Damage Detection in Free–Free Glass Fiber Fabric Composite Beams by measuring Flexural and Longitudinal Vibrations

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ABSTRACT

This paper presents the experimental investigation results of the use of the vibration method by impulse excitation of free flexural and longitudinal vibrations of composite materials. The purpose of the study is to establish the sensitivity of the method used for defect detection and localization. To realize the objective, rectangular notch-type defects were simulated at different distances and depths. The influence of the location and depth of the artificial cracks on the dynamic properties of the beams was investigated by measuring the natural frequencies of flexural and longitudinal vibrations. The conducted experiments show a change in the dynamic characteristics of the beam depending on the dimensions and location of the defects.

Keywords-Glass Fiber Composite (GFC); impulse excitation; free–free beam; flexural vibration; longitudinal vibration; natural frequencies; damage detection

I. INTRODUCTION

Natural frequency-based methods are some of the most studied methods for detecting delamination and cracks in composite structures by determining their modal shapes and natural frequencies [1-6]. Chronological reviews of frequency methods for damage detection in structures were conducted in [1, 3, 6]. The great advantage of using frequency-based methods is that they are easy to implement in practice. The limitations of their use are that in some cases the changes in frequency are very small and may not be registered due to errors or deficiencies in the measuring equipment. Additionally, changes in the mass of the structure under study or measurement temperatures can introduce uncertainty into the measured frequency changes [1]. Frequency analysis of composite materials with defects has been developed by many researchers [9-12] who concluded that the natural frequency of the beam changes in the presence of a defect. Experimental and simulation studies to determine the frequency characteristics and dynamic behavior of composites are published in [12-16, 18]. The effects of fiber orientation, fiber type, measurement method, and matrix materials on the dynamic properties of composites were investigated in [8, 14, 17]. Frequency analysis of composite plates was conducted in [12-14].

The presence of a defect leads to a local reduction in the stiffness of the structure, resulting in changes in the dynamic properties, alteration of the natural frequencies, modal shapes, and an increase in the damping decrement [1-3, 18]. From the reviewed literature, it is seen that a reliable indicator for the detection of defects or damage obtained in service is the change in resonant frequencies. However, unlike modal shape-based methods, identification of the location of the damage is not easily achieved when using frequency measurements. Many authors have studied different types of cracks [15, 16]. Important factors influencing fracture behavior and mechanisms are fracture depth and length [4, 19].

Impulse excitation is a popular technique for the diagnosis of various materials [20]. In [21], it is shown that the shape of the impactor head is an essential parameter on the low-velocity impact. The damage occurring in the composite plates is illustrated and analyzed for hemispherical and flat head shape hammer [21]. The earliest research on the subject analyzed natural frequencies; however, due to measurement errors and noise, natural frequencies in such mode shapes are not sensitive to structural damage [1, 6, 11]. Other approaches have been developed to investigate and detect cracks in various composite materials by ultrasonic methods [22-24].

In [25], an approximate analytical method for damage detection in free–free axial vibrations was proposed. The method is based on the measurement of axial natural frequencies, which are global parameters and can be easily measured from any point on the structure. In the same work, the variation of the first two natural frequencies was used to identify the crack location and size. Other studies present an analysis of the application of different vibrations for the identification of cracks in metallic materials, but no experimental results are found [26].

The purpose of the current study is to investigate the effect of cracks on the natural frequencies of excited bending and axial vibrations of GFC beams. The first natural frequencies of out-of-plane and in-plane flexural vibrations as well as the longitudinal vibrations are used to characterize a single beam with defects. The influence of the location and depth of artificial cracks is also investigated.

II. MATERIALS AND METHODS

A. Materials

A glass fiber fabric composite plate with a thickness of 5mm was considered in this study. The plate was manufactured by hot pressing the glass cloth layers, impregnated with thermo-reactive phenolic and epoxy type resins. The obtained glass fiber volume fraction was 50%. A sufficient number of beam-type specimens were cut from the composite slab with dimensions L=0.25m length, b=0.025m width and h=0.005m thickness. The threads of the glass fabric are parallel and respectively perpendicular to the x-axis (Figure 1).

Fig. 1. Specimen with an artificial rectangular channel type crack.

To establish the sensitivity of the impulse excitation vibration method for defect detection, rectangular notch type defects were simulated at different distances from the edge of the specimens (x_i), namely on 0.1L and 0.2L, 0.3L, and 0.5L. The slits are transverse, located perpendicular to the stacked layers of glazing. The simulated channels have different depths equal to 0.2h, 0.4h, 0.6h, and, in some cases, 0.8h. A schematic of the samples with the artificial defects is given in Figure 1. A sufficient number of specimens were made and examined to investigate the location of the artificial defects at a fixed defect depth.

B. METHODS

1) Free Flexural Vibrations

According to the Euler-Bernoulli theory, in flexural vibrations of a beam of constant cross section and thickness, there are innumerable natural frequencies, which are determined by:

$$\omega_n = \frac{\pi^2 n^2}{L^2}\sqrt{\frac{EJ_{y\,(z)}}{\rho A}} \tag{1}$$

where A is the cross section of the beam, $J_y(z)$ is the moment of inertia, a characteristic of the cross section, ρ is the density of the material, and E is the modulus of elasticity of the composite material. For n=1, 2, 3, 4… the natural frequencies depend on the initial conditions, i.e. the way the oscillations are excited, as well as on the boundary conditions. At n = 1 the beam bends one half wave, at n = 2 there are 2 half waves and so on.

For the fundamental frequency realization at first mode (n=1), the nodal points are located at 0.224L from each end of the beam, with the anti-nodes in the center and at each end. The first natural frequency of the two free ends of the beam can be written as [27]:

$$f_1 = \frac{1}{2\pi}\left[\frac{22.373}{L^2}\right]\sqrt{\frac{EJ_{y\,(z)}}{\rho A}} \tag{2}$$

The area of cross-section A is 1.25×10^{-4}m^2. The moment of inertia is J_y = 2.58×10^{-10}m^4. The density is ρ = 1823kg/m^3 and the modulus of elasticity is E = 2.45×10^{10}Pa. A schematic of the experimental setup for the free flexural vibration test is given in Figure 2. A pulse excitation in the middle of the specimen was used to excite bending oscillations in the specimen using a rubber mallet with flat shape. A Superlux ECM-999 type microphone located at the edge of the specimen was used to record the mechanical oscillations. A computer, a PC USB 16-bit two-channel sound card 20Hz ~20kHz, and a Real Time Acoustic Analyzer data logger was used for registration, recording, and subsequent signal processing.

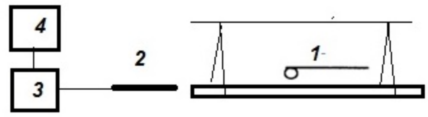
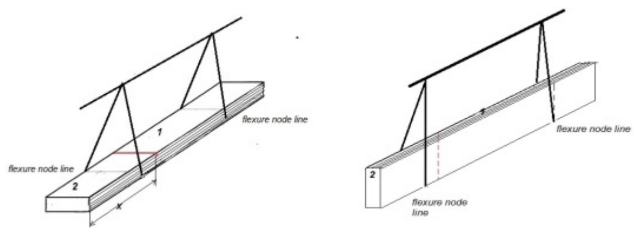


Fig. 2. Schematic of the experimental setup for the free flexural vibration mode test: 1 –rubber mallet, 2 – microphone, 3 - real time acoustic analyzer, 4 – computer.

To realize "out-of-plane" bending, in which the direction of displacement is perpendicular to the large plane of the specimen, the diagram of suspension is shown in Figure 3(a). The scheme given in Figure 3(b) is used to excite the in-plane oscillations. The microphone is placed close to the specimen above the anti-node point.



(a) (b)

Fig. 3. Schematic representation of the beam attachment for the excitation of free flexural vibration. (a) out of plane flexure mode, (b) in-plane flexure mode, 1 - impulse point, 2 - microphone point.

2) Free Longitudinal Vibrations

The equation of the motion of Euler-Bernoulli for longitudinal vibration is given by [25, 27]:

$$\frac{\partial^2 u}{\partial t^2} = c^2\,\frac{\partial^2}{\partial x^2} \tag{3}$$

where u is the longitudinal displacement of the current section dx of the beam during oscillations and $c^2 = E/\rho$ is the velocity of propagation the longitudinal elastic wave.

Equation (3) is solved by the standard method of separation of variables. The first natural resonant frequency of longitudinal vibrations is obtained by:

$$f = \frac{k}{2L}\sqrt{\frac{E}{\rho}} \tag{4}$$

where k is a natural integer.

The free longitudinal vibration test setup is shown in Figure 4. Each specimen is held by its center and struck with a plastic hammer at the edge. To analyze the acoustic response of the sample, the microphone was positioned on the opposite side of the sample. Subsequently, the corresponding vibrating sounds were recorded and analyzed by Fourier transform.

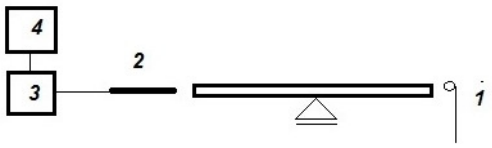


Fig. 4. Longitudinal free vibration test setup.

3) *Experimental Procedure*

Twelve tests were performed on undamaged beams and the same number was conducted on damaged beams. Table I presents the failure scenarios, which include 4 different crack locations and 3 crack depth levels at each location. The following procedure was used for the experiments: The specimen was lightly struck in the center. The test was recorded and repeated 5 times. The readings of the resonant frequency were recorded and the average value was taken. The acoustic signals were recorded and processed by signal processing methods (Fourier transform) to identify the values of the natural frequencies. They were measured for all undamaged free beams in out-of-plane and in-plane flexural and longitudinal vibration modes. A cut was then introduced into each beam at a controlled distance and depth and the natural frequencies were measured. The signals obtained from the defective and intact beams were compared in both time and spectrum. In addition, tests were made for notches in the middle of the beam with a depth of 80% of thickness.

III. RESULTS AND DISCUSSION

The damage induces a local reduction in stiffness, which determines changes in dynamic behavior. This is reflected by shifting of the natural frequencies. The change of the first resonant frequency in flexural and longitudinal vibrations in the presence of a defect can be noted by the parameter Δf:

$$\Delta f = \frac{|f_{1,o} - f_{1,d}|}{f_{1,o}}, \% \tag{5}$$

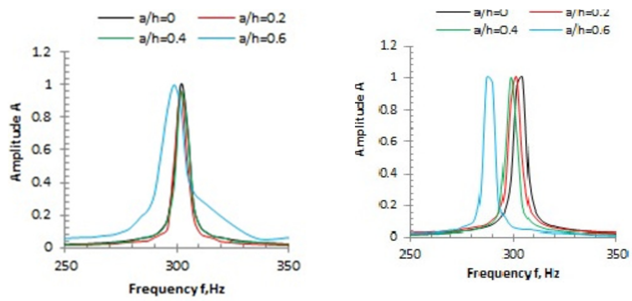
where $f_{1,o}$ is the first natural frequency of an intact beam and $f_{1,d}$ is the natural frequency of a beam with a crack.

The results of the experiments for out of plane flexure mode vibrations of GFC beams are presented in Table I. The relative first natural frequency $f_{1,d}/f_{1,o}$ and the relative frequency shift Δf in the defective beams listed in the Table. It can be seen that when the depth of the crack increases, the first resonant frequency changes significantly, especially at depth a=0.6h.

TABLE I. RESULTS OF OUT-PLANE FLEXURE MODE VIBRATIONS OF GFC BEAMS

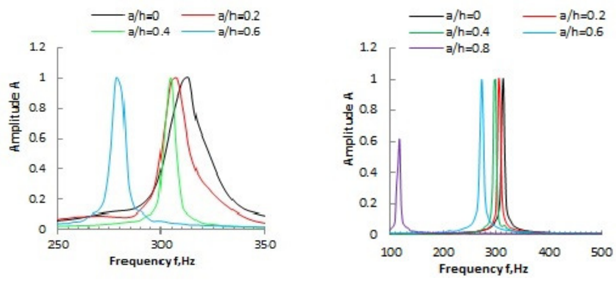
Description of samples	Dimensionless location x/L	Dimensionless depth a/h	$f_{1,d}/f_{1,o}$	Δf, %
Intact beam	0	0	1	0
Damaged beam	0.1	0.2	0.998	0.23
Damaged beam	0.1	0.4	0.997	0.31
Damaged beam	0.1	0.6	0.993	0.7
Intact beam	0	0	1	0
Damaged beam	0.2	0.2	0.990	0.96
Damaged beam	0.2	0.4	0.981	1.92
Damaged beam	0.2	0.6	0.952	4.81
Intact beam	0	0	1	0
Damaged beam	0.3	0.2	0.981	1.87
Damaged beam	0.3	0.4	0.990	2.80
Damaged beam	0.3	0.6	0.972	11.22
Intact beam	0	0	1	0
Damaged beam	0.5	0.2	0.972	2.80
Damaged beam	0.5	0.4	0.971	5.61
Damaged beam	0.5	0.6	0.921	13.08

Figures 5 and 6 show the first resonance frequencies obtained during the out-of-plane bending vibrations for intact specimens and for beams with defects located at distances x equal to 0.1L, 0.2L, 0.3L, and 0.5L at different crack depths. It can be noticed that the presence of a defect leads to a decrease in the natural frequencies.



(a) (b)

Fig. 5. First natural frequency of flexure vibration test for intact and damaged beams. (a) Notches located at 0.1L with different depths 0.2h 0.4h, 0.6h. (b) Notches located at 0.2L with different depths 0.2h 0.4h, 0.6h.

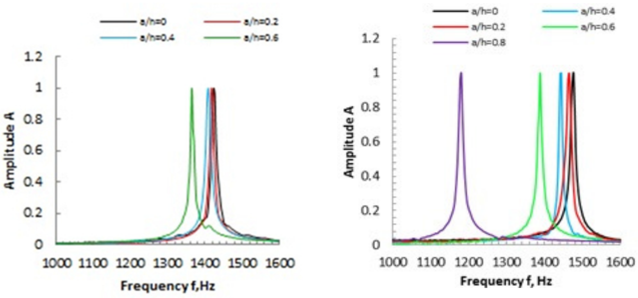


(a) (b)

Fig. 6. First natural frequency of flexure vibration test for intact and damaged beams. (a) Notches located at 0.3L with different depths 0.2h 0.4h, 0.6h. (b) Notches located at 0.5L with different depths 0.2h 0.4h, 0.6h.

As the distance to the notch increases, a larger shift of the first resonant frequency is observed. At a defect distance of

25mm, i.e. x=0.1L and a depth of 1mm, the changes in resonance frequency are very small and below 1%. At x=0.2L, the frequency shift reaches about 2%. The most significant changes in the first resonant frequency are obtained at a notch in the middle of the composite beams. At a crack depth of 3mm, the frequency shift reaches 14%. Additionally, a beam with a depth a=0.8h was investigated. The results show a relative change of about 64%. The results of the in-plane flexure tests are shown in Table II and Figure 7.



(a) (b)

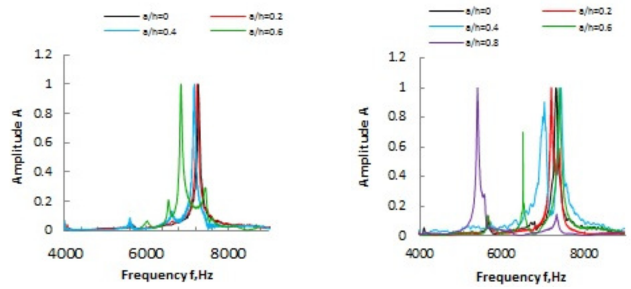
Fig. 7. First resonance frequency for in-plane flexure vibration. (a) Defect location x/L=0.3, defect depths a/h =0.2, 0.4, and 0.6. (b) Defect location x/L=0.5, defect depths a/h =0.2, 0.4, 0.6, and 0.8.

The relative shift of the first resonant frequency is not noticeable at defect location x/L= 0.1. As the x/L parameter increases, the frequency decreases and reaches 6% for the deepest crack. Figure 7 shows the frequency characteristics for slits at distances x=0.3L and 0.5L.

TABLE II. RESULTS OF IN-PLANE FLEXURE MODE VIBRATIONS OF GFC BEAMS

Description of samples	Dimensionless location x/L	Dimensionless depth a/h	$f_{1,d}/f_{1,o}$	Δf, %
Intact beam	0	0	1	0
Damaged beam	0.1	0.2	0.999	0.069
Damaged beam	0.1	0.4	0.998	0.188
Damaged beam	0.1	0.6	0.996	0.390
Intact beam	0	0	1	0
Damaged beam	0.2	0.2	0.998	0.205
Damaged beam	0.2	0.4	0.992	0.820
Damaged beam	0.2	0.6	0.988	1.230
Intact beam	0	0	1	0
Damaged beam	0.3	0.2	0.998	0.206
Damaged beam	0.3	0.4	0.990	1.029
Damaged beam	0.3	0.6	0.959	4.116
Intact beam	0	0	1	0
Damaged beam	0.5	0.2	0.992	0.793
Damaged beam	0.5	0.4	0.976	2.381
Damaged beam	0.5	0.6	0.940	5.952

The results of the longitudinal vibration mode tests are shown in Table III and Figure 8. The frequency changes for dimensionless damage location equal to 0.5 and dimensionless depths of 0.6 and 0.8 are about 11% and 26%. If the impulse impact energy is evaluated, it can be concluded that at such small impulse energy the stresses and strains are very small. For this reason, a significant reduction in resonance frequency is recorded when the material thickness at the defect site is at least 5 times smaller than the specimen thickness.



(a) (b)

Fig. 8. First resonance frequencies of longitudinal tests. (a) x/L=0.3 and defect depths 0.2h, 0.4h, and 0.6h. (b) x/L=0.5 and defect depths 0.2h, 0.4h, 0.6h, and 0.8h..

TABLE III. RESULTS OF LONGITUDINAL VIBRATION TESTS OF GFC BEAMS

Description of samples	Dimensionless location x/L	Dimensionless depth a/h	$f_{1,d}/f_{1,o}$	Δf, %
Intact beam	0	0	1	0
Damaged beam	0.1	0.2	1	0.005
Damaged beam	0.1	0.4	1	0.040
Damaged beam	0.1	0.6	0.99	0.079
Intact beam	0	0	1	1
Damaged beam	0.2	0.2	0.998	0.237
Damaged beam	0.2	0.4	0.992	0.513
Damaged beam	0.2	0.6	0.988	0.908
Intact beam	0	0	1	1
Damaged beam	0.3	0.2	0.996	0.395
Damaged beam	0.3	0.4	0.988	1.202
Damaged beam	0.3	0.6	0.942	5.757
Intact beam	0	0	1	1
Damaged beam	0.5	0.2	0.984	1.641
Damaged beam	0.5	0.4	0.961	3.886
Damaged beam	0.5	0.6	0.890	10.978

Another phenomenon that is evident in the frequency response function of damaged beams is the appearance of double peaks as shown in Figure 9. Double peaks indicate two resonances that are close. This phenomenon is called "breathing" [28]. It shows up as an irregular "broken peak" on the frequency response. Breathing causes variations in local stiffness which is time, frequency, and amplitude-dependent.

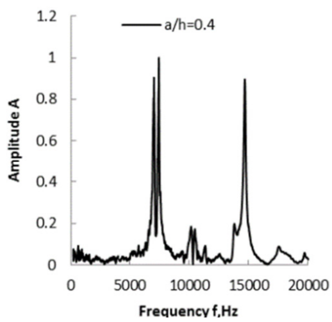


Fig. 9. The "broken peak" phenomenon in frequency response function.

Figure 10 presents the effect of defect position and depth on the resonance frequency for flexure and longitudinal vibrations. Maximum values of the Δf are obtained at the deepest crack (a=0.6h), located at a distance of x=0.5L. The frequency shifts are 13%, 6%, and 11% for the out-of-plane, in-plane and longitudinal mode vibrations, respectively.

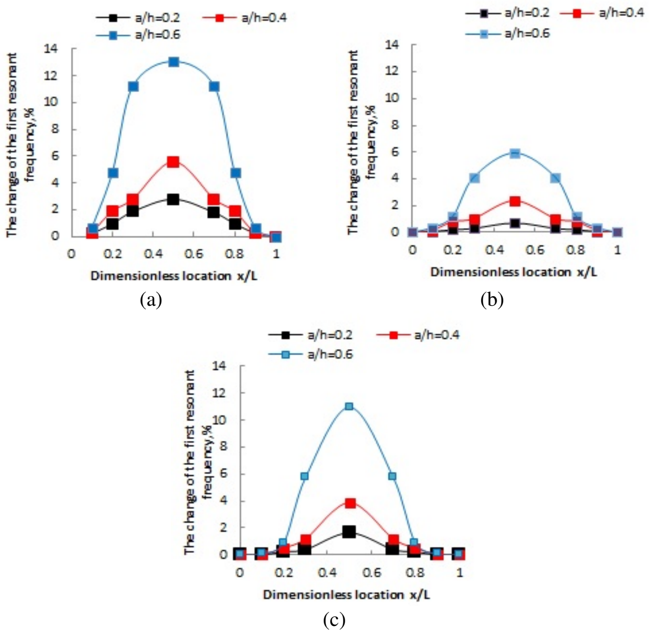


Fig. 10. Effect of defect location on frequency shift: (a) out-of-plane flexure mode, (b) in-plane flexure mode, (c) longitudinal mode.

IV. DISCUSSION

The effect of the crack on the resonant frequency is revealed differently in the oscillation modes and depends on the attachment of the beam as well as the shape, location, and depth of the crack. For transverse cracks with a specific beam location, increasing depth leads to a decrease in frequency. A crack of a certain depth and width causes different frequency changes in the experimental modes of vibrations, depending on the location of the crack. The current study confirms the opinion expressed in [18] that there are certain places where the damage hardly changes the frequency and they are close to the nodal lines. When the crack does not result in energy loss, the natural frequency in this mode of vibration is not affected. If the crack is located in places where the curvature of the modal shape has maximum values, the largest changes in frequency are obtained. The best crack detection sensitivity is obtained in tests when the crack is in the middle of the beam with out-of-plane flexure oscillations.

The contribution of the current study is the conduction of a new systematic experimental study of the damage resistance of widely used composite materials. The novelty of the investigation is the use of a combination of bending and longitudinal vibration modes that should give more accurate results in the preliminary assessment of defect areas. New data have been obtained for the studied materials, which in principle confirm the studied references [1-4, 15, 18].

V. CONCLUSIONS

In this work, the capabilities of the vibration method for identifying defects in glass fiber fabric composite beams have been investigated and verified. The results showed a change in the dynamic performance of the composite beams depending on the size and location of the defects. The shift of the first natural frequency to lower values in the presence of defects was recorded for both flexure and longitudinal vibrations. The cracks in the middle of the specimens show the most significant shift of the frequency peaks in both tests. The relative change of the first resonance frequency was used as the damage parameter. The largest frequency shifts were recorded by out-of-plane flexure, which were about 13%, and the smallest were from in-plane bending vibrations, which were 6%. In all cases, the crack depth significantly changed the structure and dynamic properties of the composite specimens. Unusual nonlinearities in the frequency response functions were found such as broken or doubled asymmetric peaks, which were are typical of cracked elements.

Important parameters of the impact events are the peak impact forces and bending, the duration of the impact event, the stresses and strains and the shape of the contact zone [21], which need to be additionally analyzed. Further research should be performed to quantify the defect position by modal shape analysis and to investigate other types of defects and the anisotropy caused by the location of glass layers in composite beams.

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