Deflection Reliability Analysis for Composite Steel Bridges

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Received: 18 June 2022 | Revised: 6 July 2022 | Accepted: 17 July 2022

Abstract-Reliability methods offer a very efficient serviceability assessment of structures with randomness due to geometry, material, and loading. Al-Awsej composite bridge in Diyala-Iraq with a span of 33.2m has been studied and its deflection reliability index for three lifespans was estimated and compared with the reliability target index. The reliability indices of the bridge have been evaluated through the First-Order Reliability Method (FORM) and Monte Carlo Simulation (MCS) method. MCS has adopted Matlab functions to generate pseudo-random numbers for the considered parameters, but it requires large sample sizes to estimate the small probabilities of failure. That leads to the use of the reduction variance methods such as the Importance Sampling (IS) method. Four cases of random loading were included: dead load and three cases of live loads, i.e. uniformly distributed load with knife-edge load, military load, and sidewalk load. Some assumptions are needed to assess the system behavior, where the bridge is represented as a parallel system with uncorrelated and perfect correlated girders. The reliability index of the composite bridge in the two cases was investigated for lifespans of 1, 10, and 50 years. For the uncorrelated case, the system shows the reliability index in the range of 5 and 4. In contrast, the correlated case offers a range between 4 and 2. With these assumptions, the results show that no failure occurs, hence the reliability index of the system is still within range of the target.

Keywords-steel girder bridge; statistical characteristics; FORM; MCs; importance sampling; parallel system; reliability

I. INTRODUCTION

Structure design is associated with a substantial level of uncertainty due to the limited information in the estimation of structural parameters. In practice, most structural engineering designs are based on deterministic parameters where structural performance is determined using a deterministic model for simplification and often ignore the variations in material properties, structure geometry, and applied loads. In previous studies, structural reliability methods that rationally evaluate the safety of complex structures or systems with unusual designs have been developed [1]. In this study, these methods will be used to assess the reliability of a bridge by determining the reliability indices of the steel girders considering the bridge is a parallel system and comparing it with target reliability that ensures that structures meet the specified safety level. Thus, reliability is an approach to determine the relationship between an element and system reliability [2].

II. COURBON'S METHOD OF ANALYSIS

Courbon's method is one of the earliest rational analyses of bridges and is very popular due to its simplicity. The reaction factor for individual longitudinal girders is given by [3]:

$$R_i = \frac{P}{N}\left[1 + \left(\frac{N \times e \times x_i}{\sum {x_i}^2}\right)\right] \quad (1)$$

where R_i is the reaction load distribution factor of the girder, P the total concentrated live load on the span, N the number of longitudinal girders, e the eccentricity of live load to the axis of the bridge, and x_i is the distance from the girder to the central axis of the bridge.

III. LOAD APPLICATION

The applied loads on the composite bridge have been determined based on the Iraqi code specifications. The self-weight of the pavement, deck, sidewalk, supporting girders, and bracing have been determined depending on the cross-section dimensions and material densities. The own weight of the handrail has been estimated based on Iraqi standard specifications for road bridges [4]. Two cases, namely lane live and military load have been considered and applied according to the Iraqi standard. For a span of 33.2m, the lane load has a value of 28.7kN/m per lane with a Knife-Edge Load (KEL) of 120kN per lane. The military load consists of 900kN (Class 100) tracked vehicles.Two tracked vehicles have applied for a carriageway width of 9m [4].

IV. RELIABILITY ANALYSIS OF THE SYSTEM

Structural reliability analysis is based on the limit state design $g(\boldsymbol{X})$, where $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ represents the set of random variables that have some statistical information. The limit $g(\boldsymbol{X}) = 0$ separates the failure domain ($g(\boldsymbol{X}) < 0$) and the safety domain ($g(\boldsymbol{X}) > 0$) [7]. The probability of failure can be defined by:

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$$P_f = P[g(\boldsymbol{X}) \leq 0] \quad (2)$$

The major source of uncertainty is actions with statistically independent maximums each year. The values of the probability of failure, P_{f_n} and of the reliability index, β_n for each reference period may be estimated using [5]:

$$P_{f_n} = 1 - (1 - P_{f_1})^n \quad (3)$$

and

$$\beta_n = -\Phi^{-1}\left(-P_{f_n}\right) \quad (4)$$

where Φ, and Φ^{-1} are the functions of the standard normal cumulative distribution.

The target reliability is a design constraint that ensures that structures meet the specified safety level. Table I shows the target values of reliability indices for various limit states and periods [6].

TABLE I. TARGET RELIABILITY INDICES

Reference period *T* (years)	**Target reliability index *β* for serviceability**	**Target reliability index *β* for moment**
1	2.9	4.7
10	2.2	4.2
50	1.5	3.8
100	1.1	3.6
200	0.6	3.5

V. RELIABILITY ANALYSIS METHODS

The evaluation of the failure probability of the structure in a closed-form is difficult and almost impossible. As a result, various analytical and numerical approaches have been developed, e.g. Taylor-series-based approaches, such as the First-Order Reliability Method (FORM), and simulation-based methods, such as Monte Carlo Simulation (MCS). The FORM approximation approaches are efficient for simple issues with few random variables, but for more complex problems with many random variables, MCS seem to be more reliable [7].

A. *First-Order Reliability Method*

It is possible to expand the original model into an infinite Taylor Series (TS) around the mean values:

$$g(X) = g(\mu_X) + (X - \mu_X)\frac{dg}{dX} + \frac{1}{2}(X - \mu_X)^2\frac{d^2g}{dX^2} + \cdots + \frac{1}{n!}(X - \mu_X)^n\frac{d^ng}{dX^n} \quad (5)$$

where the function and derivatives are evaluated at μ_X. It is common to include only linear terms, assuming that random input variables are independent. A function $g\,(\boldsymbol{X})$ of N independent random variables can be approximated by linear terms of the TS, which are [8]:

$$E(Y) \approx g\left(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}\right) \quad (6)$$

and

$$Var(Y) \approx \sum_{i=1}^{N} \sigma_{X_i}^2 \left(\frac{\partial g}{dX_i}\right)^2 \quad (7)$$

The limit state function is: $(x) = R - Q$, R, and Q both are random variables uncorrelated and assumed to be normally distributed. The reliability index β is evaluated as a function of mean and standard deviations of resistance R and load Q as given by [8]:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (8)$$

B. *Monte Carlo Method*

The failure probability in MCS is the ratio of the number of samples in the failure domain to the total number of samples:

$$P_f = \frac{N_f}{N} = \frac{1}{N}\sum_{i=1}^{N} I[g(x) \leq 0] \quad (9)$$

where the sampling points x are generated according to the probability density function, N_f is the number of sampling points such that $g(x) \leq 0$, N is the total number of sampling points, $g(x)$ is the limit state, $I[g(x)]$ is an indicator function taking values of unity when are $g(x) \leq 0$ and zero otherwise. With increasing N, the accuracy of this estimation improves, and more simulations are needed to predict a smaller failure probability [9]. The required number of trials N of MCS is approximated by:

$$N \approx \frac{(1 - P_f)}{(COV^2 \times P_f)} \quad (10)$$

where COV is the coefficient of variation of the response estimates smaller than 0.1 and P_f is usually between 10^{-2} to 10^{-6}. The total number required for that simulation is determined by:

$$N \approx \frac{(1 - 10^{-2})}{(0.1^2 \times 10^{-2})} \approx 9900 \quad (11)$$

C. *Importance Sampling*

The main idea of IS is to distribute the sampling points in the most important area so that the failure probability evaluation may be completed faster. The failure probability can be estimated as (9) is rewritten as follows:

$$P_f = \frac{1}{N}\sum_{j=1}^{N} I(x_i)\frac{fx(x_j)}{hx(x_j)} \quad (12)$$

where the sampling points x_j , $j = 1, \ldots, N$ are generated according to the distribution hx instead of fx . The effectiveness of IS is dependent on choosing an adequate $h_x(x)$ so that the probabilistic sampling in (12) may be prioritized for the most important area, resulting in a higher convergence rate. Although no overall conclusion has been reached on the best choice of $h_x(x)$, it has been suggested that the Most Probable Point (MPP) and its surroundings can be a suitable selection for the most important area [10].

VI. SYSTEM RELIABILITY

The system reliability is affected by component reliability and several other parameters, such as the correlation among the component resistances and the system type. Structural system reliability is determined by considering the system failure rather than a single component failure [11]. The series system means that the entire system fails. The formula of the failure system for the statistically independent elements is given by:

$$P_f = 1 - \prod_{i=1}^{n}(1 - P_{f_i}) \quad (13)$$

where P_{f_i} and P_f are the probability of failure for the element i and the system respectively. The equation of the system failure for perfectly correlated elements can be defined as:

$$P_f = max[P_{f_i}] \quad (14)$$

A parallel system is an overall system that fails after all its elements have failed. The probability of failure for the entire system when the elements are statistically independent and perfectly correlated respectively is defined as [11]:

$$P_f = \prod_{i=1}^{n} P_{f_i} \quad (15)$$

$$P_f = min[P_{f_i}] \quad (16)$$

VII. CASE STUDY

The Al-Awsej composite bridge is constructed in Awsej valley at Diyala town in Iraq. It has 33.2m length and 14m width and consists of three longitudinal steel girders connected by steel bracings (2L100×10). The I-steel plate girders are non-symmetrically with an upper flange of 400×30mm and two lower flanges of 510×30mm and 490×30mm respectively. The floor bridge consists of a concrete deck slab with a thickness of 250mm and pavement bitumen of 60mm extending above a carriageway width of 9m, with a 2.5m sidewalk width and handrail of 1m on each side.

(a)

(b)

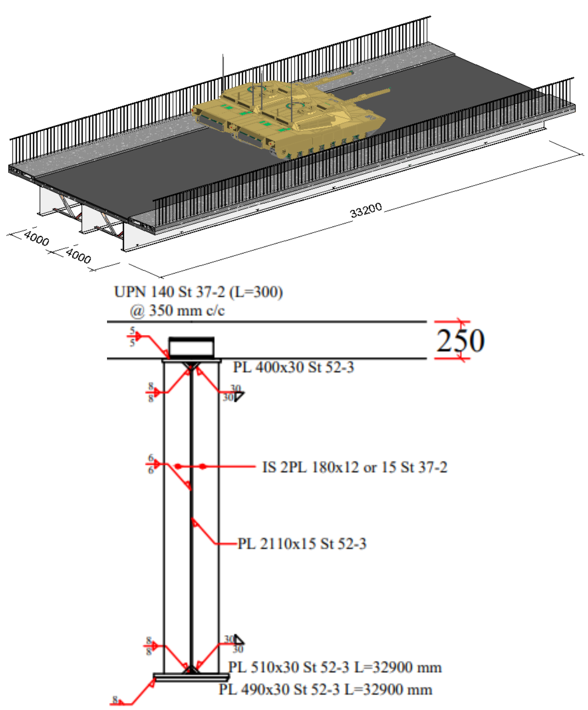

Fig. 1. The composite steel bridge.

VIII. STATISTICAL CHARACTERISTICS OF VARIABLES

In this article, the applied loads, beam span, cross-section dimensions, and modulus of elasticity are considered random variables. The statistical characteristics of these variables have been collected based on previous studies and are summarized in Table II.

TABLE II. STATISTICAL CHARACTERISTICS OF VARIABLES

Uncertainty	Mean/Nominal	COV	Distribution type
Weight of the deck slab	1.05	0.10	Normal
Weight of the girder	1.03	0.08	Normal
Weight of the pavement	1.00	0.25	Normal
Weight of the military	1.10	0.18	Lognormal
Uniform distribution with knife-edge live loads	1.10	0.18	Lognormal
Sidewalk live loads	1.10	0.18	Lognormal
Length of the girder	1.00	0.000502	Normal
Width of the Flange girder	1.00	0.00403	Normal
Thickness of the flange and web	1.05	0.044	Lognormal
Depth of the web girder	0.996	0.000486	Normal
Modulus of elasticity	0.993	0.034	Normal

IX. STATISTICAL PROPERTIES FOR THE DEFLECTION OF GIRDERS

Two methods have been used to analyze the exterior and interior girders: First-Order approximation and MCS.

A. *First-Order Approximation Method*

The determination of statistical characteristics of girder deflection has been determined due to the uncertainties in the moment of inertia I, span L, modulus of elasticity E, and loads. Self-weight and superimposed loads DL are considered uncertain due to the randomness in the weight of concrete deck slab, girders, and pavements. Randomness in the own weight of bracing, sidewalk, and handrail has been neglected. Live Load (LL) uncertainties have been considered for the cases of military, sidewalk, uniform distributed, and knife-edge loads. Table III provides the mean and standard deviation of deflection.

TABLE III. STATISTICAL CHARACTERISTICS OF THE GIRDER DEFLECTION - FORM

Loads		Nominal (mm)	Mean (mm)	Standard deviation (mm)	COV
Exterior Girder	Self-weight and superimposed loads	70.44	70.72	7.10	0.10
	Military with sidewalk live loads	52.54	56.11	10.48	0.18
	Uniformly distributed with knife-edge and sidewalk live loads	61.26	65.45	12.22	0.18
Interior Girder	Self-weight and superimposed loads	51.95	51.60	6.50	0.13
	Military and sidewalk live loads	52.54	56.11	10.48	0.18
	Uniformly distributed with knife-edge and sidewalk live loads	49.11	52.45	9.80	0.18

B. *Monte Carlo Simulation*

The MCS was utilized for deflection of the exterior and interior girders. A Matlab code has been used to generate pseudo-random sampling. Six samples, each with a size of 1000, have been generated to include the randomness in the moment of inertia I, the span of the girder L, and the modulus of elasticity E.

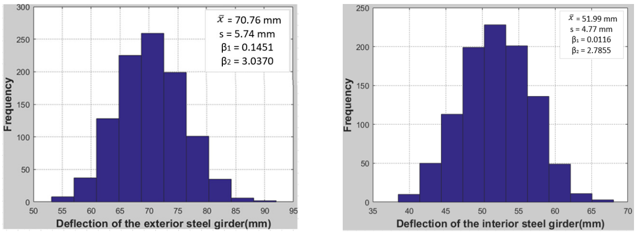

Fig. 2. Deflection sample for the case of dead and superimposed loads.

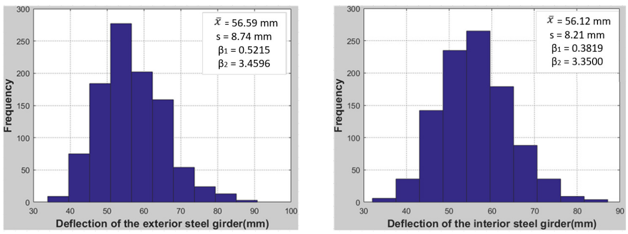

Fig. 3. Deflection sample for the case of military and sidewalk live loads.

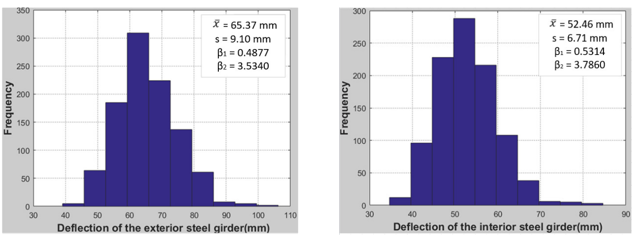

Fig. 4. Deflection sample for the case of uniform lane with knife-edge and sidewalk live loads.

Three samples have been considered for each of the exterior and interior girders. For each girder, the samples represent the randomness in dead loads, military with sidewalk loads, and uniform lane with knife-edge and sidewalk loads respectively. Table III shows that each random variable has been generated based on preselected statistical parameters and probability density function. The sample data are presented in the histogram Figures 2-4, and the statistical characteristics for the deflection of the girders are shown in Table IV.

TABLE IV. STATISTICAL CHARACTERISTICS OF THE GIRDER DEFLECTION - MCS

	Loads	Mean (mm)	Standard deviation (mm)	COV	Distribution type
Exterior Girder	Self-weight and superimposed loads	70.76	5.74	0.08	Normal
	Military and sidewalk live loads	56.59	8.74	0.15	Lognormal
	Uniformly distributed with knife-edge and sidewalk live loads	65.37	9.10	0.13	Lognormal
Interior Girder	Self-weight and superimposed loads	51.99	4.77	0.09	Normal
	Military and sidewalk live loads	56.12	8.21	0.15	Lognormal
	Uniformly distributed with knife-edge and sidewalk live loads	52.46	6.71	0.13	Lognormal

X. RELIABILITY ANALYSIS FOR STEEL GIRDER DEFLECTION

FORM and MCS methods have been adopted to estimate the reliability associated with girder deflection. Some assumptions have been adopted in the FORM, assuming all independent variables have a normal distribution. The mean and standard deviation of girders deflection in Table III have been used to evaluate the reliability index β and the probability of failure, P_f by:

$$\beta = \frac{\Delta_{Allowable\ max} - E(\Delta)}{s_\Delta} \quad (17)$$

where $\Delta_{Allowable\ max}$ represented a threshold limit of 80mm that was adopted as a camper in the design of the Al-Awsej bridge case study.

TABLE V. RELIABILITY INDEX AND PROBABILITY OF FAILURE BY FORM

	Loads	1 year		10 years		50 years	
		P_f	β	P_f	β	P_f	β
Exterior Girder	Self-weight and superimposed loads	0.09	1.31	0.63	−0.34	0.99	−2.47
	Military with sidewalk live loads	0.001	3.09	0.009	2.33	0.048	1.66
	Uniform distributed with knife edge and sidewalk live loads	0.03	1.89	0.26	0.65	0.77	−0.75
Interior Girder	Self-weight and superimposed loads	6×10^{-6}	4.37	6×10^{-5}	3.84	0.0003	3.42
	Military and sidewalk live loads	0.001	3.09	0.009	2.33	0.048	1.66
	Uniformly distributed with knife edge and sidewalk live loads	0.0001	3.68	0.001	3.04	0.005	2.52

TABLE VI. RELIABILITY INDEX AND PROBABILITY OF FAILURE BY IS METHOD

	Loads	N_f	1 year		10 years		50 years	
			P_f	β	P_f	β	P_f	β
Exterior Girder	Self-weight and superimposed loads	39	0.039	1.76	0.33	0.44	0.86	−1.09
	Military and sidewalk live loads	≈0	≈0	/	≈0	/	≈0	/
	Uniformly distributed with knife edge and sidewalk live loads	9	0.009	2.36	0.09	1.36	0.36	0.35
Interior Girder	Self-weight and superimposed loads	≈0	/	/	≈0	/	≈0	/
	Military and sidewalk live loads	≈0	/	/	≈0	/	≈0	/
	Uniformly distributed with knife-edge and sidewalk live loads	≈0	/	/	≈0	/	≈0	/

Regarding the live load deflection, R has been taken as the limit ratio of $L/375$ according to The American Association of State Highway and Transportation Officials (AASHTO) [12]. In the MCS method, importance sampling techniques have been used to reduce the number of simulations. These methods rely on random sampling from random variable distributions to obtain the response uncertainty and the numerical estimate probability of failure. The assessment of the probability of failure is based on the samples that exceed the threshold limit for girder deflection. The reliability index and probability of failure for exterior and interior girders for the three loading cases and through a lifetime of 1, 10, and 50 years are summarized in Tables V and VI. The results show that the first-order approximate analytical method has the ability to show the values of the probability of small failures, unlike MCS. Also, these Tables noted that the probability of failure increases with the passage of time thus, the reliability indices of the system decrease.

XI. Reliability Analysis for Composite Bridge Deflection

The composite bridge has been simulated as a system of elements (girders). The live load was distributed to the girders through a flexible medium and the deck slab and bracing were represented as the applied load to the girders.

In this case, the composite bridge can be considered as a parallel system. Hence, the reliability has been estimated with two instances of correlation: uncorrelated and perfect correlated. The composite bridge reliability depended on the probability of failure and the reliability index of the steel girders deflection. The results of MCS show that the probability of failure approaches zero, so the reliability of the parallel system was estimated based on the outcome of FORM (Table VII), which showed that the probability of failure increases with the passage of time, thus the reliability indices of the system decrease.

TABLE VII. RELIABILITY INDEX AND PROBABILITY OF FAILURE FOR PARALLEL BRIDGE DEFLECTION

Loads	One year		Ten year		fifty year	
	P_f	β	P_f	β	P_f	β
Uncorrelated, ρ_{ij}=0						
Self-weight and superimposed loads	5.42×10^{-8}	5.31	5.42×10^{-7}	4.88	2.70×10^{-6}	4.55
Military and sidewalk live loads	0.1×10^{-8}	5.99	0.1×10^{-7}	5.61	0.05×10^{-6}	5.33
Uniformly distributed with knife-edge and sidewalk live loads	8.41×10^{-8}	5.23	8.41×10^{-7}	4.79	4.20×10^{-6}	4.45
Perfect correlated, ρ_{ij}=1						
Self-weight and superimposed loads	6×10^{-6}	4.37	6×10^{-5}	3.84	0.0003	3.42
Military and sidewalk live loads	0.001	3.09	0.009	2.33	0.048	1.66
Uniformly distributed with knife edge and sidewalk live loads	0.0001	3.68	0.001	3.09	0.005	2.52

XII. Conlcusion

A reliability analysis of Al-Awsej composite bridge has been performed in this paper, considering the randomness in geometry, material, and applied loads. The bridge is located in Diyala-Iraq and has a span of 33.2m. The main conclusions of the current study are:

- The first-order reliability method has estimated two moments (mean and standard deviation) of the girder deflection. At the same time, the MCS method determined four moments (mean, standard deviation, skewness, and peakedness) in addition to the distribution type.
- The MCS with IS method allowed the generation of 1000 pseudo-random samples instead of a large number of samples to estimate the probability of failure of a girder's deflection by distributing the failure sampling in the most crucial area. Hence, the failure probability evaluation was completed while saving cost and time.
- With an increase in the correlation between girders, the reliability decreases and the probability of failure increases.
- The reliability index of the deflection with two cases of correlation (uncorrelated and perfect correlated) was greater than the target reliability of the deflection.

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