A critical analysis of the Mises stress criterion used in frequency domain fatigue life prediction

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ABSTRACT. Multiaxial fatigue failure criteria are formulated in time and frequency domain. The number of frequency domain criteria is rather small and the most popular one is the equivalent von Mises stress criterion. This criterion was elaborated by Preumont and Piefort on the basis of well-known von Mises stress concept, first proposed by Huber in 1907, and well accepted by the scientific community and engineers. It is important to know, that the criterion was developed to determine the yield stress and material effort under static load. Therefore the direct use of equivalent von Mises stress criterion for fatigue life prediction can lead to some incorrectness of theoretical and practical nature. In the present study four aspects were discussed: influence of the value of fatigue strength of tension and torsion, lack of parallelism of the SN curves, abnormal behaviour of the criterion under biaxial tension-compression and influence of phase shift between particular stress state components. Information contained in this article will help to prevent improper use of this criterion and contributes to its better understanding.

KEYWORDS. Mises stress; Frequency domain; Multiaxial fatigue.

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INTRODUCTION

Multiaxial fatigue life criteria are used during fatigue life prediction to compute equivalent uniaxial strain or stress state. This procedure is essential for the calculation of final fatigue life based on uniaxial fatigue characteristics. There are many propositions of multiaxial fatigue failure criteria in the literature based on stress invariants, critical plane concept, integral criteria and other [1–3]. Most of them are defined in time domain and implemented for working on stress or strain time histories or on sets of amplitudes. There are only few propositions dedicated for frequency based fatigue life assessment were the frequency domain definition of the criterion is expected [4, 5]. One of them is the criterion of Equivalent von Mises Stress (EMS), which is nowadays more and more popular and well accepted by engineers [6]. In this paper advantages and possible risks of usage of this criterion are taken under discussion. Particular four aspects were presented in details which are causing significant effect on computed fatigue life:

– differences between fraction of fatigue strength of pure tension and torsion and the value of square root of 3,
– lack of parallelism of the S-N curves for pure tension and torsion,

– abnormal behaviour of the criterion under biaxial tension-compression loading condition in comparison to the experimental results presented in the literature,
– influence of phase shift between particular stress state components on the value of equivalent stress.

The aim of this paper is the clear presentation of the theoretical limitation and the area of practical application of the EMS criterion. Examples based on technical experiments are recalled which confirms the correctness of discussed limitations. Provided information are highly important for engineers which are using equivalent von Mises stress in frequency domain as well as for researchers who are working on new multiaxial fatigue failure criteria where von Mises stress is using as a part of the definition.

EQUIVALENT VON MISES STRESS CRITERION IN FREQUENCY DOMAIN

The main reason for creating of this document are increasingly frequent, uncritical application of the Equivalent von Mises Stress (EMS) criterion in engineering calculations in the field of fatigue assessment. Since M.T. Huber in 1904 [7] and R. von Mises in 1913 [8] publish the theoretical background of the possibility of measurement of material effort by specific work of strain EMS criterion has become the most widely used in industry and science, among others in determining equivalent uniaxial stress, yield strength limit or other parameters used for example in constitutive equations [9]. Without going into details about derivation, which are well presented in many publications, the final equation for equivalent stress EMS criterion can be written as follow:

$$\sigma_{EMS} = \sqrt{\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+\left(\sigma_{zz}-\sigma_{xx}\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]} \tag{1}$$

or

$$\left(\sigma_{EMS}\right)^2 = \left(\sigma_{xx}\right)^2+\left(\sigma_{xx}\right)^2+\left(\sigma_{xx}\right)^2-\sigma_{xx}\sigma_{xx}-\sigma_{yy}\sigma_{zz}-\sigma_{zz}\sigma_{xx}+3\tau_{xy}^2+3\tau_{yz}^2+3\tau_{zx}^2 \tag{2}$$

using proper components of the stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \tag{3}$$

In 1994 Preumont and Piefort [10] represent a breakthrough adapt of the von Mises criterion for determining the material fatigue directly in frequency domain. They propose to calculate of the power spectral density of equivalent von Mises stress directly in frequency domain as follow

$$G_{EMS}(f) = \text{Trace}\left\{\mathbf{Q}_M \mathbf{G}(f)\right\} \tag{4}$$

However in the referenced paper [10] plane stress state was analysed presented method is well applicable also in spatial stress state with following power spectral density matrix:

$$\mathbf{G}(f) = \begin{bmatrix} G_{xx,xx}(f) & \cdots & G_{xx,yz}(f) \\ \vdots & \ddots & \vdots \\ G_{yz,xx}(f) & \cdots & G_{yz,yz}(f) \end{bmatrix} \tag{5}$$

defined according to vector of stress tensor components

$$\mathbf{S} = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \tau_{xy} \quad \tau_{yz} \quad \tau_{zx}] \tag{6}$$

and with von Mises coefficient matrix [11]

$$\mathbf{Q}_M = \begin{bmatrix} 1 & -0.5 & -0.5 & 0 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ -0.5 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \quad (7)$$

After solving Eq. (4) using (5) and (7) following expression for PSD of EMS can be written

$$\begin{aligned} G_{EMS}(f) = {} & G_{xx,xx}(f) + G_{yy,yy}(f) + G_{zz,zz}(f) + \\ & 3G_{xy,xy}(f) + 3G_{yz,yz}(f) + 3G_{zx,zx}(f) + \\ & -\mathrm{Re}\left(G_{xx,yy}(f)\right) - \mathrm{Re}\left(G_{yy,zz}(f)\right) - \mathrm{Re}\left(G_{zz,xx}(f)\right) \end{aligned} \quad (8)$$

PSD of equivalent von Mises stress in the form of Eq. (8) can be useful while programing this criterion in low level programing languages where matrix operation (4) cannot be easily realised.

LIMITATIONS ON THE USE OF EMS CRITERION IN THE FATIGUE CALCULATION

Fraction of Fatigue Strengths of Tension and Torsion

Let us set the loading of an abstract structure and the reference axes in such a way as to obtain only one a non-zero shear stress component

$$\mathbf{S}_{tor.} = [\, 0 \quad 0 \quad 0 \quad \tau_{xy} \quad 0 \quad 0 \,] \quad (9)$$

In practice this can be a stress state observed at surface of round specimen under pure torsion. In such a case PSD matrix will possess only one nonzero component – the component $G_{xy,xy}(f)$. According to the Eq. (8) equivalent PSD function of stress reduce to the following form

$$G_{EMS}(f) = 3G_{xy,xy}(f) \quad (10)$$

Generally speaking PSD function of stress describes how power of a stress history is distributed over frequency. 'The power' should be understood as the variance of the stress history, what can be expressed as follow

$$\mu = \int_0^\infty G(f)df \quad (11)$$

where μ is the variance of the stress history. Also, it is possible to calculate the expected signal amplitude for a specified small frequency range of a width of Δf

$$\sigma_a = \sqrt{2\mu} = \sqrt{2 \cdot \int_{fr}^{fr+\Delta f} G(f)df} \quad (12)$$

Analysing the Eq. (10) it can be seen that according to the von Mises stress criterion for computing PSD of equivalent uniaxial stress is to multiply PSD for pure torsion times 3. Transforming this action to the stress amplitude following the relationship presented in Eq. (12) we get the equation:

$$\sigma_{af} = \sqrt{3}\tau_{af} \quad \Rightarrow \quad \frac{\sigma_{af}}{\tau_{af}} = \sqrt{3} \approx 1.7321 \tag{13}$$

what is also valid for basic von Mises criterion in time domain, Eqs. (1) and (2). According to numerous reported experimental results such an equality is fulfilled only for few materials. Usually the ratio (13) varies between 1 and 2 for fatigue strength and fatigue limit as well. It is important to know how much influence have a deviation from the specified square root of three value (13) on calculated fatigue life. In order to present the scale of the problem fatigue life was calculated for round specimen under random, narrow-banded and Gaussian, pure torsion loading. In such a kind of loading the Probability Density Function (PDF) of amplitudes describe Rayleigh distribution [12]. On the Fig. 1a) PDF for shear stress amplitudes and equivalent tension amplitudes according EMS criterion were presented. It was also assumed that the Miner rule is applicable and constant amplitude SN curves for pure torsion and tension are known and described as follow

$$\sigma_a = S_f \left(\frac{N}{N_\sigma}\right)^{\frac{-1}{m_\sigma}}, \qquad \tau_a = T_f \left(\frac{N}{N_\tau}\right)^{\frac{-1}{m_\tau}} \tag{14}$$

where: S_f and T_f – fatigue limits for tension and torsion; N_σ and N_τ – number of cycles for knee points; m_σ and m_τ – slopes of the SN curves. On the Fig. 1b) two SN curve are presented which satisfy the $S_f / T_f = 1.7321$ condition, Eq. (13). Computed fatigue life T_1 according PDF of shear stress amplitudes and SN curve for torsion are equal to T_2 computed from PDF of equivalent tension amplitudes and SN curve for tension. For materials that do not meet the condition (13) computed fatigue life T_1 and T_2 differ significantly. Such a case is presented on Fig. 1c) for $S_f / T_f = 1.5$ what results in $T_1 / T_2 = 3.16$.

Lack of Parallelism of the SN Curves

Lack of parallelism is a special situation of the problem discussed in previous section. In this case the equality (13) cannot be fulfilled in whole range of cycles to failure. Depends on the m_σ and m_τ slopes of SN curves different deviation from $T_1 / T_2 = 1$ can be obtained. This effect was illustrated in Figs. 1d) 1e) 1f) where computed results obtained with the same procedure as described in previous section were presented.

Abnormal Behaviour of the EMS Criterion under Biaxial Tension-Compression

Biaxial tension-compression is a plane stress state where for specific reference axes only the shear component is constant and equal zero

$$\mathbf{S}_{bi.} = [\sigma_{xx} \quad \sigma_{yy} \quad 0 \quad 0 \quad 0 \quad 0\ ] \tag{15}$$

Pure biaxial tension-compression is rarely found in practice but it is used for verification of multiaxial fatigue failure criteria. Such kind of stress state is released on so called cruciform specimens through loading of two sets of arms in perpendicular direction [13]. This two loading components can be of any type, for example in-phase, out-of-phase or random with given correlation coefficient. In biaxial tension-compression the PSD of equivalent stress can be computed as follow

$$G_{EMS}(f) = G_{xx,xx}(f) + G_{yy,yy}(f) - \mathrm{Re}[G_{xx,yy}(f)] \tag{16}$$

Real part of cross spectral density $\mathrm{Re}[G_{xx,yy}(f)]$ is equal 0 for fully uncorrelated loading components $\sigma_{xx}(t)$ and $\sigma_{yy}(t)$. There are some interesting results published in the literature which are showing basic biaxial fatigue behaviours of tested materials. Cláudio et al. [13] presenting results from which it appears that the EMS criterion does not fit the real behaviour of material. This can be observed on the Fig. 2 where EMS criterion for correlated data gives lower stress amplitudes and for out-of-phase loading higher stress amplitudes than expected.

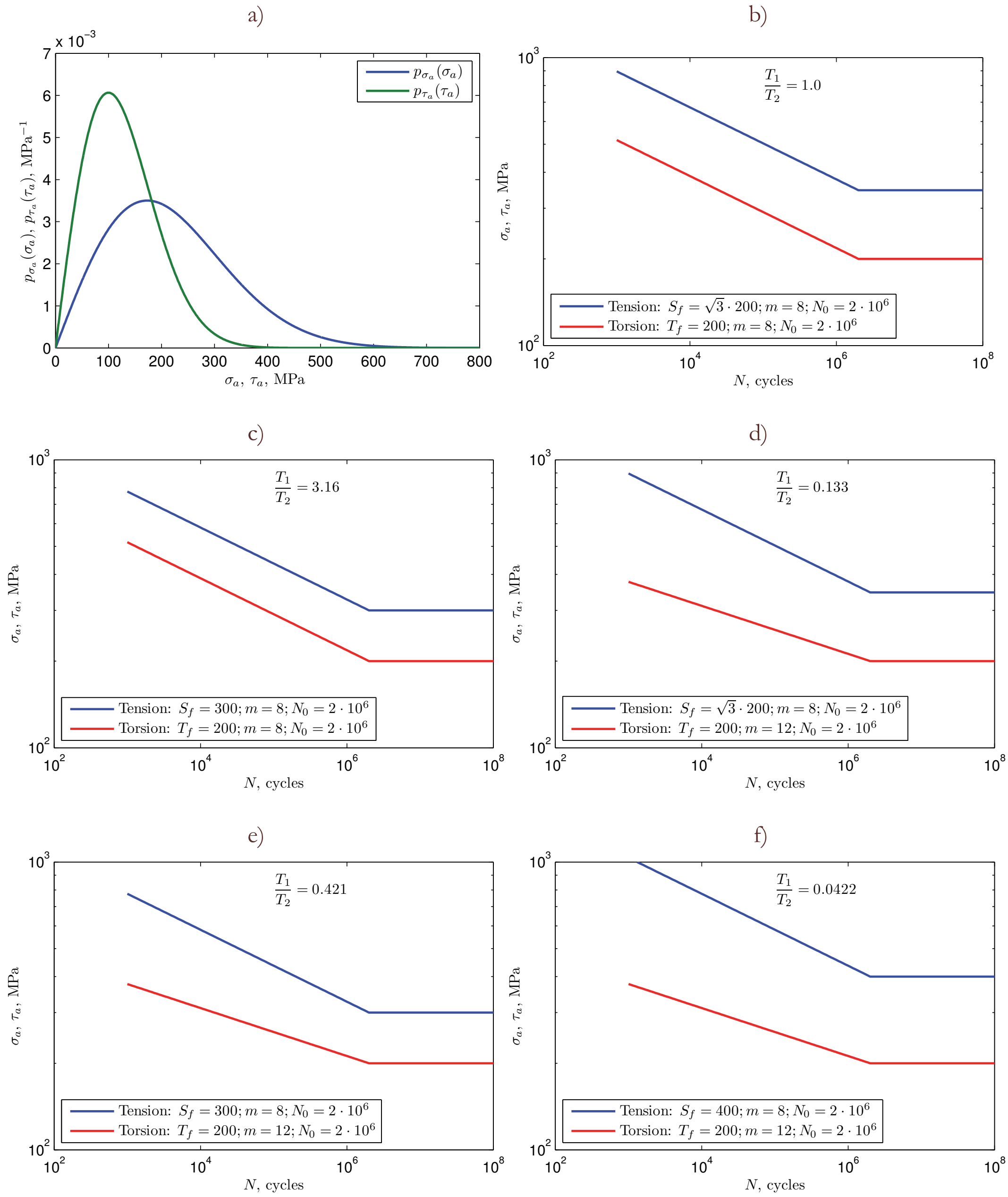

Figure 1: PDF of amplitudes used for computation of fatigue life T1 and T2 (a) and five sets of SN curves for torsion and uniaxial tension (b), (c), (d), (e), and (f).

Influence of Phase Shift between Particular Stress State Components

It is well known under fatigue community that the phase shift between particular components of the multiaxial loading are influencing the fatigue life.

A special interest among scientists and researchers has a combination of tension-compression or bending and torsion, as it is commonly encountered in a responsible machine elements such as shafts. Many test results have been published in this area. All of them show a significant effect of the phase shift fatigue. Analysed criterion does not have such properties. Under a combination of tension and torsion

$$\mathbf{S}_{tt.} = [\sigma_{xx} \quad 0 \quad 0 \quad \tau_{xy} \quad 0 \quad 0\ ] \tag{17}$$

the Eq. (8) for PSD of EMS simplify to following expression

$$G_{EMS}(f) = G_{xx,xx}(f) + 3G_{xy,xy}(f) \tag{18}$$

As we can see cross power spectrum is not present. This function is the only one which includes information about correlation (phase shift for one harmonic component) what can be treat as a proof of the omission of phase shift effect.

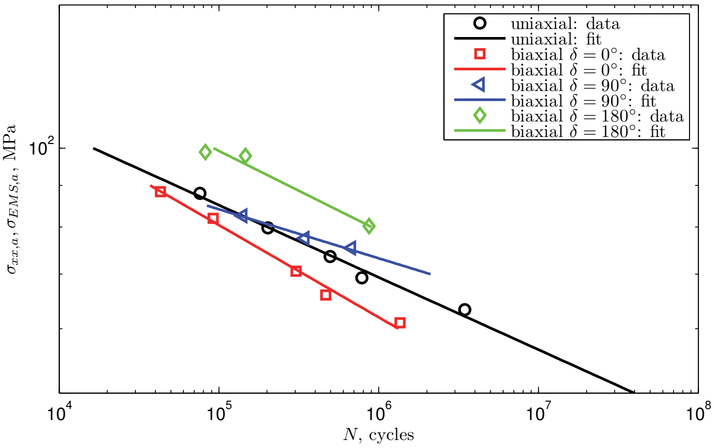

Figure 2: SN curves uniaxial and biaxial data presented by Cláudio et al. [13].

REMARKS AND FINAL CONCLUSIONS

1. It is not recommended to use the EMS criterion in a case where the shear stresses dominate, and the ratio of fatigue limits is different from the square root of three.
2. Abnormal behaviour of EMS criterion it can be observed in comparison to experimental results under biaxial tension-compression. For example uniaxial and in-phase biaxial loading give the same equivalent amplitude according this criterion but the test results showing shortening of the fatigue life, see Fig. 2.
3. The impact of non-parallelism of fatigue characteristics on calculated life is significant and depends on loading level.
4. Correlation between normal and shear stress components are neglected. Therefore, this criterion can be used for materials that do not show sensitivity to the phase shift between these components.

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