A Study on Ductility of Prestressed Concrete Pier Based on Response Surface Methodology

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Abstract—**The ductility of prestressed concrete pier is studied based on response surface methodology. Referring to the pervious prestressed concrete pier, based on Box-Behnken design, the ductility of 25 prestressed concrete piers is calculated by numerical method. The relationship between longitudinal reinforcement ratio, shear reinforcement ratio, prestressed tendon quantity, concrete compressive strength and ductility factor is gotten. The influence of the longitudinal reinforcement ratio, the shear reinforcement ratio, the prestressed tendon quantity and concrete compressive strength to curvature ductility is discussed. Then the ductility regression equation is deduced. The result showed that the influence of the prestressed tendon quantity to the ductility of prestressed concrete pier is significant. With the increasing of the prestressed tendon quantity, the curvature ductility curved reduces. With the increasing of shear reinforcement ratio and compressive strength of concrete, the curvature ductility increases linearly. And the influence of the longitudinal reinforcement ratio to ductility of the prestressed concrete pier is insignificant.**

Keywords-response surface methodology; experiment design; prestressed concrete; pier; ductility

I.   INTRODUCTION

The application of PreStressed Concrete (PRC) pier has increased because of its efficiency and high quality. Precast segmental construction methods can cut construction costs by reducing construction time while maintaining quality.   In addition, because of the self-centering capability of prestressed tendon, PRC pier could meet the performance requirement during the normal use stage as well as improve the seismic performance of a whole bridge [1]. Many researchers have investigated the seismic performance of PRC pier. Hewes and Priestley investigated the performance of unbonded post-tensioned precast concrete segmental bridge columns under lateral earthquake loading [2]. In [3], authors studied the seismic performance, identify the key design variables, and evaluate the effect of different ground motions and different column configurations for a self-centering reinforced concrete column with unbonded prestressing strand placed at the center of the cross section. In [4], authors investigated the seismic

performance of unbonded prestressed hollow concrete columns constructed with precast segments. In [5], authors tested several different pier bents in a four-span bridge earthquake simulation study. In [6], authors investigated the response of segment joints using detailed non-linear time-history analyses. A suite of ten near field earthquake records was used to determine the median joint response as well as to quantify the effect of vertical motion on the joint response. The authors showed that a prestressed bar could increase pier self reset capability and decrease the residual displacement of the bridge pier under earthquake. The mechanical properties of PRC pier are relation to some parameters, such as the longitudinal reinforcement ratio, the shear reinforcement ratio, the prestressed tendon quantity and compressive strength of concrete. The parameters analysis can be conducted with the statistical analysis methodology.

Response Surface Methodology (RSM) represents a collection of statistical and mathematical techniques and it is often used for development, improvement and optimization of various processes, where certain response is influenced by several variables. In [7], authors used RSM to investigate the performance of corroding under-reinforced beams. In [8], authors adopted he RSM to create response surface functions of the specific energy for thin-walled columns. In [9], authors used RSM to estimate the representative fragility curves for horizontally curved steel, I-girder bridges in conjunction with Monte Carlo simulation. Experimental design is widely used for controlling the effects of parameters in many processes. Its usage decreases the number of experiments, using time and material resources. Central Composite Design (CCD) and Box Behnken Design (BBD) are usually adopted in RSM. CCDs are a factorial or fractional factorial design with center points, augmented with a group of axial points [10]. BBD is a type of response surface design that does not contain an embedded factorial or fractional factorial design [11]. In [12], BBD was emp[loyed to optimize the indomethacin-loaded chitosan nanoparticle size. In [13], BBD was used to optimize the nanoscale retrograded starch formation. In [14], BBD was applied for fabrication of titanium alloy and 304 stainless steel joints.

The present paper investigates the ductility of prestressed concrete pier through response surface methodology. The

influence of the longitudinal reinforcement ratio, the shear reinforcement ratio, the prestressed tendon quantity and concrete strength grade to curvature ductility is discussed.

II. RESPONSE SURFACE METHODOLOGY

Response surface methodology (RSM) is a collection of statistical and mathematical methods that are useful for modeling and analysis engineering problems. Response surface methodology was developed by Box and collaborators in the 1950s [15]. The design procedure of response surface methodology is as follows [11, 16]:

- Designing of a series of experiments for adequate and reliable measurement of the response of interest.

- Choosing of the experimental design and carrying out the experiments according to the selected experimental matrix.

- Getting the experimental results by serial experiments.

- Mathematic–statistical treatment of the obtained experimental data through the fit of a polynomial function.

A. Mathematical Model

The simplest model which can be used in RSM is based on a linear function [17].

$$y=\beta_0+\sum_{i=1}^{k}\beta_i x_i+\varepsilon \qquad (1)$$

Where β_0, β_1 represents the coefficients of the linear parameters, x_i represents the variables, k is the number of variables, and ε is the residual associated with the experiments. To evaluate curvature, a second-order model must be used. In order to determine a critical point (maximum, minimum, or saddle), it is necessary for the polynomial function to contain quadratic terms according to the equation presented below[17]:

$$y=\beta_0+\sum_{i=1}^{k}\beta_i x_i+\sum_{i=1}^{k}\beta_{ii} x_i^2+\sum_{1\le i\le j}^{k}\beta_{ij} x_i x_j+\varepsilon \qquad (2)$$

where b_{ii}, β_{ij} represents the coefficients of the quadratic parameter.

B. Experimental design

The parameters analysis could be conducted with statistical analysis methodology. In this study, BBD was chosen. For BBD, the design points fall at combinations of the high and low factor levels and their midpoints. Box and Behnken suggested how to select points from the three-level factorial arrangement, which allows the efficient estimation of the first- and second-order coefficients of the mathematical model. BBDs have treatment combinations that are at the midpoints of the edges of the experimental space and require at least three continuous factors [18], shown in Figure 1. Because BBDs often have fewer design points, they can be less expensive to do than central composite designs with the same number of factors. Table I contains the coded values of the factor levels for BBD on three factors.
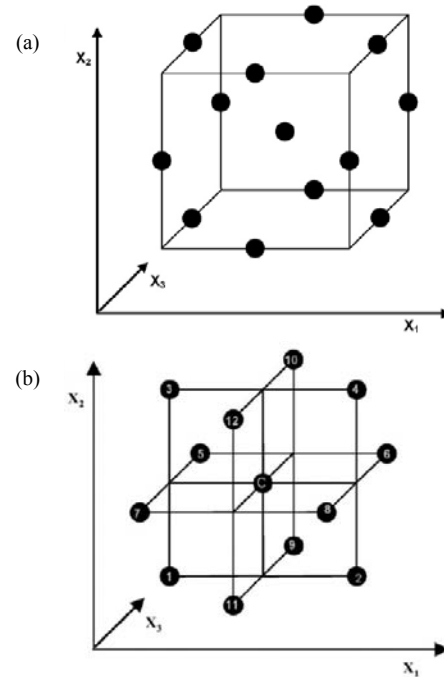

Fig. 1.   BBD cube for 3 factors: (a) cube for BBD and three interlocking 3 factorial design, (b) points for BBD and three interlocking 3 factorial design:

TABLE I.   BBD TABLE FOR 3 FACTORS

Number	x_1	x_2	x_3
1	-1	-1	0
2	1	-1	0
3	-1	1	0
4	1	1	0
5	-1	0	-1
6	1	0	-1
7	-1	0	1
8	1	0	1
9	0	-1	-1
10	0	1	-1
11	0	-1	1
12	0	1	1
C	0	0	0

III. THE DUCTILITY CAPACITY OF PRESTRESSED CONCRETE PIER

A. Constitutive Relations

1) Concrete

The concrete damage plasticity (CDP) model was adopted [19]. A general constitutive relationship for CDP model is shown in Figure 2, where ε_{un}^{m} and σ_{un}^{m} presents strain and stress of m-th tipping point, ε_{pl}^{m} is the concrete compressive plastic strain on m-th loading, d_c is compressive damage factor, and $\underline{d_t}$ is tensile damage factor. Compression section was defined as [20]:

$$\sigma = \begin{cases} f_c\left[2\dfrac{\varepsilon}{\varepsilon_0} - \left(\dfrac{\varepsilon}{\varepsilon_0}\right)^2\right] & \varepsilon \leq \varepsilon_0 \\[2mm] f_c\left[1 - 0.15\dfrac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0}\right] & \varepsilon_0 \leq \varepsilon \leq \varepsilon_u \end{cases} \qquad (3)$$

where f_c=uniaxial compressive strength of concrete, ε_0=yield strain, ε_u=ultimate compressive strain. Tension section was defined as

$$\sigma = f_t \frac{\dfrac{\varepsilon}{\varepsilon_t}}{a_t\left(\dfrac{\varepsilon}{\varepsilon_t} - 1\right)^{1.7} + \dfrac{\varepsilon}{\varepsilon_t}} \qquad \varepsilon \geq \varepsilon_t \qquad (4)$$

where $a_t = 0.312 f_t^2$, f_t= uniaxial ultimate tensile stress of concrete, ε_t= peak tensile strain of concrete [21].
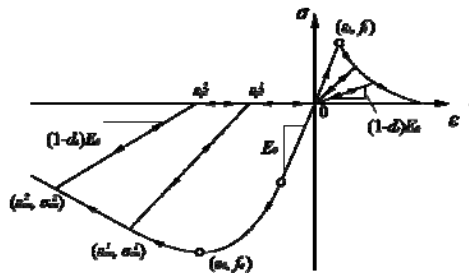

Fig. 2.   Constitutive relationship for CDP model

2)   Reinforcement
Bilinear kinematic (BKIN) hardening of material was used to define the behavior of the steel bar. Material properties for the bar were as follows [20]: elastic modulus E_s=200GPa; yield stress f_y=335MPa; yield strain ε_y=0.00168 and 0.1 E_s as the slope of the hardening phase. BKIN model for the steel bar behavior for unloading and reloading branches is shown in Figure 3.
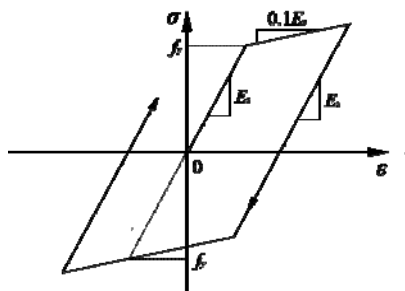

Fig. 3.   Constitutive relationship for reinforcement

3)   PT strands
In this study, the mechanical model of the PT strands was also defined as a BKIN model [20], where elastic ratio of

prestressing tendons E_s=195GPa, yield stress f_y=1860MPa; yield strain ε_y=0.00954 and 0.1 E_s as the slope of the hardening phase. Effective tensile stress σ_{con}=800MPa which was equivalent to 43% of the ultimate tensile strength, and axial compression ratio u=22.7%.

B.   Definition of ductility

Because of inelastic deformation capacity of reinforced concrete ductility members depends on cross section of plastic rotation capacity in plastic hinge zone, through responding to the ductility capacity of prestressed concrete could compute cross section curvature ductility coefficient [22]. A measure of the ductility of structures with regard to seismic loading is the displacement ductility factor defined as Δu/Δy ,where Δu is the lateral deflection at the end of the post-elastic range and Δy is the lateral deflection at first yield. A rotational ductility factor for members has been calculated by some dynamic analyses as [23]

$$\mu_\varphi = \frac{\varphi_u}{\varphi_y} \qquad (5)$$

where φ_u=maximum curvature at the section and φ_y=curvature of the section at first yield, as shown in Figure 4.
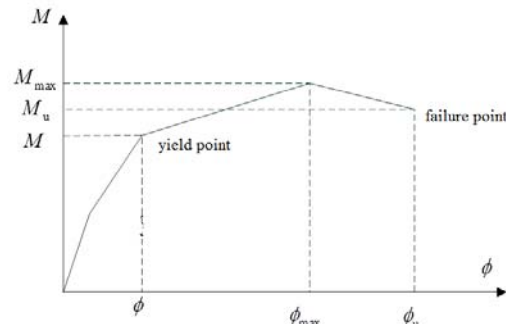

Fig. 4.   Moment-Curvature relationship

The curvature ductility coefficient of a structure is usually much bigger than its displacement ductility factor in the plastic hinge zone. The reason is that rotation of plastic hinge becomes the main deformation while yield occurring [24]. In this paper, members will yield if the outermost longitudinal tensile plain reinforcement of reinforced concrete members reach to the initial yield curvature. The ductility of PRC pier are relation to the longitudinal reinforcement ratio, the shear reinforcement ratio, the prestressed tendon quantity and compressive strength of concrete. Referring to pervious prestressed concrete pier, based on BBD, it is operable to analyze ductility of PRC pier with FEM.

IV.   CASE STUDY

One bridge pier circular cross section is 2 meters in diameter, just as shown in Figure 5. The ratio of longitudinal reinforcement x_1 would take the values of 0.80%, 1.00% and 1.20%. The shear reinforcement ratio x_2 would take the values of 0.03%, 0.035% and 0.04%. The prestressed tendon x_3 would

take the values of Φ15.27-10, Φ15.27-15 and Φ15.27-20. The concrete strength grade x_4 would take the values of 30, 40 and 50. There are four factors with three level. Base on BBD, the experiment design and ductility coefficient results y are shown in Table II.
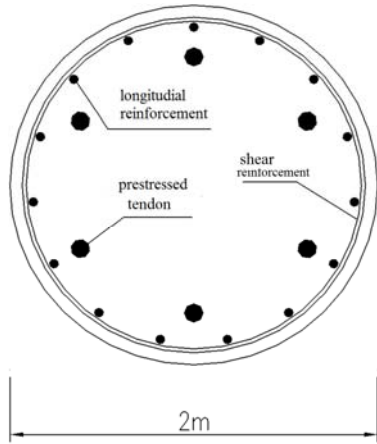

Fig. 5.   Geometrical characteristic of the tested pier section

TABLE II.   THE EXPERIMENT DESIGN AND RESULTS

No.	X_1(%)	X_2(%)	X_3	X_4(MPa)	y
1	1.0	0.040	20	40	4.204
2	1.0	0.040	15	30	4.378
3	0.8	0.035	15	50	6.998
4	0.8	0.035	20	40	4.109
5	1.0	0.030	15	30	3.822
6	1.0	0.035	10	50	11.67
7	1.0	0.035	10	30	6.979
8	1.0	0.040	15	50	7.270
9	1.0	0.035	20	50	4.778
10	1.2	0.030	15	40	5.260
11	1.0	0.030	10	40	8.720
12	1.0	0.040	10	40	9.304
13	1.2	0.035	10	40	8.995
14	1.0	0.035	15	40	5.476
15	0.8	0.035	10	40	8.940
16	0.8	0.030	15	40	5.681
17	1.0	0.030	15	50	6.687
18	1.0	0.030	20	40	3.971
19	0.8	0.040	15	40	6.098
20	1.0	0.035	20	30	2.777
21	1.2	0.035	15	50	7.042
22	0.8	0.035	15	30	4.278
23	1.2	0.040	15	40	5.648
24	1.2	0.035	20	40	4.146
25	1.2	0.035	15	30	3.989

With statistical analysis and ignore insignificant quadratic term, the ductility regression equation is deduced.

$$y=9.48720-4.05461x_1-55.14804x_2$$
$$-1.04795x_3+0.35360x_4$$
$$-3.51000x_2x_3-0.013450x_3x_4+1.81397x_1^2 \qquad (6)$$
$$+2197.35294x_2^2+0.039947x_3^2$$

Based on (6), the influence of each factor to curvature ductility is discussed. The results are shown in Figures 6-8.
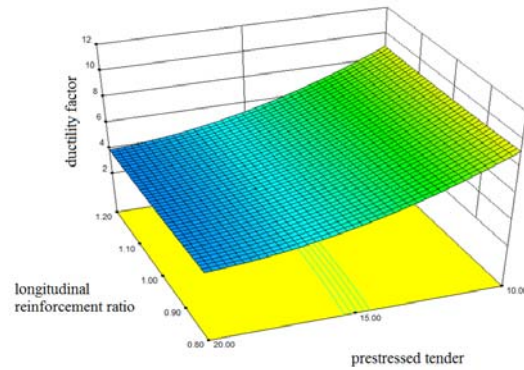

Fig. 6.   Relationship between longitudinal reinforcement ratio, prestressed tendon and ductility factor
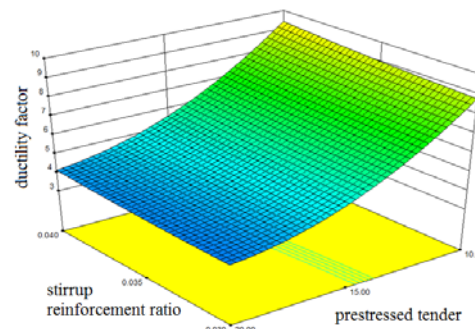

Fig. 7.   Relationship between shear reinforcement ratio, prestressed tendon and ductility factor
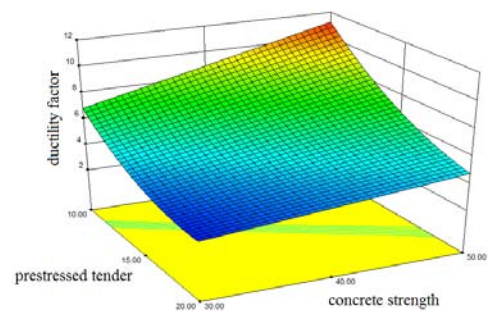

Fig. 8.   Relationship between concrete strength, prestressed tendon and ductility factor

The results show that the influence of the prestressed tendon quantity to ductility of prestressed concrete pier is significant and the influence of the longitudinal reinforcement ratio to ductility of prestressed concrete pier is insignificant. With the increasing of the prestressed tendon quantity, the curvature ductility curved reduces. With the increasing of shear

reinforcement ratio and compressive strength of concrete, the curvature ductility linear increases.

V. CONCLUSION

Based on response surface methodology, the ductility of prestressed concrete pier is studied. According to Box-Behnken design, the ductility regression equation is deduced with statistical analysis. The influence of the longitudinal reinforcement ratio, the shear reinforcement ratio, the prestressed tendon quantity and concrete strength grade to curvature ductility is discussed. The results show that:

- The prestressed tendon quantity to ductility of prestressed concrete pier is significant. With the increasing of the prestressed tendon quantity, the curvature ductility curved reduces.

- With the increasing of shear reinforcement ratio and compressive strength of concrete, the curvature ductility linear increases.

- The influence of the longitudinal reinforcement ratio to ductility of prestressed concrete pier is insignificant.

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