# An Experimental Study of a Combined Oblique Cylindrical Weir and Gate Structure 

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Received: 2 January 2023 | Revised: 17 February 2023 | Accepted: 20 February 2023


#### Abstract

In the present study, the effects of different oblique angles and diameters on the cylindrical weir and below a gate as a combined device have been studied experimentally. For this purpose, sixteen models of combined cylindrical weir-gate structures have been tested in a laboratory flume. These models had four different oblique angles $\alpha\left(\mathbf{3 0}^{\mathbf{0}}, \mathbf{4 5}^{\mathbf{0}}, \mathbf{6 0 ^ { \circ }}\right.$, and $\mathbf{9 0}^{\mathbf{}}$ ). For each angle, the diameter of cylindrical weir-gate changed four times $(4,7.3,9$, and 11 cm$)$. The results of all models indicate that the theoretical discharge $\left(\mathrm{Q}_{\mathrm{th}}\right)$ is inversely proportional to the ratio of diameter to height $(\mathrm{D} / \mathrm{h})$ and length to height $(\mathrm{L} / \mathrm{h})$. As the diameter increases and oblique angles decrease, the actual discharge ( $\mathrm{Q}_{\mathrm{ac}}$ ) increases. A general expression was established linking $\mathrm{Q}_{\mathrm{th}} / \mathrm{g}^{1 / 2} h^{5 / 2}$ and the discharge coefficient $\mathrm{C}_{\mathrm{d}}$ with $\mathrm{D} / \mathrm{h}, \mathrm{L} / \mathrm{h}$, and alfa. The discharge coefficient ranged from 0.55 to 0.99 for various oblique angles and decreased as the angle increased. A strong correlation was observed between the estimated and the calculated values.


Keywords-combined flow; cylindrical weir; gate; discharge coefficient; theoretical discharge

## I. INTRODUCTION

Weirs and gates are the most commonly used and important structures for measuring water flow, regulating flow in irrigation channels, and redirecting it from the main channel to secondary channels [1]. Weirs are structures that regulate water surface and flow control in water conveyance channels and hydraulic structures. One of their demerits is that they need to be cleaned of sediment and trash periodically. Gates are used extensively for flow control and water measurement. One disadvantage of the gates is that they retain floating materials. In order to maximize their advantages, weirs and gates can be combined together in one device, so that water could pass over the weir and below the gate simultaneously. It may reduce the effect of sediments and floating materials upstream and may affect scoring downstream [2-4]. One such combination is the cylindrical weir-gate structure. Regarding the form of the combined cylindrical weir-gate structures, it has some advantages including easy design, sediments and floating material flow, higher flow discharge coefficient, lower cost, and the combination of those structures in one device can minimize the disadvantages of separate use of each device.

The performance of this combination has been studied extensively. Authors in [5] presented the effects of hydraulic and geometrical parameters, viscosity, and surface tension in the combined flow over rectangular weirs and rectangular sharp-crested gates. They found that the trend of variation in sloping bed cases was similar to that in horizontal beds. Authors in [6] studied the characteristics of free flow through a combined triangular weir and rectangular gate. They found that the values of theoretical discharge were inversely proportional to the geometrical dimensionless parameters and directly proportional to the distance between the bottom of the weir and the upper edge of the gate. Authors in [7] investigated the coefficient of discharge $\left(\mathrm{C}_{\mathrm{d}}\right)$ for a combined rectangular weir with semi-circular gate. They showed that the value of $\mathrm{C}_{\mathrm{d}}$ increases when the width of the weir increases and that the average value of $\mathrm{C}_{\mathrm{d}}$ was equal to 0.695 . Also, it was found that the value of $\mathrm{C}_{\mathrm{d}}$ increases with increasing distance between the weir and the gate with an average value of $\mathrm{C}_{\mathrm{d}}$ equal to 0.74 . Authors in [8, 10] proposed new methods to improve the performance of sharp crested triangular and curved plan form weirs. In these studies, the authors varied the vertex angles and
kept the weir height constant, and conducted tests to determine the discharge coefficients. Based on the results of these tests, they proposed two empirical equations for estimating the discharge coefficients. Authors in [11] studied the flow over a sharp crested weir and under a gate in a combined oblique structure as flow measurement structure. They found that the major parameters affecting significantly on discharge coefficient were $h / a, \alpha, L / B, D / a$, while the value of $C_{d}$ ranged from 0.623 to 0.403 . Authors in [12] experimentally studied the effects of canal size on the discharge coefficients of cylindrical weir-gate by using two flumes. They found that the value of $\mathrm{C}_{\mathrm{d}}$ in large canals ranged from 0.75 to 1.05 and in small canals from 0.55 to 0.9 . Authors in [13] studied the effect of vertical movement of a cylindrical weir-gate on free flow hydraulics. The experimental results showed that the gate opening change was in inverse relation with the discharge coefficient change, so that the maximum and minimum discharge coefficient were visible in the cylindrical weir and cylindrical gate, respectively. Furthermore, in a constant diameter and constant discharge, discharge coefficient changes had an increasing process by the decreases of the gate opening height. Besides, in a constant discharge and gate opening height, the discharge coefficient decreased with an increase in the structure diameter.

Authors in [14] investigated the effect of weir height on the performance of sharp crested rectangular plan form weirs. They found that weirs with lower heights had higher capacity and better performance compared to those with higher heights. Authors in [15] experimentally examined the behavior of sharp crested triangular plan form weirs under free flow conditions using a range of weir models that varied in vertex angle and weir height. They found that the discharge coefficient decreases with an increase in the relative head above crest and that weirs with small vertex angles have low discharge coefficients. They presented an empirical equation for predicting discharge coefficients based on the relative head above crest and vertex angle, and concluded that weirs with small vertex angles and low heights have higher efficiency and better performance. In an extensive experimental study, authors in [16] evaluated the hydraulic performance of circular crested triangular weirs under free flow conditions using multiple models with different vertex angles, weir heights, and crest diameters. They found that the discharge coefficient decreases with an increase in the relative head above crest, and that weirs with small vertex angles, low heights, and small crest diameters have better performance. They presented a simple power equation for estimating the discharge coefficient based on the ratio of water depth above crest to weir height, the ratio of head above crest-to-crest diameter, and the vertex angle. In a more recent study, authors in [17] examined the performance of semi-circular crested normal and oblique weirs with flood embankments and found that the discharge coefficient decreases when the crest radius and height increase. Authors in [18] conducted an optimization study on the shape of labyrinth spillways in order to minimize construction costs and determine the discharge coefficient in terms of side wall angle and relative head above crest. Noori, [19] attempted to increase the discharge capacity of oblique weirs by rounding their crests. He presented an empirical expression to determine $\mathrm{C}_{\mathrm{d}}$ in terms of H/P, H/D, and a (in radians), with a correlation
coefficient of 0.90 and a standard error of 0.01 . The weir with the highest discharge capacity had $\mathrm{a}=20, \mathrm{P}=20 \mathrm{~cm}, \mathrm{D}=4 \mathrm{~cm}$, and a $\mathrm{D} / \mathrm{P}$ ratio of 0.2 , and showed a percentage increase in discharge ranging from $147 \%$ to $175 \%$ compared to a sharp crested normal weir. Noori's results indicate that circular crested oblique weirs generally have higher discharge capacity than other weir shapes.

The main goal of the current research is to study the impact of various oblique angles and diameters on the hydraulic properties of a combined cylindrical oblique weir and gate structure. Additionally, this study aims to establish relationships for the prediction of flow rate and the coefficient of discharge.

## II. THEORETICAL DISCHARGE

The total theoretical discharge through a combined cylindrical weir and gate structure for free flow conditions can be obtained by adding the discharge under the gate to the discharge over the weir.

$$
\begin{align*}
& \mathrm{Q}_{\mathrm{th}}=\mathrm{Q}_{\mathrm{g}}+\mathrm{Q}_{\mathrm{w}}  \tag{1}\\
& \mathrm{Q}_{\mathrm{th}}=\mathrm{La} \sqrt{2 \mathrm{gH}}+\frac{2}{3} \mathrm{~L} \sqrt{\frac{2}{3} \mathrm{~g}^{3 / 2}} \tag{2}
\end{align*}
$$

where $Q_{\text {th }}$ is the total theoretical discharge, $Q_{g}$ is the discharge passing under the gate, $\mathrm{Q}_{\mathrm{W}}$ is the discharge passing over the weir, and $\mathrm{H}=\mathrm{h}+\mathrm{d}+\mathrm{a}$ the total head above the flume bed L . The variables used in the dimensional analysis were chosen to represent the experimental conditions, see Figure 1. Therefore, the factors affecting the flow in a combined structure are expected to be:
$Q_{\text {th. }}=f_{1}(h, a, D, L, B, g, \rho, \mu, \sigma, \alpha)$
where $Q_{t h}$ is the total theoretical discharge $\left(L^{3} / T\right)$, a is the gate height ( L ), D is the diameter of the weir or vertical distance between the lower edge of the weir and the upper edge of the gate $(\mathrm{L}), \mathrm{L}$ is the length of the weir $(\mathrm{L}), \mathrm{B}$ is the width of the flume ( L ), $g$ is the acceleration due to gravity $\left(\mathrm{L} / \mathrm{T}^{2}\right), \rho$ is the mass density of the fluid $\left(\mathrm{M} / \mathrm{L}^{3}\right), \mu$ is the dynamic viscosity of the fluid (M/TL), $\sigma$ is the surface tension $\left(\mathrm{M} / \mathrm{T}^{2}\right)$, and $\alpha$ represents the oblique angles from the direction of flow in degrees.

The general relationship among all variables is:

$$
\begin{equation*}
\mathrm{f}_{2}(\mathrm{Q}, \mathrm{~h}, \mathrm{a}, \mathrm{D}, \mathrm{~L}, \mathrm{~B}, \mathrm{~g}, \rho, \mu, \sigma, \alpha)=\mathrm{constant} \tag{4}
\end{equation*}
$$

Using the Buckingham's Pi-theorem, and selecting h, Q, and $\rho$ as repeating variables, (4) becomes:

$$
\begin{equation*}
\mathrm{f}_{3}\left(\frac{\mathrm{a}}{\mathrm{~h}}, \frac{\mathrm{D}}{\mathrm{~h}}, \frac{\mathrm{~L}}{\mathrm{~h}}, \frac{\mathrm{~B}}{\mathrm{~h}}, \frac{\sigma \mathrm{~h}^{3}}{\rho \mathrm{Q}^{2}}, \frac{\mu \mathrm{~h}}{\rho \mathrm{Q}}, \frac{\mathrm{~h}^{5} \mathrm{~g}}{\mathrm{Q}^{2}}, \alpha\right)=\text { constant } \tag{5}
\end{equation*}
$$

where $\mu h / \rho Q$ is the Reynolds number $R_{e}$ and $\sigma h^{3} / \rho Q^{2}$ is the Weber number $\left(W_{e}\right)$. The values of the Weber number and Reynolds number can be neglected at turbulent flow due to the neglected surface tension and viscosity. The values of B and a are constant during the experimental program, therefore can be dropped. Inversing and taking the square root of $h^{5} g / Q^{2}$, (5) can be written as:

$$
\begin{equation*}
\frac{\mathrm{Q}_{\mathrm{th}}}{\sqrt{\mathrm{~g} h^{5 / 2}}}=\mathrm{f}_{4}\left(\frac{\mathrm{D}}{\mathrm{~h}}, \frac{\mathrm{~L}}{\mathrm{~h}}, \alpha\right) \tag{6}
\end{equation*}
$$

The coefficient of discharge $\left(C_{d}\right)$ is determined by comparing the amount of fluid that is actually discharged in an experiment to the amount of fluid that would be discharged under ideal conditions.

$$
\begin{equation*}
\mathrm{C}_{\mathrm{d}}=\frac{\mathrm{Q}_{\mathrm{act}}}{\mathrm{Q}_{\mathrm{th}}} \tag{7}
\end{equation*}
$$

where $C_{d}$ is the coefficient of discharge, and $Q_{\text {act }}$ is the actual discharge $\left(\mathrm{L}^{3} / \mathrm{T}\right)$. Equation (7) can be written as:

$$
\begin{equation*}
C_{d}=f_{5}\left(\frac{D}{h}, \frac{\mathrm{~L}}{\mathrm{~h}}, \alpha\right) \tag{8}
\end{equation*}
$$



Fig. 1. Combined weir and gate structure.
TABLE I. DETAILS OF THE TESTED CYLINDRICAL WEIRGATE MODELS

| Model No. | Oblique angle ( $\alpha$ ) | Diameter (cm) |
| :---: | :---: | :---: |
| 1 | $90^{0}$ | 4 |
| 2 |  | 7.3 |
| 3 |  | 9 |
| 4 |  | 11 |
| 5 | $60^{0}$ | 4 |
| 6 |  | 7.3 |
| 7 |  | 9 |
| 8 |  | 11 |
| 9 | $45^{0}$ | 4 |
| 10 |  | 7.3 |
| 11 |  | 9 |
| 12 |  | 11 |
| 13 | $30^{0}$ | 4 |
| 14 |  | 7.3 |
| 15 |  | 9 |
| 16 |  | 11 |

## III. EXPERIMENTAL SET UP

The laboratory experiments were carried out in a rectangular free surface water flume, having a 2.5 m long working section with a prismatic cross section 7.6 cm wide and 25 cm deep with toughened plastic walls as shown in Figure 2. The flow rate was measured by a flow meter with a range of $0.5-2.5 \mathrm{~L} / \mathrm{s}$. During the experimental program, 16 combined
weir-gate models, made from PVC pipe, with 4 different oblique angles to the longitudinal axis of the channel $\left(30^{\circ}, 45^{\circ}\right.$, $60^{\circ}$, and $90^{\circ}$ ), were tested. For each angle, the diameter of cylindrical weir-gate changed 4 times $(D=4,7.3,9$, and 11 cm ). Details of the experimental program are shown in Table I. Each model was placed at 1 cm above the flume bed, at 1.32 m distance from the upstream of the flume. In each run, the flow rate was recorded and the flow head over the model crest, using a point gauge, was measured at a distance of (3-4) h upstream of the model. The zero on the gauge must correspond to the level of the model crest. A total of 173 experiments were conducted.


Fig. 2. A model sample during operation.

## IV. RESULTS AND DISCUSSION

## A. Variation of $Q_{\text {act }}$ with $H$

After all models were tested and the data were collected the relationships between $\mathrm{Q}_{\text {act }}$ and total upstream head H for different oblique angle and diameter values are shown in Figure 3. It is indicated that the discharge passing the cylindrical weir gate increases with the increase of the total upstream head H and decreased oblique angle.

## B. Variation of $Q_{t h}$ with $D / h$ and $L / h$

The variation of $Q_{t h}$ with $D / h$ and $L / h$ was studied for all models with different oblique angles $\left(\alpha=90^{\circ}, 60^{\circ}, 45^{\circ}\right.$, and $30^{\circ}$ ). The results are plotted in Figures 4 and 5 for $\mathrm{D} / \mathrm{h}$ and L/h, respectively. From the obtained figures, one may observe that the decrease in the value of the discharge due to the increase in $\mathrm{D} / \mathrm{h}$ and $\mathrm{L} / \mathrm{h}$ is an inevitable result when the diameter of the pipe and the angle $\alpha$ (the structure length L ) are constant. The increase of these two variables follows a decrease in the load over the weir (h) and thus the discharge decreases. Similar curve patterns were obtained experimentally in $[6,12]$.


Fig. 3. Relation between the actual discharge and the total head H for different oblique angles and diameters.

## C. Predication of Combined Discharge

The relationship between the variation of $Q_{\text {th }} / \sqrt{\mathrm{g}} h^{\frac{5}{2}}$ and the structural parameters $\mathrm{D} / \mathrm{h}$ and $\mathrm{L} / \mathrm{h}$ for a flow in a combination of different oblique angles was represented as a function in (6). To investigate the impact of D/h and L/h on $\mathrm{Q} / \sqrt{\mathrm{g}} \mathrm{h}^{\frac{5}{2}}$, the results of experiments on flow in a combination
of cylindrical weirs and gates of varying diameters and angles were used as input in a regression analysis program. This led to the development of the following empirical power expression:

$$
\begin{equation*}
\frac{Q_{\mathrm{t} h}}{\sqrt{g} h^{5 / 2}}=0.252\left(\frac{D}{h}\right)^{0.714}\left(\frac{L}{h}\right)^{1.729} \tag{9}
\end{equation*}
$$





Fig. 4. Relation between $\mathrm{Q}_{\mathrm{th}}$ with $\mathrm{D} / \mathrm{h}$ for different angles.

Equation (9) was obtained with a determination coefficient $\left(\mathrm{R}^{2}\right)=0.998$. A comparison between values of $Q_{t h}$ predicted by (9) and those observed experimentally show quite good agreement, as shown in Figure 6.


Fig. 5. Relation between $Q_{t h}$ and $L / h$ for different angles.


Fig. 6. The correlation between the predicted and the observed values from the experiments.

## D. Variation of $C_{d}$ with the Combined Effects of $L / h, D / h$, and

 aThe values of $\mathrm{C}_{\mathrm{d}}$ from combined flow condition were calculated based on (8) and are discussed for the case when the oblique angle of weir and gate structure increase and $C_{d}$ decreases. The obtained average values of $C_{d}$ were 0.50 , $0.69,0.72$, and 0.98 for $\alpha=30^{\circ}, 45^{\circ}, 60^{\circ}$, and $90^{\circ}$, respectively. The experimental results of the 16 combined weir-gate models are utilized as input data in the regression program SPSS in order to acquire an empirical power expression of the form:

$$
\begin{equation*}
C_{d}=0.0448(\alpha)^{0.602}\left(\frac{D}{h}\right)^{0.052}\left(\frac{L}{h}\right)^{0.088} \tag{10}
\end{equation*}
$$

with a correlation coefficient $\mathrm{R}=0.88$. A comparison between the values of $C_{d}$ predicted by (10) and those observed experimentally are shown in Figure 7.


Fig. 7. Predicted vs experimentally observed values of $\mathrm{C}_{\mathrm{d}}$ for all test runs.

## V. CONCLUSION

The current work aimed to experimentally study the effect of oblique angle and diameter on the flows over oblique cylindrical weir-gates. Sixteen models were designed and tested with angles varying 4 times from $30^{\circ}$ to $90^{\circ}$ and diameters from 4 to 11 cm . The discharge ranged between 0.5 and 2.5 LPM . The results from the study were discussed and the following conclusions can be drawn:

- The main parameters that effect $\frac{\mathrm{Q}_{\text {th }}}{\sqrt{\mathrm{g}} \mathrm{h}^{2.5}}$ and Cd were $\frac{\mathrm{D}}{\mathrm{h}}, \frac{\mathrm{L}}{\mathrm{h}}$, and $\alpha$.
- The theoretical discharge $Q_{t h}$ decreases with increasing D/h and $\mathrm{L} / \mathrm{h}$ for different angles and diameters.
- The actual discharge increases as the diameter of combined weir -gate increases and the oblique angle decreases.
- When the oblique angle increased, the discharge coefficient decreased, with a range of values from 0.5 to 0.98 .
- The values produced by the developed equation were found to closely match the observed values.
- Two empirical expressions, (9) and (10), in the form of power functions were obtained with correlation coefficients of 0.998 and 0.88 , respectively.


## ACKNOWLEDGMENT

The authors acknowledge the assistance of the water resources engineers Rojan M. Shareef, Payman M. Mahmood, Bishang N. Mohammed, and Jihan Sleman, in the laboratory works.

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