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WÖHLER DIAGRAM CALCULATION FOR PURPOSES RELATED TO CRACK PROPAGATION

Liviu Stoicescu
“Dunarea de Jos” University of Galati,
Faculty of Naval Architecture, Galati, Domneasca
Street, No. 47, 800008, Romania,
E-mail: liviu.stoicescu@ugal.ro

Roman Pirvulescu
Danube Delta National Institute,
Babadag Street, No. 165, Tulcea, Romania,
E-mail: roman.pirvulescu@gmail.com

ABSTRACT

The topic of this paper is the crack propagation analysis used for the Wöhler diagram calculation. There is developed an analytical model valid for positive pulsating cycles where the cracks faces remain open at all times.

Keywords: crack propagation, Wöhler diagram.

1. INTRODUCTION

One of the long term structural evaluation criteria is the fatigue assessment. For this criteria there are used the idealized Wöhler diagrams, based on the crack propagation according to the material and structure types. In this paper an analytical model for particular structural and loading cases is developed.

2. THE ANALYTICAL MODEL

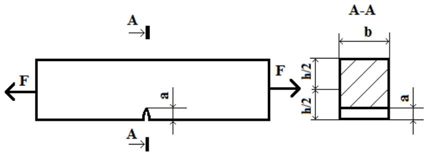

Fig. 1

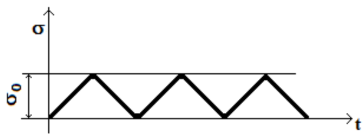

Fig. 2

Fatigue or durability calculations are made by using Wöhler diagram, a diagram which is obtained experimentally through specific procedures applied for a long time. One should consider a rectangular specimen (Fig. 1) subjected to a pulsatile load of two equal and opposite forces F (t).

In the middle of the specimen there is a crack perpendicular to the force direction, whose length a(t) grows in time between $a_{min}=a_i$ (originally existing technologically on the specimen surface due to the imperfect surface processing) to a final length a_f, at which the specimen breaks when left with only the strength of the shaded surface. If the failure is due only to the insufficient strength of this surface, for a material resistant to bending and axial loads in the elastic range, the maximum stress in the crack section is

$$\sigma_{max} = F/A_f + Fe/W_f$$

where:

$$A_f = b(h-a)$$

is the failure section area,

$$W_f = b(h-af)^2/6$$

is the minimum section modulus,

$$F = \sigma_{0f} bh \quad e = a/2 \quad .$$

It thus results:

$$\sigma_{max} = \sigma_{0f} h \frac{(h+2af)}{(h-af)^2} \tag{1}$$

where the maximum pulsatile flow is written as (see Fig. 2)

$$\sigma_0 = F_{max}/A_0 = F/bh$$

and σ_{0f} represents the material tensile strength.

According to Paris-Erdogan law for the crack growth regime [4], [5]:

$$\frac{d_a}{d_N} = C(\Delta K)^m = C\sqrt{\pi a(N)}Y(a)\sigma_{max}(N) \quad (2)$$

where N is the number of applied pulsatile load cycles; C and m are material constants (for mild steel C = 3.5-6, m = 3)

$$Y(a) = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{m/2}}$$

$$\alpha = a(N)/h \quad (3)$$

By integrating the Paris-Erdogan differential formula between a_i and a_f it results:

$$N = \frac{1}{c}\int_{a_i}^{a_f} \frac{1}{Y^m \sigma_{max}^m \left(\sqrt{\pi}\right)^m} \frac{da}{a^{m/2}} \quad (4)$$

If y^m and σ_{max} are considered as almost constant, then the integral can be analytically obtained to be: (5)

$$N_f = \frac{1}{CY^m\sigma_{max}^m\left(\sqrt{\pi}\right)^m}\int_{a_i}^{a_f}\frac{da}{a^{m/2}} = \frac{a_f^{(1-m/2)} - a_i^{(1-m/2)}}{CY^m\sigma_{max}^m\left(\sqrt{\pi}\right)^m(1-m/2)}$$

By changing to the dimensionless variables

$$\beta = \sigma_{max}/\sigma_r > 1 \quad (6)$$

$$\alpha_f = \frac{a_f}{h} < 1 \quad (7)$$

Equation (1) becomes:

$$\beta = \frac{1+2\alpha_f}{(1-\alpha_f)^2} \quad (8)$$

α_f can be found based on β (8) as:

$$\alpha_f^2\beta - 2\alpha_f(\beta+1) + \beta - 1 = 0 \quad (9)$$

Considering only the positive solution of the equation (9) it results:

$$\alpha_f = 1 + \beta - \sqrt{3\beta+1} \quad (10)$$

By substituting in (8) α_f from (7) and the σ_{max} from (6) considering static breaking upon reaching $\sigma_{max} = \sigma_r$, it results: (11)

$$\sigma_0^m N_f = \frac{h^{(1-m/2)}}{CY^m\beta^m\pi^{m/2}}\left[(1+\beta-\sqrt{3\beta+1})^{(1-m/2)} - \alpha_i^{(1-m/2)}\right]$$

Equation (10) allows the calculation of Wöhler curve, considering the state of the surface of the specimen (usually a_i=0.1 mm) and α_i =a/h, $\beta = \sigma_r/\sigma_0$.

One may compare Wöhler curves as calculated according to the methodology described above with those determined experimentally specified in the rules and guides of societies, as per DNV and GL [2], [3] and [6]. One may notice that in the above formula h also appears, usually representing plate's thickness. In [3] an experimental correction based on thickness is given, which outlines the influence of the plate's thickness on the Wöhler curve.

The above formula has been obtained by analytically integrating the variation of the crack's length "a" on the number of pulsating load cycles N, while considering both σ_{max} and Y as constants during crack propagation. If one takes into account that the form factor Y depends on α = a/h and β depends on α through a formula similar to formula (8), then the integral in formula (4) can be solved numerically by using equation (3) for Y (α), (12)

$$\sigma_0^m N_f = \frac{h^{(1-m/2)}}{C\left(\sqrt{\pi}\right)^m}\int_{a_i}^{a_f}\frac{d\alpha}{Y^m(\alpha)\beta^m(\alpha)} = \frac{h^{(1-m/2)}}{C\pi^{m/2}}\left|I(\alpha)\right|_{a_i}^{a_f}$$

The integral I(α) is solved by using Y(α) from (3) and $\beta = (1+2\alpha_f)/(1-\alpha_f)^2$, the final limit α_f being calculated through formula (10).

3. CONCLUSION

The calculation procedure described in section 2 is valid for positive pulsating cycles where the cracks faces remain open at all times.

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