Multiple and non-planar crack propagation analyses in thin structures using FCPAS

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ABSTRACT. In this study, multiple and non-planar crack propagation analyses are performed using Fracture and Crack Propagation Analysis System (FCPAS). In an effort to apply and validate FCPAS procedures for multiple and non-planar crack propagation analyses, various problems are solved and the results are compared with data available in the literature. The method makes use of finite elements, specifically three-dimensional enriched elements to compute stress intensity factors (SIFs) without special meshing requirements. A fatigue crack propagation criterion, such as Paris-Erdoğan equation, is also used along with stress intensity factors to conduct the simulation. Finite element models are generated within ANSYS™ software, converted into and solved in FRAC3D program, which employs enriched crack tip elements. Having computed the SIFs for a given crack growth increment and using a growth criterion, the next incremental crack path is predicted and the fracture model is updated to reflect the non-planar crack growth. This procedure is repeated until cracks reach a desired length or when SIFs exceed the fracture toughness of the material. It is shown that FCPAS results are in good agreement with literature data in terms of SIFs, crack paths and crack growth life of the structure. Thus, accuracy and reliability of FCPAS software for multiple and non-planar crack propagation in thin structures is proven.

KEYWORDS. Fracture; Nonplanar Crack Growth; Crack Propagation.

INTRODUCTION

Although in-plane crack propagation under mode-I loading conditions is very common for machinery parts and structures and is still a popular research subject, some parts can fail under mixed mode loading, causing nonplanar crack surface. This type of crack propagation can occur under mixed mode loading conditions or when crack surface is not perpendicular to an axial load. Under mixed mode conditions, multiple cracks growing in a nonplanar manner can even be more critical for the structure due to interaction effects. Cracks can accelerate each other, coalesce or change direction due to the interaction effects. This situations can be seen in thin walled structures such as sheet or integral panels of aircrafts or engine parts that are subjected to high temperatures. Other examples can also be given in applications in the areas of transportation, energy and aviation. Therefore, accurate prediction of nonplanar crack propagation in machine parts or structures is very important to assure safety, efficiency and reliability of some engineering structures.

In the literature, there are several numerical and experimental studies that deal with fracture and propagation analyses of multiple cracks and nonplanar crack growth. One of the numerical studies is by Wessel et.al [1]. In that study, Wessel et.al used boundary element method (BEM) and compared their results to experimental data. Jonesa et.al. [2] analyzed interacting multiple cracks using finite element method (FEM) and a hybrid formulation which represents stiffness changes. Yan analyzed interacting multiple cracks and complex crack configurations in linear elastic media using an effective numerical method which is an extended form of Bueckners' princible [3]. Leonel et.al used two-dimensional BEM method for multiple crack propagation analyses [4]. They used maximum circumferential stress theory for evaluating stress intensity factors (SIF) and propagation angle, and Paris' law to predict structural life. Another 2D linear elastic fracture mechanics (LEFM) problem is analyzed by Yan [5] using BEM method for propagating multiple cracks. Yan also used maximum circumferential stress theory and Paris' law. A Java-based boundary element program front end was developed by Hsieh et.al. for fracture analysis of multiple curvilinear cracks in general anisotropic materials [6]. Citarella et.al. compared DBEM and FEM methods by 3D fatigue crack growth of two anti-symmetric cracks [7]. Price and Trevelyan analyzed two eccentric crack that propagate nonplanarly in a thin geometry [8]. Bouchard and Chastel used maximum circumferential stress criterion, strain energy density fracture criterion and maximum strain energy release rate criterion for single and multiple nonplanar crack growth analyses [9].

The objective of this study is to apply, demonstrate and validate usage of FCPAS for multiple cracks propagating in a non-planar manner under fatigue loading. In the study, finite element method with enriched elements is used to obtain stress intensity factors (SIF) [10]. SIF values are calculated during nodal displacement calculation which is a step of finite element (FE) solution. By using enriched element method, SIFs are calculated accurately and no special re-processing or meshing techniques and post-processing of results are needed. It is shown that FCPAS results for multiple non-planar cracks agree well with those from the literature.

METHOD

For nonplanar crack propagation analyses, FCPAS finite element software is used. Also ANSYS™ [11] commercial FE software is used for generating FE model of the cracked geometry. All calculations for SIFs and propagation process are performed by FRAC3D solver of FCPAS software. Crack propagation process with FCPAS software consists of FE modeling of cracked geometry, solution step, propagation of the cracks and best ellipse fit for the propagated cracks. These steps are repeated until failure or some other geometric limit. Work flow scheme of FCPAS analysis is shown in Fig. 1.

FCPAS solver, FRAC3D, uses enriched element method for calculating SIFs [12]. A general form of displacements for enriched elements is given in Eqs. 1-3.

$$\begin{aligned} u(\xi,\eta,\rho) = \sum_{j=1}^{m} N_j(\xi,\eta,\rho) u_j &+ Z_0(\xi,\eta,\rho)\left(f_u(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) f_{uj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_I^i \right) \\ &+ Z_0(\xi,\eta,\rho)\left(g_u(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) g_{uj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{II}^i \right) \\ &+ Z_0(\xi,\eta,\rho)\left(h_u(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) h_{uj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{III}^i \right) \end{aligned} \tag{1}$$

$$\begin{aligned} v(\xi,\eta,\rho) = \sum_{j=1}^{m} N_j(\xi,\eta,\rho) v_j &+ Z_0(\xi,\eta,\rho)\left(f_v(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) f_{vj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_I^i \right) \\ &+ Z_0(\xi,\eta,\rho)\left(g_v(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) g_{vj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{II}^i \right) \\ &+ Z_0(\xi,\eta,\rho)\left(h_v(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) h_{vj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{III}^i \right) \end{aligned} \tag{2}$$

$$
\begin{aligned}
w(\xi,\eta,\rho) = \sum_{j=1}^{m} N_j(\xi,\eta,\rho) w_j &+ Z_0(\xi,\eta,\rho)\left(f_w(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) f_{wj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_I^i \right) \\
&+ Z_0(\xi,\eta,\rho)\left(g_w(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) g_{wj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{II}^i \right) \\
&+ Z_0(\xi,\eta,\rho)\left(h_w(\xi,\eta,\rho) - \sum_{j=1}^{m} N_j(\xi,\eta,\rho) h_{wj} \right)\left(\sum_{i=1}^{ntip} N_i(\Gamma) K_{III}^i \right)
\end{aligned}
\tag{3}
$$

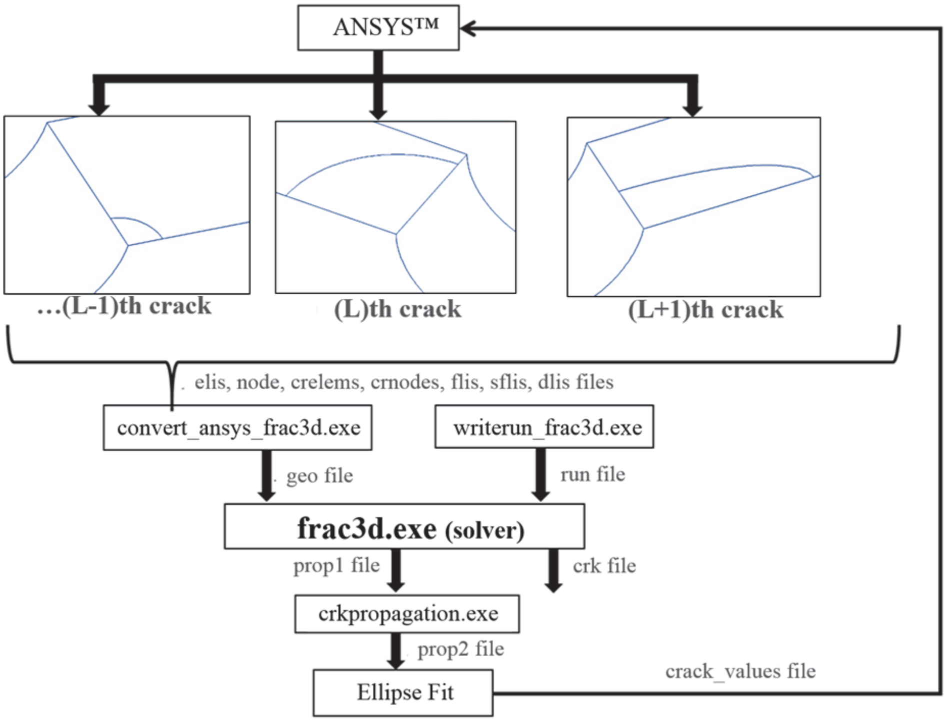

Figure 1: Work scheme of FCPAS finite element software for analysis of multiple cracks.

In Eqs. 1-3, ξ, η and ρ are local coordinates in enriched elements, u_j, v_j, *and* w_j are the nodal displacement, N_j are regular finite element shape functions, Z_0 is a zeroing function which varies between 0 and 1, m is number of nodes in the element, f_j, g_j and h_j represent the mode I, II and III components of crack tip displacements, K_I, K_{II} and K_{III} are the unknown SIFs. Finally Γ is the local isoparametric coordinate along the crack front that varies between -1 and 1.

After FE solution and SIF calculation, cracks are propagated taking into account crack interaction effects. A modified form of Paris-Erdoğan equation (Eq. 4) is used for this process.

$$
\Delta a_i = \Delta a_{\max} \left(\frac{\Delta K_i}{\Delta K_{\max}} \right)^n \tag{4}
$$

In the next step, if cracks are to be represented in elliptical form, a best ellipse fit method is applied to propagated crack tip nodes. Then ellipse parameters are given to ANSYS software as new crack dimensions. For through the thickness cracks in thin walled structures, there is no need for ellipse fitting and any crack tip node coordinates are used for new crack locations. After all propagation analyses are completed, a crack growth law such as Paris-Erdoğan formulation (Eq. 5) is used for prediction of crack propagation life [10].

$$
\frac{da}{dN} = C\left(\Delta K\right)^n \tag{5}
$$

CRACK PROPAGATION ANALYSES

Corner, surface and through-thickness cracks under mode-I conditions in different geometries are analyzed and results are compared to those from the literature which are experimental test data and/or numerical results. FCPAS results showed very good agreement with the literature data in terms of SIFs, crack profiles/paths and propagation lives. Some of these studies can be found in [13] and [14].

In addition to the above mode-I propagation analyses including multiple mode-I cracks, several multiple-non-planar crack propagation analyses are also performed using FCPAS. In this study, two different nonplanar crack propagation analyses are performed. First set of analyses represent the study by Yan [15]. A thin plate which contains two equal sized cracks is subjected to tensile stress. One of the cracks is perpendicular to the loading axis of the plate, and the second crack makes a 45-degree angle. Details of the geometry and loading conditions can be seen in Fig. 2.

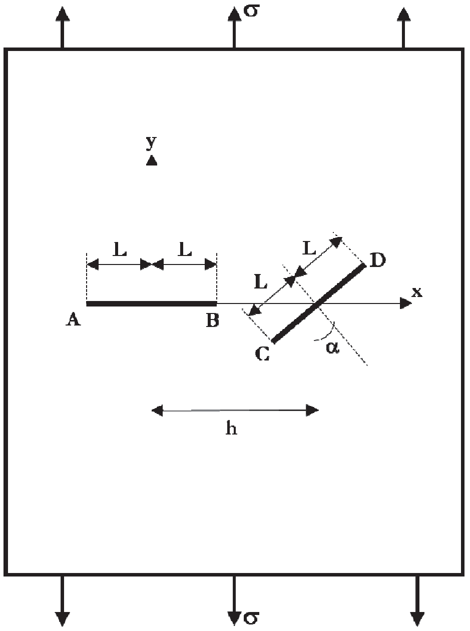

Figure 2: Details of the cracked model [15].

In Fig. 2, L=4 mm, α=45 degree, h=8 mm, width and height of the plate are taken as 160 mm to simulate large plate conditions relative to the crack sizes and the thickness of the plate is taken as 5 mm to assure plane stress conditions. Load and loading ratio (R) are 150.42 MPa and 0.048, C constant of the material is 1.039E-10, constant n is 2.7438 [15]. Modulus of elasticity and Poisson's ratio are used as 70000 MPa and 0.321 respectively. Same analysis as that of Yan's is performed using FCPAS and results are compared. Crack path comparison is shown in Fig. 3. It should be noted that for the incremental crack propagation analyses presented in this paper, the propagation angles are computed using the maximum tangential stress (MTS) criterion proposed by Erdoğan and Sih [16]. According to this criterion, under mixed mode loading, crack extends in a direction such that mode-II SIF becomes zero.

Figure 3: Crack path comparison between FCPAS and Yan's analysis.

Although the loading is uniaxial in the problem considered, because of the shear stress which occurs on the slanted crack plane, K_{II} stress intensity factors are generated at its crack tips causing the tips to grow initially in a non-planar manner.

Thus, initially, this effect is only near the tips of the slanted crack (tips C and D). It can be seen from the above paths that, the 45-degree slanted crack becomes nearly perpendicular to the loading direction after the first step of the crack propagation analysis. However, it also seen that as the cracks keep growing, especially between crack tips B and C, stress re-distributions take place, which cause these crack tips to continue changing directions. This can be seen in Figs. 4 and 5, where K_I and K_{II} SIFs are plotted as a function of absolute (curved) crack length. It is seen from these figures that crack tips A and D keep propagating purely in mode-I after the initial mixed-mode deflection, i.e., K_{II} SIF is very close to zero for these crack tips during crack growth. It should also be noted that during the analyses presented in this paper, the maximum crack length increment among all propagation steps is taken to be nearly one-tenth of the largest crack length and that these steps can further be refined to further check the convergence of the crack path and life predictions. However, the chosen steps yield good agreement with Ref. [15] in terms of the paths of the crack tips.

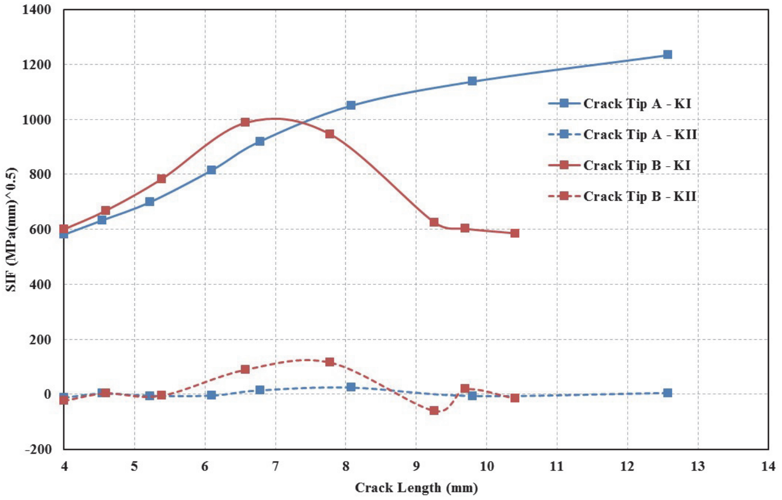

Figure 4: Mode-I and mode-II stress intensity factors during crack growth – crack tips A and B.

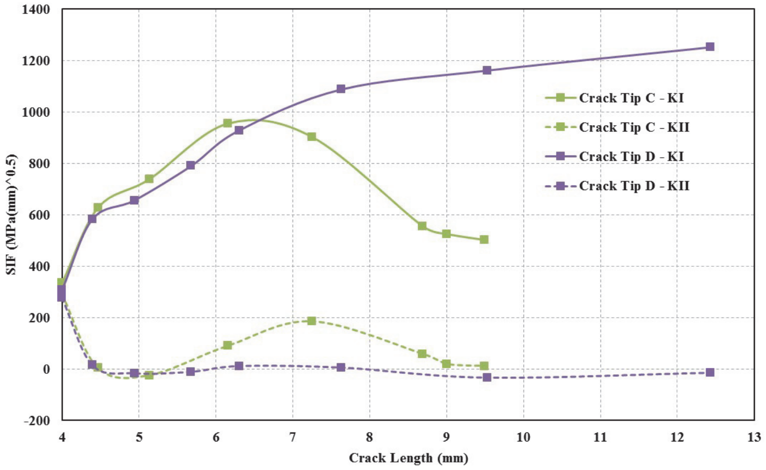

Figure 5: Mode-I and mode-II stress intensity factors during crack growth – crack tips C and D.

In an effort to compare the predictions of the current study with those of Yan’s, equivalent SIFs vs. number of cycles are plotted in Fig. 6. To be able to make comparisons, equivalent SIFs are calculated in the same way as Ref. [15].

$$\Delta K_e = \frac{1}{2}\cos\frac{\theta_0}{2}\left[\Delta K_I\left(1+\cos\theta_0\right) - 3\Delta K_{II}\sin\theta_0\right] \tag{6}$$

In Eq. (6), θ_0 is the crack growth angle, K_I and K_{II} are mode-I and mode-II stress intensity factors.

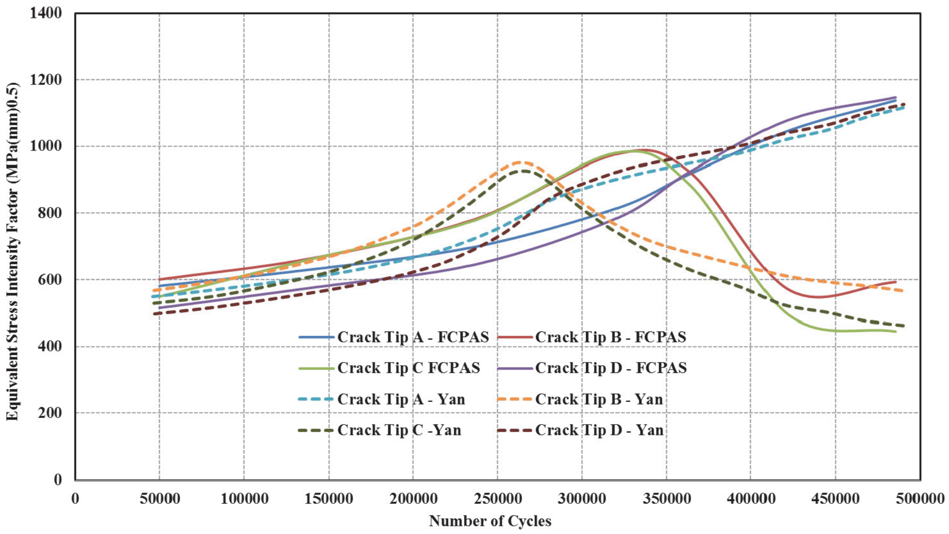

Figure 6: Equivalent SIF comparisons between FCPAS and Yan's [15] numerical results.

While there is no information about the unit of constant C of the material in Yan's study, load and material constant C used in the current simulations are, 15.333 N/mm^2 (value of Ref. [15] given in kg unit divided by 9.81) and 1.02E-9 (value of Ref. [15] multiplied by 9.81) to match the results with the same number of cycles. In that case, it is concluded from this application that Fig. 3 shows very good agreement in terms of crack path predictions and that Fig. 6 also shows good agreement, especially in the beginning and end of the simulation.

Another non-planar crack propagation analysis performed in this study is about a plate under uniaxial stress containing a hole and multiple cracks [17]. Judt and Ricoeur's study [17] contains three cracks analyzed by interaction integral. The same multiple nonplanar cracks problem is analyzed by FCPAS. Geometry, loading and boundary condition details of the problem can be seen in Fig. 7.

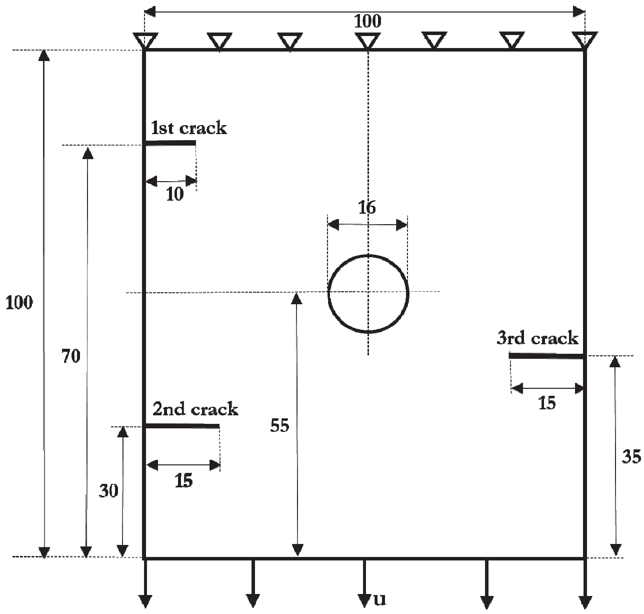

Figure 7: Geometry and boundary condition details of three cracked model (dimensions are in mm)[17].

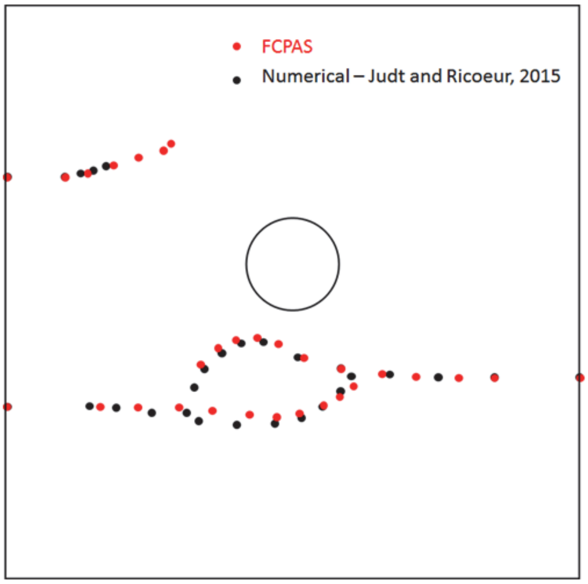

Figure 8: Crack path comparison between FCPAS and the literature data [17].

The plate is under displacement controlled cyclic loading. 0.005 mm displacement is applied at the bottom area of the plate to ensure tension loading and top area is fixed. Al-7075 is chosen as material of the plate and modulus of elasticity and Poisson's ratio are 72000 MPa and 0.33, respectively. Material constant n is 1.34 and fracture toughness is 23.9 $MPa\sqrt{m}$ for the material [17]. Thickness of the plate is taken as 3 mm to simulate plane stress conditions. Having performed crack propagation analyses using the same procedure employed in the first application, obtained results are compared to those from Judt and Ricoeur's [17] study. Comparison of crack paths between FCPAS and Ref. [17] is shown in Fig. 8.

As can be seen from Fig. 8, eccentricity of the cracks causes shear stress and therefore, K_{II} SIFs. Thus, cracks change their direction from the horizontal plane. Vertical distance between 2nd and 3rd cracks are very small and therefore, SIFs in this region are higher than the region around the 1st crack. As a result, 2nd and 3rd cracks propagate faster than the 1st crack, which propagate at a slower rate. Comparisons of equivalent SIF vs. number of crack growth steps, as given in Ref. [17], are shown in Fig. 9. It should be noted that, in the current analyses, much less number of increments (17 steps) are used compared to Ref. [17] (325 steps). Equivalent SIFs are calculated using Eq. (7) [17].

$$K_e = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 4\left(\alpha_1 K_{II}\right)^2} \qquad (7)$$

α_I=1.155 [17], K_I and K_{II} are mode-I and mode-II stress intensity factors.

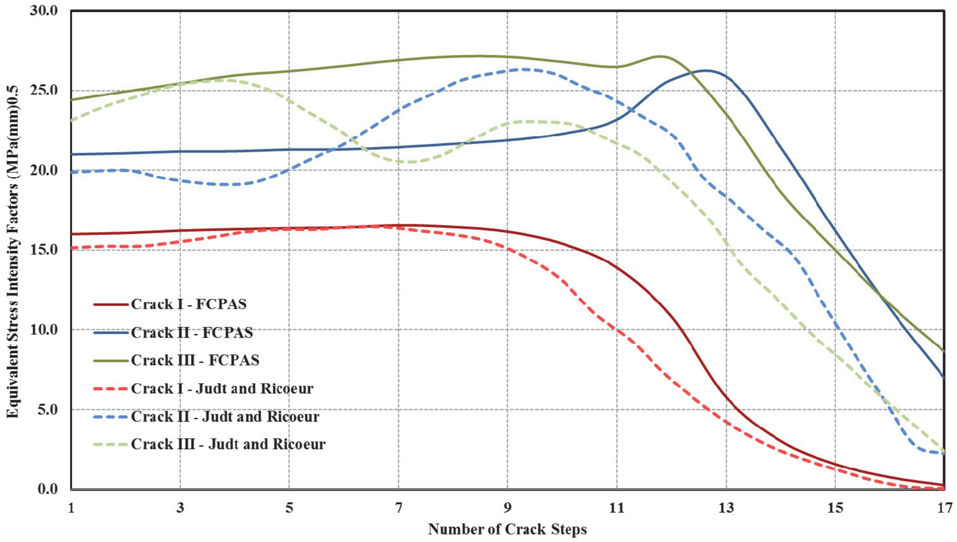

Figure 9: SIF comparison between FCPAS and Ref. [17].

Fig. 8 and Fig. 9 show that FCPAS results are very close to those of Judt and Ricoeur [17] in terms of crack paths and SIF vs. number of crack growth steps. Therefore, this case serves as a second validation problem for application of FCPAS crack propagation analysis procedures to multiple cracks in thin-walled structures growing in a non-planar manner.

CONCLUSIONS

In this study, three-dimensional non-planar crack propagation analyses for thin-walled structures were performed using FCPAS (Fracture and Crack Propagation Analysis System). Two case studies related to multiple cracks with non-planar growth were presented. Results of these studies in terms of crack paths and stress intensity factor variations with number of cycles showed good agreement with analysis data from the literature. Thus, it was concluded that non-planar growths of multiple cracks in thin-walled structures can accurately be simulated using FCPAS. Further developments in FCPAS to be able to analyze non-planar crack growth of surface and corner cracks under mode-I, II and III conditions are planned as future studies.

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