbn.shzdi@gmail.com

Resumen

La distribución de función de potencias es ampliamente usada. Dada su importancia, es necesario estimar sus parámetros de manera precisa. En este artículo, los momentos TL de la distribución de función de potencias son derivados así como sus casos especiales tales como los momentos L, LL y LH. Los coeficientes de variación, sesgo y curtosis son obtenidos a partir de los momentos L y TL. Los parámetros desconocidos son estimados y los momentos lineales son comparados con el método de momentos y estimadores máximo verosímiles en la base del sesgo, raíz del error cuadrático medio a través de un estudio de simulación. Los momentos L permiten obtener estimaciones más precisas y esta conclusión es verdad para diferentes valores paramétricos y tamaño de muestra.

***Palabras clave*:** distribución de función de potencias, estadísticas de orden, estimación de parámetros, momentos, simulación de Monte Carlo.

1. Introduction

The Power Function Distribution (PFD) is a flexible distribution as it is able to model the various types of data. It is usually used for the reliability analysis, life time and income distribution data. Meniconi & Barry (1996) compare the PFD with Exponential, Lognormal and Weibull distribution to measure the reliability of electrical components. They conclude that the PFD is the best distribution to model such types of data. Similarly many probability models are also used to assess the pattern of the income distribution but these models are mathematically more complex to handle. The PFD on the other hand is quite handy in this regard. Ahsanullah & Kabir (1974), Meniconi & Barry (1996), Ali, Woo & Nadarajah (2005), Chang (2007), Sinha, Singh, Singh & Singh (2008) and Tavangar (2011) define the characteristics of the PFD. Saran & Pandey (2004) estimate the parameters of PFD and they also characterize this distribution. Rahman, Roy & Baizid (2012) applied the Bayesian estimation method to estimate the parameters of PFD.

TL-moments are frequently used for modelling a variety of data set particularly in hydrologic research and especially to model the flood frequency data. L-moments derived by Hosking (1990) and TL-moments, LL-moments and LH-moments introduced by Elamir & Seheult (2003), Bayazit & Onoz (2002) and Wang (1997) respectively. LL-moments have been used in hydrology for the distribution of low flows and LH-moments for the distribution of peak flood discharges. Recently Shahzad & Asghar (2013), Shabri, Ahmad & Zakaria (2009), Abdul-Moniem (2009), Abdul-Moniem & Selim (2009), Asquith (2007), Abdul-Moniem (2007) and Karvanen (2006) are those who had worked on these moments. Linear moments of PFD are not discussed before in the literature according to our knowledge. We have derived these moments and compared their performance with traditional methods.

The rest of the paper is organised as follows. In Section 2 have discuss the method of moments (MM), L-moments (LM), Trimmed L- moments (TLM), LL-moments (LLM) and LH-moments (LHM). Section 3 is about PFD. The first four moments by LM, TLM, LLM and LHM of the PFD are derived in Section 4. In Section 5, the expression of coefficient of variation, coefficient of skewness and coefficient of kurtosis by MM, LM and TLM are derived. In Section 6, Monte Carlo simulation study is carried out to estimate the parameter of PFD by MM, LM, TLM and Maximum likelihood estimation (MLE). To find out the most suitable method of estimation we evaluate all the above methods on the basis of bias, RMSEs, coefficient of variation, coefficient of skewness and coefficient of kurtosis. Finally this study is concluded in Section 7.

2. Generalized TL-Moments

Let $Y_1, Y_2, \ldots, Y_n$. be a random sample of size n having density function $f(y)$ and further let $Y_{(1:n)}, Y_{(2:n)}, \ldots, Y_{(n:n)}$ denote the order statistics. Elamir & Seheult (2003) defined the rth generalized TLM with t_1 smallest and t_2 largest trimming as follows

$$L_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} E\left(Y_{r-k+t_1:r+t_1+t_2}\right), \tag{1}$$

$t_1, t_2 = 0, 1, 2, \ldots; r = 1, 2 \ldots$ The expression of the expected value of the $(r+t_1-k)th$ order statistics of the random sample of size $(r+t_1+t_2)$ is as

$$\begin{aligned} E\left(Y_{r-k+t_1:r+t_1+t_2}\right) &= \frac{(r+t_1+t_2)!}{(r-k+t_1-1)!(t_2+k)!}\int_0^1 y(F)^{r-k+t_1-1}[1-F(y)]^{t_2+k}dF \\ &= \frac{(r+t_1+t_2)!}{(r-k+t_1-1)!(t_2+k)!}\int_{-\infty}^{\infty} yf(y)[F(y)]^{r-k+t_1-1}[1-F(y)]^{t_2+k}dy \end{aligned} \tag{2}$$

Where $y(F)$ is the quantile function of random variable Y with distribution function $F(y)$.

The generalized TLM ratio such as coefficient of variation, coefficient of skewness and coefficient of kurtosis computed from the first four generalized TL-moments. These ratios are defined as $\tau_1 = L_2^{(t_1,t_2)}/L_1^{(t_1,t_2)}$, $\tau_3 = L_3^{(t_1,t_2)}/L_2^{(t_1,t_2)}$ and $\tau_4 = L_4^{(t_1,t_2)}/L_2^{(t_1,t_2)}$ respectively.

2.1. L-Moments

Hosking (1990) introduced LM and derived these moments for well-known distributions. He also proved many theoretical advantages of LM over ordinary moments. LM can be defined for any random variable whose mean exists. This method is used as a summary statistic, estimation of parameters and hypothesis testing for probability distributions. LM also provides better identification of the parent distribution than the conventional moments. Furthermore, LM are less sensitive in the case of outliers in data (Vogel & Fennessey 1993). The r^{th} LM is defined by (1), when $t_1 = t_2 = 0$.

$$L_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(Y_{r-k:r}\right), \quad r = 1, 2, 3 \dots \tag{3}$$

Substituting $r = 1$ and $r = 2$ in (3) provides measure of location and dispersion respectively. The LM ratios are denoted by τ_{LM-1}, τ_{LM-3} and τ_{LM-4}.

According to Hosking (1990), LM are linear combinations of probability weighted moments (PWM) and the unbiased sample estimators of the PWMs are defined by the Hosking & Wallis (1995). The r^{th} sample estimators of the PWMs are as follows:

$$b_r = \frac{1}{n}\sum_{k=0}^{r-1}\frac{(j-1)(j-2)\cdots(j-r)}{(n-1)(n-2)\cdots(n-r)} y_{j:n}$$

Using the above PWMs sample estimators one can find the sample LM(l_r) estimators. The first four sample LM estimators are defined as $l_1 = b_0$, $l_2 = 2b_1 - b_0$, $l_3 = 6b_2 - 6b_1 + b_0$ and $l_4 = 20b_3 - 30b_2 + 12b_1 + b_0$. Sample estimates for LM ratios can be computed as $t_{tm-1} = l_2/l_1$, $t_{tm-3} = l_3/l_2$ and $t_{tm-4} = l_4/l_2$.

2.2. TL-Moments

Elamir & Seheult (2003) introduced TLM which are more robust than LM. In the TLM a predetermined percentage of data is trimmed by assigning zero weight before estimating the moments. These moments are possible even if the mean of the distribution does not exist. Elamir & Seheult (2003) derived TLM for Cauchy distribution, as its mean does not exist. These moments are also used to obtain the most fitted distribution and to estimate the parameters of probability distributions. Elamir & Seheult (2003) defined r^{th} population TLM, when $t_1 = t_2 = t$ in (1)

$$L_r^{(t)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(Y_{r-k+t:r+2t}\right), \quad r = 1, 2, 3 \dots \tag{4}$$

The r^{th} sample TLM is as

$$l_r^{(t)} = \frac{1}{r}\sum_{j=t+1}^{n-1}\left[\sum_{k=0}^{r-1}\frac{\binom{r-1}{k}\binom{j-1}{r+t-k-1}\binom{n-j}{t+k}}{(-1)^{-k}\binom{n}{r+2t}} E\left(Y_{r-k+t:r+2t}\right)\right] Y_{j:n}. \tag{5}$$

2.3. LL-Moments

LLM are the linear functions of the expectations of the lowest order statistic (El-Magd & Noura 2010) and introduced by Bayazit & Onoz (2002). These moments assign more weight to the smaller observations and use only some portion of data to model the lower part instead of complete data. Therefore, the estimates from these moments provide a better fit of the lower part of the data. For the derivation of r^{th} LLM, Bayazit & Onoz (2002) proposed the following relationship that is also obtained by (1) when $t_1 = 0$ and $t_2 = m$

$$L_{Lr}^{(m)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(Y_{r-k:r+m}\right), \quad r = 1, 2, 3\ldots \tag{6}$$

LLM can be estimated for various orders and the preferable order is from one to four ($m = 1, 2, 3, 4$).

2.4. LH-Moments

LHM are due to the Wang (1997), these moments are based on linear combinations of higher-order statistics. These moments are the modified versions of L-moments and defined only for the upper part of the data. So, LHM are recommended for defining the characteristics of the larger observations. The general expression of r^{th} LHM given by Wang (1997) is as follows:

$$L_{Hr}^{(s)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(Y_{r+s-k:r+s}\right), \quad r = 1, 2, 3\ldots \tag{7}$$

Equation (7) is obtained by replacing $t_1 = s$ and $t_2 = 0$ in (1). Wang (1997) suggested the order of the s up to only four. All the coefficients of different orders are expressed in a usual way.

3. Power Function Distribution

PFD is one of the members of the Beta distributions family, commonly used to model the income distribution. The probability density function of PFD type-II (PFD-II) with shape parameter θ and scale parameter β is given as below:

$$f(y) = \theta y^{(\theta-1)}/\beta^{\theta}, \qquad 0 \leq y \leq \beta \tag{8}$$

If the scale parameter takes the value 1.0 in (8) then PFD-II becomes PFD type-I (PFD-I) with only shape parameter as

$$f(y) = \theta y^{(\theta-1)}, \qquad 0 \leq y \leq 1 \tag{9}$$

The cumulative distribution function, mean and r^{th} moment about origin are $y^{\theta}/\beta^{\theta}$, $\theta\beta/(\theta+1)$ and $\theta\beta^r/(\theta+r)$ for PFD-II y^{θ}, $\theta/(\theta+1)$ and $\theta/(\theta+r)$ for PFD-I respectively.

4. Linear Moments of PFD

In this section, diversity of linear moments (LM, TLM, LLM and LHM) are derived for PFD using the general form given in (1). To obtain moments for PFD-I simply substitute $\beta = 1$ in equations of the following sub-section.

4.1. L-Moments for PFD

Let the continuous random variable Y have a probability density function of PFD-II and $Y_{1:n} \leq Y_{2:n} \leq Y_{3:n} \leq \cdots Y_{n:n}$ be a sample of n order statistics. The general expression of LLM is given in (6), and by using it the LLMs for PFD-II are derived as follows:

$$E(Y_{j:n}) = \frac{n!\theta\beta}{(j-1)!(n-j)!} \sum_{k=0}^{n-j} \binom{n-j}{i} \frac{(-1)^i}{j\theta + i\theta + 1} \tag{10}$$

First four LM of PFD-II are derived using (3) and (10)

$$\begin{aligned} L_1 &= \frac{\theta\beta}{(\theta+1)} \\ L_2 &= \frac{\theta\beta}{(\theta+1)(2\theta+1)} \\ L_3 &= \frac{\theta(1-\theta)\beta}{(\theta+1)(2\theta+1)(3\theta+1)} \\ L_4 &= \frac{\theta(2\theta^2-3\theta+1)\beta}{(\theta+1)(2\theta+1)(3\theta+1)(4\theta+1)} \end{aligned}$$

4.2. TL-Moments for PFD

To derive the first four TLM for PFD-II the expression (4) and (10) are used and finally obtain the moments as given below

$$\begin{aligned} L_1^{(1)} &= \frac{6\theta^2\beta}{(2\theta+1)(3\theta+1)} \\ L_2^{(1)} &= \frac{6\theta^2\beta}{(2\theta+1)(3\theta+1)(4\theta+1)} \\ L_3^{(1)} &= \frac{20\theta^2\beta(1-\theta)}{3(2\theta+1)(3\theta+1)(4\theta+1)(5\theta+1)} \\ L_4^{(1)} &= \frac{30\theta^2\beta(2\theta^2-3\theta+1)}{4(2\theta+1)(3\theta+1)(4\theta+1)(5\theta+1)(6\theta+1)} \end{aligned}$$

4.3. LL-Moments for PFD

LLM are the alternates of LM, it gives more weight to lower values of the data (Bayazit & Onoz 2002). The general expression of LLM given in (6), using the PFD-II are found as

$$L_{L1}^{(m)} = \theta\beta\left\{\sum_{i=0}^{m}\begin{pmatrix} m \\ i \end{pmatrix}\frac{(m+1)\,(-1)^{i}}{(i\theta+\theta+1)}\right\}$$

$$L_{L2}^{(m)} = \theta\beta\left\{\sum_{p=0}^{1}\sum_{i=0}^{m+p}\begin{pmatrix} 1 \\ p \end{pmatrix}\begin{pmatrix} m+p \\ i \end{pmatrix}\frac{(m+2)!\,(-1)^{p+i}}{2\,(m+p)!\,(2\theta-p\theta+i\theta+1)}\right\}$$

$$L_{L3}^{(m)} = \theta\beta\left\{\sum_{p=0}^{2}\sum_{i=0}^{m+p}\begin{pmatrix} 2 \\ p \end{pmatrix}\begin{pmatrix} m+p \\ i \end{pmatrix}\frac{(m+3)!\,(-1)^{p+i}}{3\,(2-p)!\,(m+p)!\,(3\theta-p\theta+i\theta+1)}\right\}$$

$$L_{L4}^{(m)} = \theta\beta\left\{\sum_{p=0}^{3}\sum_{i=0}^{m+p}\begin{pmatrix} 3 \\ p \end{pmatrix}\begin{pmatrix} m+p \\ i \end{pmatrix}\frac{(m+4)!\,(-1)^{p+i}}{4\,(3-p)!\,(m+p)!\,(4\theta-p\theta+i\theta+1)}\right\}$$

4.4. LH-Moments for PFD

According to Wang (1997) the LHM assigns more weight to larger values of the data. By using (7) the LHM of PFD-II are defined as

$$L_{H1}^{(s)} = \frac{(s+1)\theta\beta}{\theta(s+1)+1}$$

$$L_{H2}^{(s)} = \frac{(s+2)!\theta\beta}{2}\left[\frac{1}{(s+1)!\,(\theta(s+2)+1)} - \frac{1}{s!}\left\{\sum_{p=0}^{1}\begin{pmatrix} 1 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+1)+1}\right\}\right]$$

$$L_{H3}^{(s)} = \frac{(s+3)!\theta\beta}{3}\left[\frac{1}{(s+2)!\,(\theta(s+3)+1)} - \frac{2}{(s+1)!}\left\{\sum_{p=0}^{1}\begin{pmatrix} 1 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+2)+1}\right\}\right.$$
$$\left. + \frac{1}{2(s!)}\left\{\sum_{p=0}^{2}\begin{pmatrix} 2 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+1)+1}\right\}\right]$$

$$L_{H4}^{(s)} = \frac{(s+4)!\theta\beta}{4}\left[\frac{1}{(s+3)!\,(\theta(s+4)+1)} - \frac{3}{(s+2)!}\left\{\sum_{p=0}^{1}\begin{pmatrix} 1 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+3)+1}\right\}\right.$$
$$\left. + \frac{3}{2(s+1)!}\left\{\sum_{p=0}^{2}\begin{pmatrix} 2 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+2)+1}\right\} - \frac{1}{6(s!)}\left\{\sum_{p=0}^{3}\begin{pmatrix} 3 \\ p \end{pmatrix}\frac{(-1)^{p}}{\theta\,(s+p+1)+1}\right\}\right]$$

5. Coefficients of Power Function Distribution

The coefficient of variation (CV), coefficient of skewness (Sk) and coefficient of kurtosis (Kr) are considered important characteristics of a data set. Abdul-Moniem (2012) and Hosking (1990) proved that the sample moments ratios are the form of the CV, Sk and Kr and are more accurate than the MM coefficients. These coefficients are derived for PFD using MM, LM and TLM and compiled in Table 1.

Table 1: CV, Sk and Kr by MM, LM and TLM of Power Function distribution.

Moments	CV	Sk	Kr
		PFD-I	
MM	$\frac{1}{\sqrt{\theta(\theta+2)}}$	$\frac{2(1-\theta)\sqrt{\theta+2}}{(\theta+3)\sqrt{\theta}}$	$\frac{3(2+\theta)(3\theta^2-\theta+2}{\theta(\theta+3)(\theta+4)}$
LM	$\frac{1}{2\theta+1}$	$\frac{1-\theta}{3\theta+1)}$	$\frac{(2\theta-1)(\theta-1)}{(3\theta+1)(4\theta+1)}$
TLM	$\frac{1}{4\theta+1}$	$\frac{10(1-\theta)}{9(3\theta+1))}$	$\frac{5(2\theta-1)(\theta-1)}{4(5\theta+1)(6\theta+1)}$
		PFD-II	
MM	$\frac{1}{\sqrt{\theta(\theta+2)}}$	$\frac{2(1-\theta)\sqrt{\theta+2}}{(\theta+3)\sqrt{\theta}}$	$\frac{3(2+\theta)(3\theta^2-\theta+2}{\theta(\theta+3)(\theta+4)}$
LM	$\frac{1}{2\theta+1}$	$\frac{1-\theta}{3\theta+1)}$	$\frac{(2\theta-1)(\theta-1)}{(3\theta+1)(4\theta+1)}$
TLM	$\frac{1}{4\theta+1}$	$\frac{10(1-\theta)}{9(3\theta+1))}$	$\frac{5(2\theta-1)(\theta-1)}{4(5\theta+1)(6\theta+1)}$

6. Monte Carlo Simulation Study

In this section, a simulation study is carried out to compare the properties of the MM Estimators, LM Estimators and TLM Estimators for PFD-I and PFD-II distributions. Our results are similar to those of Hirano & Porter (2003) about the MLE. As the support of the PFD-II depend on the parameter of interest, so standard asymptotic distribution theory does not apply it. So far MLE estimates are compared with the moments estimates only in the case of PFD-I because its support does not depend on the parameter. This comparison is based on Biases, Root Mean Square Errors (RMSEs), CV, Sk and Kr. MATLAB-7 software is used for this analysis.

In simulation experiment different sample sizes $n \in$ (10; 25; 50; 100; 250; 500; 1,000; 5,000) for the different values of each parameter $\theta \in$ (0.5, 1.0, 3.0, 7.0, 10, 15) and $\beta \in$ (1.5, 5.0, 7.0) are considered. The estimates are calculated from 10,000 repeated samples. LL-moments and LH-moments estimators are not evaluated numerically because both of these moments do not use the full data.

In Figure 1, the first six sub figures show a pattern of bias in estimated parameter θ for different methods. Each sub figure is drawn for different values of θ and sample sizes n = 10, 25, 50, 100. Figure 1, shows that the bias increases as the value of the parameter increases. When $\theta = 0.5$ maximum bias is 0.06, and for $\theta = 1$ maximum bias is 0.12. So doubling the parameter value, bias increases tremendously. The same increasing trends for the remaining higher values of the parameters are also observed, even when $\theta = 15$ the bias increases up to 9 times. It is also observed that, as the sample size increases the bias reduces to zero. The same pattern was observed in the last sub figure in which the sample size is n = 250; 500; 1,000; 5,000. In all the cases for different parameter value and sample size the MLE estimator produces more biased results as compared to the MM, LM and TLM estimator. MM and LM estimator are equally good and provide more unbiased results than TLM estimator, because these two moments produce the same estimator and estimates for the parameter.

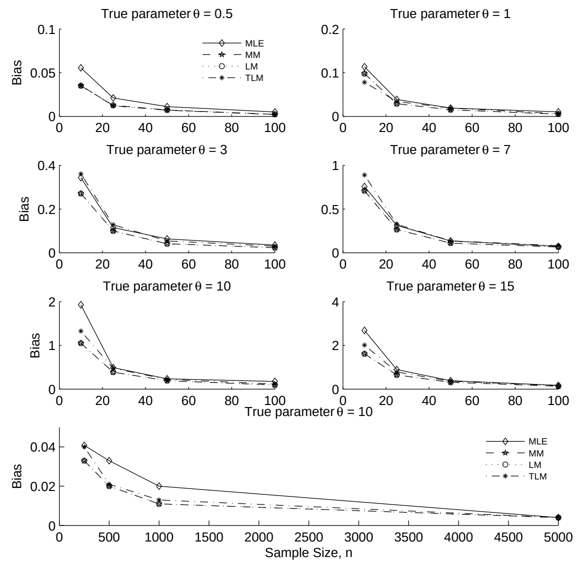

Figure 1: Biasness of the MLE, MM, LM and TLM estimator for the parameter θ of PFD-I with different sample size.

Figure 2 shows that the RMSE increases as increase the parametric value but decreases with the sample size increment. These results also indicate that the least preferable method for the parameter estimation is MLE for the PFD-I. The MME and LME estimator are equally more preferable than the TLM estimator because they have small RMSEs for all sample sizes from 10 to 5,000.

In Figure 3, the pattern of the Sk and Kr are displayed for PFD-I. The Sk and Kr for both the LM and TLM are approximately zero and the Kr is as usual greater than the Sk of the MM.

In the simulation study of PFD-II, the samples of sizes $n \in (10; 25; 50; 100; 250; 500; 1{,}000; 5{,}000)$ are considered. We have considered all the combinations of $\theta \in (0.5,\ 1.0,\ 3.0,\ 7.0,\ 10,\ 15)$ and $\beta \in (1.5,\ 5.0,\ 7.0)$ to generate the data and then use all the considered moments to evaluate the best moments for the parameter estimation. So the analysis is done for all combinations of parametric value for all the sample sizes and some of these results are presented in Figure 4, while the remaining results almost show the same pattern. It is observed that when the parameter values and sample sizes are small than only at that time TLM estimators provide comparatively more biased results, as the first sub figure shows. For remaining all pairs of parameters bias of LM estimators $<$ TLM estimators $<$ MM estimators. So LMEs are the most preferable for all sample sizes as they produce the least biased results.

Figure 5 presents the trend of the RMSEs, skewness and kurtosis of selected results among all calculated results. RMSEs of the parameter β with all methods

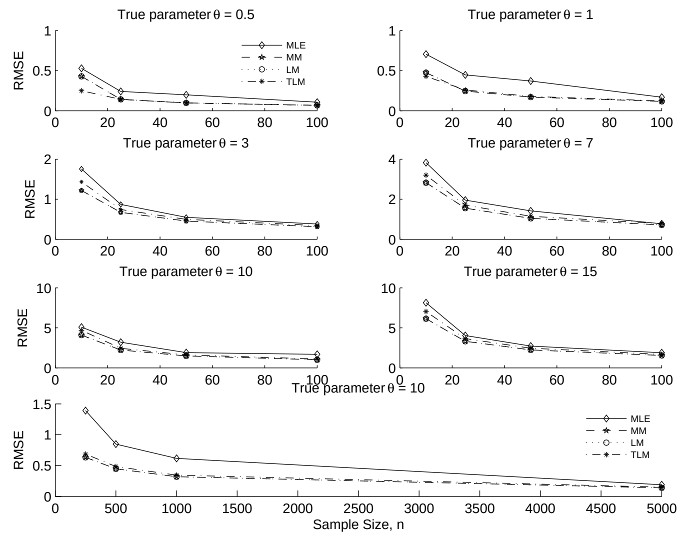

FIGURE 2: RMSE of the MLE, MM, LM and TLM estimator for the parameter θ of PFD-I with different sample size.

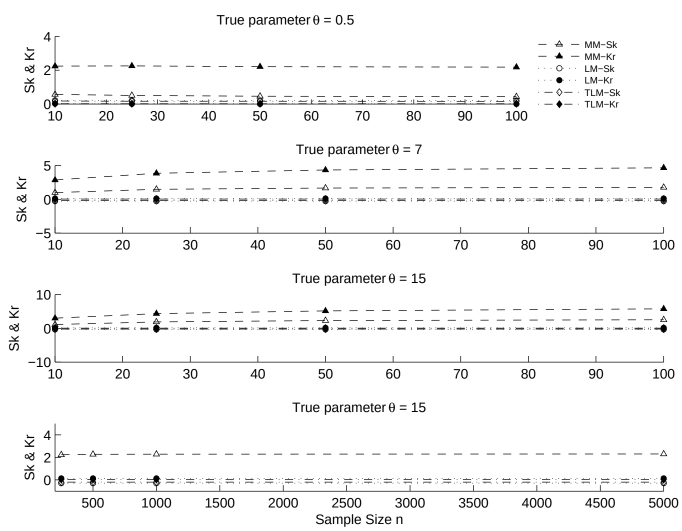

FIGURE 3: Skewness and kurtosis of the MM, LM and TLM of PFD-I with different sample size.

are less but LM estimators produce the most minimum value. And for the parameter θ MM estimators produce most biased results and LM estimators produce minimum biased results as compared to all considered moments. The skewness and kurtosis of MM is greater than LM and TLM as is indicated by the 2^{nd} sub figure in Figure 5.

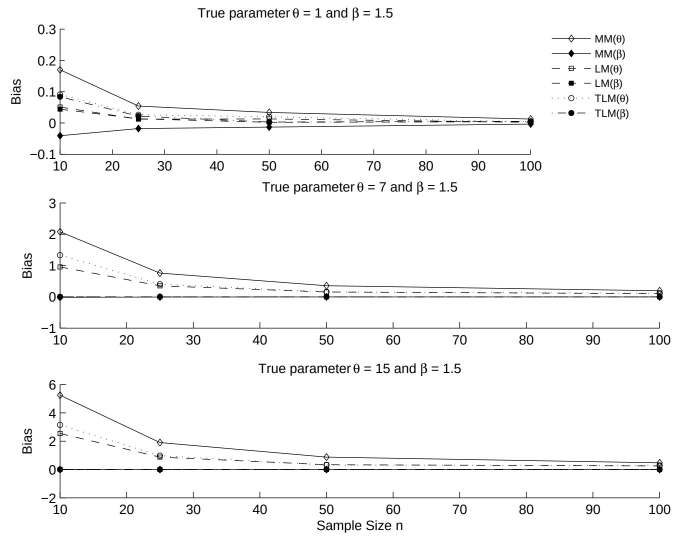

FIGURE 4: Biasness of the MM, LM and TLM estimators for the parameters (θ, β) of PFD-II with different sample size.

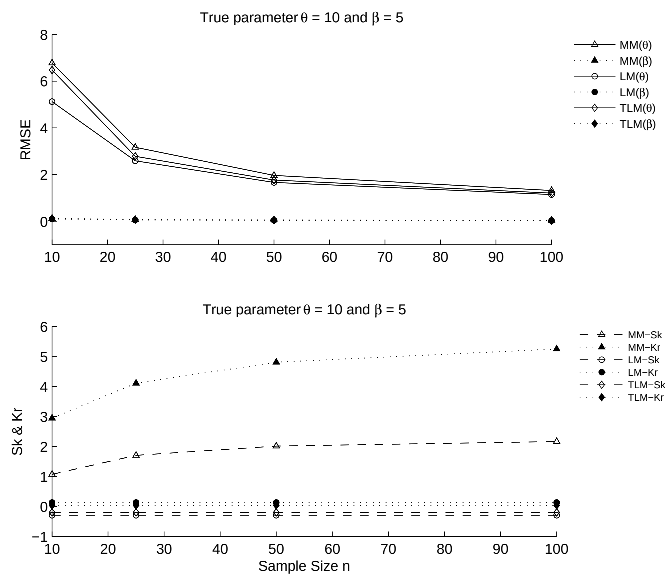

FIGURE 5: RMSE, skewness and kurtosis of the MM, LM and TLM estimators of PFD-II with different sample size.

7. Conclusion

In this study, expressions of the first four linear moments, CV, Sk and Kr are derived for PFD (type I and II). Parameters are estimated by MM, LM, TLM and MLE and following interesting results have been observed from the simulation study. The parameter estimates of MM and LM are the same for the PFD-I because these moments have produced the same estimators. The results of the TLM estimator are slightly higher than the MM and LM estimator while MLE results are more biased among all moments. Higher values of parameter produce more biased results but biasedness gets reduced with higher sample size. The RMSEs also support the MM and LM estimators as compared to the TLM and MLE estimators. Both the shape and scale parameter of PFD-II are over estimated by the MM estimators with high RMSEs but as the sample size increased the estimates become closer to the parameters and bias reduces to zero. The TLM estimator provides better results than the MM estimator but not as efficient as the LM estimator. Similarly MM and TLM estimators are overestimated for higher values of parameters in small sample sizes. The estimates are close to the parametric values by LM even in small sample size. On the basis of RMSEs criteria LM emerges as the best procedure. The coefficients of skewness and kurtosis are more consistent for LMEs than MMEs and TLMEs for all considered sample sizes and parameter values. So keeping in mind all of the above the description method of LM is the best in all aspects. We suggest its use while estimating the parameter of PFD.

Acknowledgement

We acknowledge the support of the anonymous reviewers, whose constructive comments and suggestions helped to improve the quality of the paper.

[Received: March 2014 — Accepted: November 2014]

References

Abdul-Moniem, I. B. (2007), 'L-moments and TL-moments estimation for the exponential distribution', *Far East Journal of Theoretical Statistics* **23**(1), 51–61.

Abdul-Moniem, I. B. (2009), 'TL-moments and L-moments estimation for the Weibull distribution', *Advances and Application in Statistics* **15**(1), 83–99.

Abdul-Moniem, I. B. (2012), 'L-moments coefficients of variations', *International Journal of Statistics and Systems* **7**(2), 97–100.

Abdul-Moniem, I. & Selim, Y. M. (2009), 'TL-moments and L-moments estimation for the generalized Pareto distribution', *Applied Mathematical Sciences* **3**(1), 43–52.

Ahsanullah, M. & Kabir, A. L. (1974), 'A characterization of the power function distribution', *The Canadian Journal of Statistics* **2**(1), 95–98.

Ali, M. M., Woo, J. & Nadarajah, S. (2005), 'On the Ratic X/(X+ Y) for the Power Function Distribution', *Pakistan Journal of Statistics* **21**(2), 131–138.

Asquith, W. H. (2007), 'L-moments and TL-moments of the generalized lambda distribution', *Computational Statistics and Data Analysis* **51**(9), 4484–4496.

Bayazit, M. & Onoz, B. (2002), 'LL-moments for estimating low flow quantiles', *Hydrological sciences journal* **47**(5), 707–720.

Chang, S. (2007), 'Characterization of the power function by the independence of record values', *Journal of the Chungcheong Mathematical Society* **20**(2), 140–145.

El-Magd, A. & Noura, A. T. (2010), 'TL-moments of the exponentiated generalized extreme value distribution', *Journal of Advanced Research* **1**(4), 351–359.

Elamir, E. A. & Seheult, A. H. (2003), 'Trimmed L-moments', *Computational Statistics and Data Analysis* **43**(3), 299–314.

Hirano, K. & Porter, J. R. (2003), 'Asymptotic Efficiency in Parametric Structural Models with Parameter-Dependent Support', *Econometrica* **71**(5), 1307–1338.

Hosking, J. R. M. (1990), 'L-moments: Analysis and estimation of distributions using linear combinations of order statistics', *Journal of the Royal Statistical Society. Series B* **52**, 105–124.

Hosking, J. R. M. & Wallis, J. R. (1995), 'A comparison of unbiased and plotting-position estimators of L moments', *Water Resources Research* **31**(8), 2019–2025.

Karvanen, J. (2006), 'Estimation of quantile mixtures via L-moments and trimmed L-moments', *Computational Statistics and Data Analysis* **51**(2), 947–959.

Meniconi, M. & Barry, D. M. (1996), 'The power function distribution: A useful and simple distribution to assess electrical component reliability', *Microelectronics Reliability* **36**(9), 1207–1212.

Rahman, H., Roy, M. K. & Baizid, A. R. (2012), 'Bayes estimation under conjugate prior for the case of power function distribution', *American Journal of Mathematics and Statistics* **2**(3), 44–48.

Saran, J. & Pandey, A. (2004), 'Estimation of parameters of a power function distribution and its characterization by kth record values', *Statistica* **64**(3), 523–536.

Shabri, A., Ahmad, U. N. & Zakaria, Z. A. (2009), 'TL-moments and L-moments estimation for the generalized Pareto distribution', *Journal of Mathematics Research* **10**(10), 97–106.

Shahzad, M. & Asghar, Z. (2013), 'Comparing TL-Moments, L-Moments and conventional moments of dagum distribution by simulated data', *Revista Colombiana de Estadistica* **36**(1), 79–93.

Sinha, S., Singh, P., Singh, D. & Singh, R. (2008), 'Power function distribution characterized by dual generalized order statistics', *Journal of Reliability and Statistics Studies* **1**(1), 18–27.

Tavangar, M. (2011), 'Power function distribution characterized by dual generalized order statistics', *Journal of Iranian Statistical Society* **10**(1), 13–27.

Vogel, R. M. & Fennessey, N. M. (1993), 'L-moment diagrams should replace product moment diagrams', *Water Resources Research* **29**(6), 1745–1752.

Wang, Q. J. (1997), 'LH moments for statistical analysis of extreme events', *Water Resources Research* **33**(12), 2841–2848.