A statistically self-consistent fatigue damage accumulation model including load sequence effects under spectrum loading

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ABSTRACT. A probabilistic methodology is proposed to evaluate fatigue damage accumulation and fatigue lives of specimens under variable amplitude loading. With probabilistic modifications in the present model, the calculative consistency is achieved between fatigue damage and fatigue life. The load sequence effects on fatigue damage accumulation are properly accounted for variable amplitude loading. The developed damage model overcomes the inherent deficiencies in the linear damage accumulation rule, but still preserves its simplicity for engineering application. Based on the Monte Carlo sampling method, numerical verification of this model is conducted under two kinds of spectrum loading. The predicted probabilistic distributions of fatigue lives are validated by fatigue tests on Al-alloy straight lugs.

KEYWORDS. Fatigue damage; Fatigue life; Probabilistic statistical model; Load sequence effect; Statistical self-consistency.

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INTRODUCTION

Fatigue damage is one of the most common failure modes encountered in engineering structures. Therefore fatigue failure prediction is widely carried out during various processes, such as engineering structural design, safety assessment and optimization. In the past decades, many damage models have been presented to quantitatively analyze fatigue damage and failure of materials and components. However, the mechanisms and process of fatigue failure under variable amplitude loading conditions are random in nature owing to a variety of indeterminable factors in materials as well as external environments.

The linear cumulative damage rule, firstly proposed by Palmgren in 1924 and subsequently by Miner in 1945 [1], is widely used for fatigue analyses of structures under variable amplitude loading. It can be expressed as: $D=\sum(ni/Ni)$. After that, many developments have been made to get more accurate predictions, including the nonlinear damage rules, such as damage curve based method [2], ductility exhaustion model [3], continuum damage mechanics approach [4] and energy-based damage method [5]. The fatigue damage accumulation within these models and approaches are mostly based on deterministic concepts, whereas in practice, damage accumulation is usually of stochastic nature.

Several probabilistic approaches have been established for damage accumulation evaluation. Many issues related to statistical characteristics of fatigue have been involved, such as the combined randomness of loading process and fatigue resistance of materials [6], probabilistic density function of failure cycles [7], complexity of loading conditions [8], random critical damage [9], frequency domain approach [10] and time domain approach [11]. However, few of them focused both on the load sequence effect and the probabilistic nature of fatigue damage. Based on Morrow's nonlinear plastic work interaction damage rule, W.-F.Wu and T.-H. Huang [8] proposed an approach to predict the fatigue damage and fatigue life under random loading considering the loading history and the statistical feature.
In this paper, a new model is proposed to predict fatigue damage and fatigue lives of specimens under spectrum loading. The effect of high load retardation on damage accumulation is taken into account by introducing an exponential function exp(-fj) in the model. The independent variable fj is related to the stress ratio of high and low stress levels. The statistically self-consistency between the probabilistic distributions of fatigue damage and failure cycles is also achieved by introducing a consistent index b (greater than one) and a random disturbance Δ.
In the damage model, the fatigue damage and fatigue life are predicted using the probabilistic sampling method supported by test data. As for the critical damage Dc, i.e. the damage value at failure, there are mainly two points of view: (1) Dc is deterministic, equal to or smaller than unit; (2) Dc is a random variable, with mean value equal to unit. In the present paper, the second view point is adopted. The fatigue life N is assumed to follow the two-parameter Weibull distribution, W(α,β) [12]. The shape parameter α is set as 4 [13] for aluminum alloy according to a lot of fatigue test data. In order to verify the reliability and feasibility of this model, the predicted fatigue life is compared with that of the fatigue test results.

A STATISTICALLY SELF-CONSISTENT MODEL FOR FATIGUE DAMAGE AND FATIGUE LIFE PREDICTION

For constant amplitude loading, Palmgren-Miner's linear damage accumulation rule, which is widely used in engineering, is expressed as

$$D_c = \sum_{i=1}^{n_f} \frac{1}{N_i} = 1 \tag{1}$$

where N_i is the fatigue life under the ith stress level, which actually obeys some distribution; n_f is the cycle number to failure; and the critical damage value D_c is assumed to be one.
Obviously, when N is taken as a constant, Eq. (1) can be simplified to the form of n_f/N=1, which is the most common form of Miner's damage criterion. However, in actual situations, N has probability characteristics, thus n_f is also correspondingly statistical rather than deterministic.
With a given distribution of N, the equation can be called statistically consistent if n_f obtained from Eq. (1) with the Monte Carlo sampling method is approximate to N with respect to probability distribution. In practice, fatigue life N is exactly reflected in the number of load cycles to failure, so it is necessary that the damage criterion be statistically consistent. However, it can be verified that Eq. (1) is statistically inconsistent as follows.
Through numerical simulation with the Monte Carlo sampling method [14], it can be found that the cycle number n_f obtained from Eq. (1) is not identical to the original fatigue life N considering probability distribution. Assuming $N \sim W(a, \beta)$, with a=2.1, β=1129, the mean value μ=1000, and the standard deviation σ=500, sufficient random sampling(2,000 times) of Eq. (1) is performed. As a result, 2000 different n_fs can be obtained and then statistics of n_f are calculated as: μ=670, σ=41. It should be noted that both the mean value and standard deviation are smaller than those of the original N, especially for standard derivation. Thus it can be concluded that Eq. (1) is statistically inconsistent.
In order to enlarge the mean value and the standard deviation of n_f, a consistent index b (greater than one) and a random disturbance Δ with mean value equal to zero are introduced to the left and right sides of Eq. (1) respectively, which makes the linear fatigue damage prediction model statistically consistent, given by

$$\sum_{i=1}^{n_f} \left(\frac{1}{N_i} \right)^b = 1 + \Delta, \qquad \Delta = \frac{N_i - E(N)}{E(N)} \tag{2}$$

where, N_i is a random fatigue life from certain distribution, which corresponds to the ith load cycle, and $E(N)$ is the mean value of the life distribution.

The self-consistent index b is related to statistical parameters a and β of fatigue life distribution. It can be solved by comparing the mean values of n_f with N through numerical simulation. The right-hand side term $(1+\Delta)$ indicates that the critical damage is a random variable. Assuming $N \sim W(a, \beta)$, $(1+\Delta)$ obeys $W(a, \beta/\mu)$ distribution, with its mean value equal to unit and standard derivation equal to σ/μ, respectively. Here, σ is the standard derivation of fatigue life N.
Assuming $N \sim W(a, \beta)$, a=2.1, β=1129 and accordingly μ=1000, σ=500, through large number of Monte Carlo sampling tests, it is found that if the consistent index b is taken as 1.065, the mean values of n_f and N can be comparable. Repeating Monte Carlo sampling of Eq. (2) for 2000 times, the statistics of n_f are calculated as: a=2.1, β=1139, μ=1009, σ=513. Comparing with the original fatigue life, it can be noted that n_f is close to N with respect to the mean value and standard deviation, as well as the maximum likelihood estimations of parameters a and β. Furthermore, in the goodness of fit test with the original Weibull distribution, n_f has passed K-S test and W^2 test quite well. Therefore, the new cumulative damage criterion expressed in Eq. (2) can be considered statistically consistent.
Through some appropriate transformation of Eq. (2), a statistically consistent fatigue damage model that can quantitatively calculate the statistical properties of damage under spectrum loading conditions can be achieved. For simplicity, the case of constant amplitude loading is first presented. It should be mentioned that in practice, for the case of constant amplitude loading, the concept of loading block generally does not exist. Here, it is used for the generalization to the case of variable amplitude loading.
Assuming there are n load cycles in one loading block, the equation to calculate the damage D_B caused by one loading block can be established by moving the random disturbance Δ to the left-hand side of Eq. (2), written as

$$D_B = \sum_{i=1}^{n}\left(\frac{1}{N_i}\right)^b - \frac{N_i - E(N)}{E(N)} \tag{3}$$

Then, the fatigue life can be predicted by

$$N_f = \frac{1}{D_B} \cdot n \tag{4}$$

Only if the predicted fatigue life N_f obtained by Eq. (4) is approximate to the original N, can the fatigue damage model be called statistically consistent. A coefficient relative to the cycle number n must be added in front of Δ to make D_B satisfy the boundary conditions: (1) damage value is zero when the cycle number is zero; (2) the mean value of damage is approximate to 1 when the cycle number equals to the mean value of fatigue life. In addition, the coefficient should increase with the number of cycles. Through the large number of Monte Carlo sampling tests, it is found that when the coefficient in front of Δ is defined as n/N_i, N_f can be close to N for the probabilistic properties.
Thus, the damage caused by one loading block D_B can be given by

$$D_B = \sum_{i=1}^{n}\left(\frac{1}{N_i}\right)^b - \frac{n}{N_i}\frac{N_i - E(N)}{E(N)} \tag{5}$$

Eq. (5) can be extended to the case of spectrum loading naturally, written as

$$\begin{aligned} D_B &= \sum_{j=1}^{m}\left[\sum_{i=1}^{n_j}\left(\frac{1}{N_{ji}}\right)^{b_j} - \frac{n_j}{N_j}\frac{N_j - E(N_j)}{E(N_j)}\right] \\ N_f &= \frac{1}{D_B} \times \sum_{j=1}^{m} n_j \end{aligned} \tag{6}$$

where D_B is the damage introduced by one loading block with m stress levels; m is the number of stress levels in a spectrum loading; n_j is the cycle number under the jth stress level; a_j is the self-consistent exponent dependent on the fatigue life distribution under jth load level , which makes the mean value of n_f (Eq.(2)) approximate to the original N; N_j

is a random number obeying the distribution of fatigue life under jth stress level; $E(N_j)$ is the mean value of fatigue life under the jth load level and N_f is the random fatigue life under spectrum loading.

For the case of constant amplitude loading, the self-consistency of fatigue damage and fatigue life prediction model can be verified by comparing the failure cycles n_f obtained from model with the original fatigue life N. However, this rule does not work when it comes to the case of spectrum loading, because the distributions of original fatigue lives are not unique. In order to verify the fatigue damage and fatigue life under spectrum loading predicted by Eq. (6), two engineering assumptions are firstly made on the basis that there are no load sequence effect: 1) the fatigue life distribution under spectrum loading, N_f, should be closer to N_j with larger damage ratio $n_j/E(N_j)$, and the mean value of N_f should be in the range of $[E(N_j)_{min}, E(N_j)_{max}]$; 2) the mean value of N_f should be very close to the one calculated by Palmgren-Miner's linear damage accumulation rule.

Numerical sampling test and parameter estimation are carried out according to Eq. (6) under two-level spectrum loading. The parameters of fatigue life distribution for each load level are shown in Tab. 1. Four projects with different combinations of load cycles and stress levels are employed in this test and the corresponding simulation results are listed in Tab. 2. The probabilistic density function(PDF) of different fatigue life distributions are compared with each other, as shown in Fig. 1.

j	α_j	β_j	$E(N_j)$	σ	exponent a_j
1	2.7	1125	1000	400	1.038
2	3.7	11079	10000	3000	1.014

Table 1: Parameters of fatigue life distribution for each load level.

project	n_1	n_2	n_1/E_1	n_2/E_2	μ_{N_f}	$\bar{N}$	$\mu_{N_f}/\bar{N}$
1	200	100	0.2	0.01	1386	1429	0.97
2	400	100	0.4	0.01	1211	1220	0.99
3	100	3000	0.1	0.3	7344	7750	0.95
4	100	5000	0.1	0.5	8278	8500	0.97

Table 2: Simulation results of 4 projects of two-level spectrum loading.

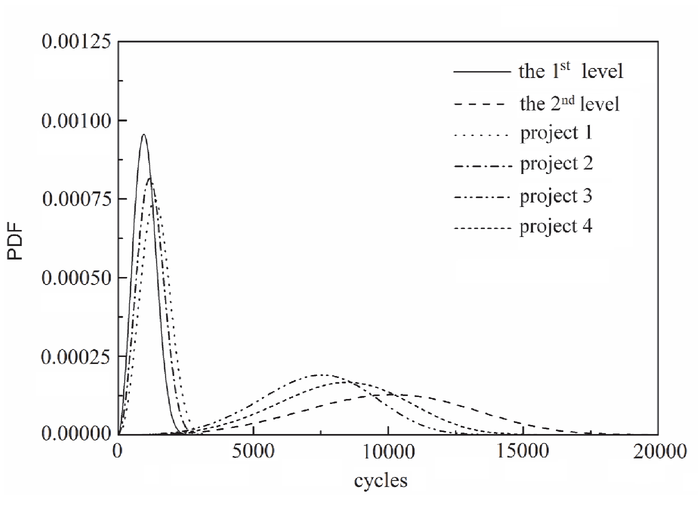


Figure 1: Comparisons of fatigue life distributions of different projects with the original constant amplitude loading.

From Fig. 1, the PDFs of project 1(N_{f1}) and project 2(N_{f2}), which have larger damage ratios of n_1/E_1, are closer to the 1st level(N_1) rather than to the 2nd level(N_2). Furthermore, the PDF of project 2 is closer to the 1st level than project 1. The PDFs of project 3(N_{f3}) and project 4(N_{f4}), which have larger damage ratios of n_2/E_2, are closer to the 2nd level(N_2) rather than the 1st level. Furthermore, the PDF of project 4 is closer to the 2nd level than project 3. The mean values of N_f are in the range of $[E(N_1), E(N_2)]$. This simulation results verifies the first assumption. In Tab. 2, the mean value of fatigue life calculated by this model is almost the same as the one given by Palmgren-Miner's linear damage accumulation rule, which is consistent with the second assumption. Thus, Eq. (6) is verified to be a reasonable model for fatigue damage prediction based on numerical calculation. However, it should be further validated by experimental tests before practical application.

STATISTICAL DISTRIBUTION OF FATIGUE LIFE UNDER CONSTANT AMPLITUDE LOADING

In order to calculate the fatigue damage and fatigue life by Eq. (6) for structures subjected to variable amplitude loading, statistical distributions of fatigue lives under constant amplitude loading are needed as a baseline data. Axial fatigue test on straight lugs specimens was conducted by using INSTRON-1342 fatigue test machine at a frequency of 20Hz at room temperature with the stress ratio R of 0.06.The geometry of the specimen is shown in Fig.2 and the test results are listed in Tab. 3 [15].

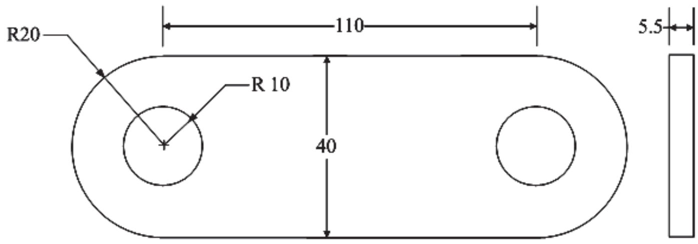


Figure 2: Geometry of straight lugs specimen for fatigue testing, dimensions in mm.

Stress level	S_a (MPa)	R(stress ratio)	fatigue life(cycles)
1	38.46	0.06	365105,171327,230701,119173,408225,346068
2	42.73	0.06	264258,114391,184623,94016,128481,274699
3	47.00	0.06	142472,153949,58218,61666,77398,125822

Table 3: Fatigue test results of straight lugs subjected to constant amplitude loading.

For median fatigue life ranging from 10^4 to 10^6 cycles, the relations of S-N_p and S-β could be written as

$$\lg\beta=B_1\lg S+A_1 \tag{7}$$

$$\lg N_p=B_2\lg S+A_2 \tag{8}$$

where A_1, B_1, A_2 and B_2 are constant coefficients; S is the stress level of the constant amplitude loading; β is the characteristic life of test sample and N_p is the fatigue life of the test sample with reliability of p. The values of A_1, B_1, A_2 and B_2 can be determined by least square method with given S, β and N_p.

Due to the time and cost, only six fatigue life data is acquired for each stress level. To get more reliable information from these limited numbers of tests, three statistical methods are employed to analyze the data in Tab. 3, i.e. Bayesian Jerry prior method (BJPM), Bayesian conjugate prior method (BCPM) and the Bootstrap method (BTPM). The estimation of characteristic life($\hat{\beta}$), the estimation of characteristic life with confidence level of c (β_c) and the fatigue life with 95% reliability and 95% confidence level ($N_{95/95}$) are all obtained. It is also verified whether the original fatigue test data fall into the 95% confidence intervals of the probabilistic distributions based on the above three statistical methods (Tab. 4). From a statistical point of view, the more test data fall into the 95% confidence interval, the more reasonable the probabilistic distribution is. Fatigue life distributions under constant amplitude loading based on the three statistical methods are shown in Fig. 3, where the original test data is marked on the curve. The fatigue life with 95% confidence interval is denoted by the vertical lines.

Stress level	Statistical methods	$\hat{\beta}$	β_c	$N_{95/95}$	Number of data out of the 95% confidence interval
1	BJPM	321095	279087	132816	1
	BCPM	318112	295985	140858	1
	BTPM	329156	297991	141813	1
2	BJPM	212586	184774	87933	0
	BCPM	207824	186917	88953	0
	BTPM	217875	197179	93837	0
3	BJPM	121001	105170	50050	0
	BCPM	119321	109461	52092	0
	BTPM	124085	112271	53429	0

Table 4: Estimated parameters of straight lugs fatigue tests under constant amplitude loading based on BJPM, BCPM and BTPM.

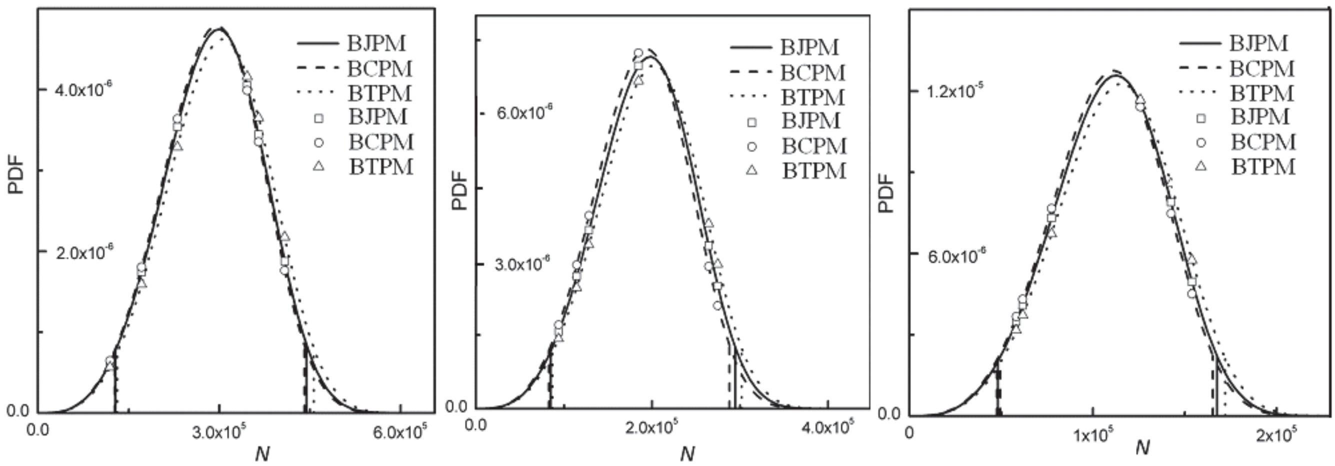


Figure 3: Fatigue life distributions under constant amplitude loading based on BJPM, BCPM and BTPM.

All the statistical results obtained through the three statistical methods are similar and the probabilistic distributions are very close to each other. Meanwhile, almost all the test data fall into the 95% confidence interval of the probabilistic distribution, indicating that the probabilistic distributions based on the three statistical methods are reasonable and reliable enough. Moreover, the BJPM is more conservative and simple to use, without super parameters, sample size enlarging or resampling. Thus, the test results in the following are processed by BJPM.
The fitted values of A_1, B_1, A_2, B_2 in Eq. (7) and Eq. (8) are listed in Tab. 5, where r is the correlation coefficient of the linear fitting.

A_1	B_1	A_2	B_2	r
13.2024	-4.8468	12.8190	-4.8468	0.9930

Table 5: Coefficients of S-N curves.

For an arbitrarily stress level S_j with constant stress ratio of 0.06, the corresponding statistical parameters, i.e. β and $N_{95/95}$, can be calculated by Eq. (7) and Eq. (8). Then, the probabilistic distribution of fatigue life N_j corresponding to S_j can be obtained by numerical sampling based on the Weibull distribution, which provides necessary parameters for the fatigue damage and fatigue life prediction under variable amplitude loading.

The consistent index b is closely related to statistical parameters α and β of fatigue life distribution and can be solved by Monte Carlo sampling test according to Eq. (2). When α is assumed as 4, the quantitative relation between index b and scale parameter β can be obtained through large number of sample tests, the results are shown in Tab. 6. Note that the general trend is that b decreases with β and b stay constant in some interval of β instead of changing continuously with β.

β	$(8\times10^3,10^4]$	$(10^4,5\times10^4]$	$(5\times10^4,10^5]$	$(10^5,10^6]$
b	1.011	1.010	1.009	1.008
β	$(10^6,6\times10^6]$	$(6\times10^6,5\times10^7]$	$(5\times10^7,5\times10^8]$	$(5\times10^8,10^{10}]$
b	1.007	1.006	1.005	1.004

Table 6: Quantitative relation between the consistent index b and scale parameter β(α=4).

EXPERIMENTAL VERIFICATION OF THE STATISTICAL FATIGUE DAMAGE MODEL UNDER SPECTRUM LOADING

Fatigue tests under spectrum loading

The specimens are the same as those used in the fatigue tests under constant amplitude loading [15]. The load spectra and fatigue life results are listed in Tab. 7.

spectrum	Load order	1	2	3	4	5	Cycles to failure
5-level	Sa(MPa)	17.09	27.77	42.73	36.32	21.36	500826, 686156, 2445490, 1201897, 2253944
	cycles	1000	500	200	600	900	
3-level	Sa(MPa)	27.77	42.73	36.32	\	\	371112, 522772, 670298, 507405, 426219, 364397
	cycles	500	302	600	\	\	

Table 7: Fatigue test results of straight lugs under spectrum loading.

Using the data in Tab. 7, the statistical parameters of fatigue life under spectrum loading can be obtained by BJPM, including $\hat{\beta}$,E, σ, $N_{95/95}$ and the interval of 95% fatigue life (β_1, β_2), (E_1,E_2) and (σ_1,σ_2) . Here σ is the standard deviation of the test results, demonstrating the scatter characteristics of fatigue lives.

Experimental verification of the fatigue damage model

Statistical parameters for each stress level of the multilevel spectrum are obtained via the S-N curves described in section 3, including characteristic life β, the average life E, $N_{95/95}$ and the self-consistent index a, listed in Tab. 8.

	Sa(MPa)	17.09	27.77	42.73	36.32	21.36
statistical parameters	β	158271738	1605853	198884	437239	6180778
	E	143457895	1455549	180269	396314	5602272
	$N_{95/95}$	65467281	664242	82266	180859	2556608
	self-consistent index a	1.006	1.007	1.008	1.008	1.008

Table 8: Statistical parameters of fatigue life distributions under constant amplitude loading.

Two thousand values of N_f can be acquired using Eq. (6) from 2000 groups of random sampling based on the fatigue life distributions under constant amplitude loading, as shown in Tab. 8. The steps of random sampling are as follows:

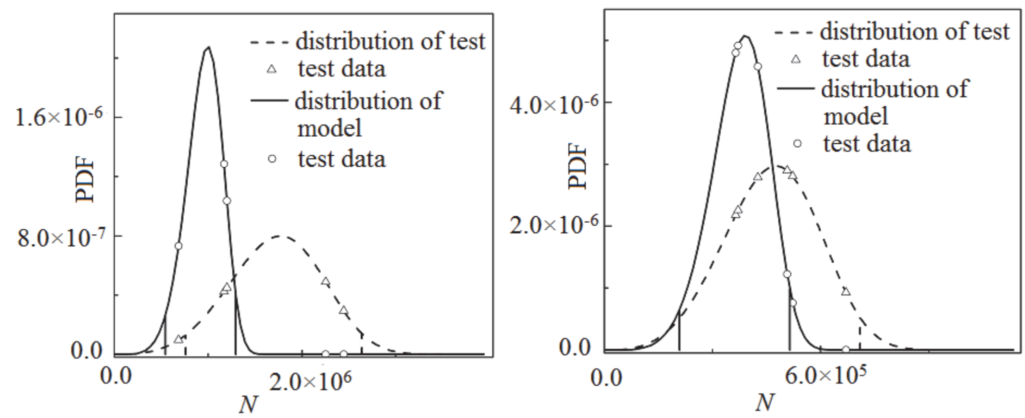
1) For the jth stress level, do n_j times of sampling according to the probabilistic distribution of fatigue life N_{ji} using parameters in Tab. 8. The damage caused by the jth level of stress(D_{Bj}) is obtained through Eq. (6).
2) The whole fatigue damage D_B in one loading block can be obtained by summing up D_{Bj} from the 1st level to the mth level.
3) Fatigue life(N_f) under spectrum loading is calculated using the second formula of Eq. (6).
4) Repeating steps 1), 2)and 3) for 2000 times.

Based on 2000 values of N_f, the statistical parameters of fatigue lives, including $\hat{\alpha}$ (shape parameter), $\hat{\beta}$, E, σ and $N_{95/95}$, are estimated by the method of maximum likelihood. The ratios of parameters between model and test data are indicated by R_E, R_σ and R_{N95} respectively. The results are listed in Tab. 9. N_1 is the number of test data outside the 95% confidence interval of the model's probability distribution, which reflects the rationality of the model. Whether E and σ from test data, respectively, fall into the interval (E_1,E_2) and (σ_1,σ_2) from the model also indicates the validity of the prediction, as listed in the last 2 columns of Tab. 9.

load level	$\hat{\alpha}$	$\hat{\beta}$ ($\times10^6$)	E ($\times10^5$)	R_E	σ ($\times10^5$)	R_σ	$N_{95/95}$ ($\times10^5$)	R_{N95}	N_1	*Fall into* (E_1, E_2)	*Fall into* (σ_1, σ_2)
5	5.833	1.035	9.585	0.555	1.906	0.394	6.180	0.772	2	no	no
3	5.283	0.404	3.717	0.801	0.810	0.622	2.284	1.061	2	no	no

Table 9: Comparisons of parameters based on the statistical fatigue damage model with those from fatigue tests.

The predicted PDFs of fatigue life under spectrum loading are compared with the test results, as shown in Fig. 4. The comparisons indicate that significant discrepancy exists in the mean values, standard deviation and lateral deviations, especially for the 5-level spectrum loading. In addition, as shown in Fig.4, the schematic PDFs of predicted fatigue life N_f and the test results are quite different. Load sequence effect is believed to be the main reason for the difference between prediction and tests [16]. Therefore, improvements have to be brought to this model since interactions among variable stress levels should be considered.



a) fatigue life distributions under 5-level spectrum loading.

b) fatigue life distributions under 3-level spectrum loading.

Figure 4: Fatigue life distributions under spectrum loading.

A MODIFIED FATIGUE DAMAGE ACCUMULATION MODEL UNDER SPECTRUM LOADING CONSIDERING LOAD SEQUENCE EFFECT

The difference between the model and the tests under spectrum loading is mainly attributed to the load sequence effects, i.e. the acceleration effect and the retardation effect, as demonstrated by many researches in various fatigue tests [17, 18]. This direct effect of loading sequence plays an important role in the process of damage accumulation if the difference between load levels is large enough. Usually the retarding effect is more remarkable than the acceleration effect and easier to be defined quantitatively. Therefore, the retardation effect of high stress level is taken into account only in this work.

The retardation effect is due to the high stress level in the loading spectrum. The fatigue damage accumulation rate of specimens under a low stress level slows down since a high stress level may leads to micro-plasticity of material and compressive residual stress. The retardation effect is apparently related to the disparity between high and low load levels. In a variable amplitude stress condition, according to Morrow's plastic work interaction damage rule [19], the fatigue damage caused by the stress amplitude S_k can be written as:

$$D_k = \frac{n_k}{N_k}\left(\frac{S_k}{S_{\max}}\right)^d \tag{9}$$

where S_{max} is the maximum stress amplitude in the stress history; n_k is the cycle number of stress peak at level S_k; N_k is the number of stress peak to cause failure if the constant amplitude S_k is considered; and d is Morrow's plastic work interaction exponent which can be considered as the stress sequence effect on the fatigue damage.

Using the similar principle as Morrow's plastic work interaction damage rule, an exponential function $\exp(-f_j)$ is introduced to modify the fatigue damage caused by the low stress level, seen in Eq. (10). As the independent variable , f_j, in this exponential function is larger than one and related to the stress ratio of high and low load level, i.e. $\frac{S_{\max}^{j-1}}{S_j}$, where S_j is the stress peak of the jth stress level and $S_{\max}^{j-1}$ is the maximum stress peak among the stress levels from S_1 to S_{j-1}. In addition, a constant coefficient v is defined to accommodate the model to the test data, which can be fitted by comparing the calculated fatigue life through large number of Monte Carlo sampling with the test data. In order to reduce the damage accumulation rate of the lower stress levels, negative one is set as a multiplier to the independent variable f_j. The modified fatigue damage model with load retardation effect under multilevel spectral loading is given as:

$$\begin{aligned}
D_B &= \sum_{j=1}^{m}\left\{\left[\sum_{i=1}^{n_j}\left(\frac{1}{N_{ji}}\right)^{a_j}\right]\cdot\exp\left(-f_j\right) - \frac{n_j}{N_j}\cdot\frac{N_j - E\left(N_j\right)}{N_j}\right\} \\
N_f &= \frac{1}{D_B}\times\sum_{j=1}^{m} n_j \\
f_j &= \begin{cases} 0, \quad j=1 \quad or \quad S_j > S_{\max}^{j-1} \\ v\cdot\dfrac{S_{\max}^{j-1}}{S_j}, \qquad S_j < S_{\max}^{j-1} \end{cases} \quad and, S_{\max}^{j-1} = \max\left(S_1 \sim S_{j-1}\right)
\end{aligned} \tag{10}$$

where v is a constant depending on the material itself and the assumption of fatigue life distribution.

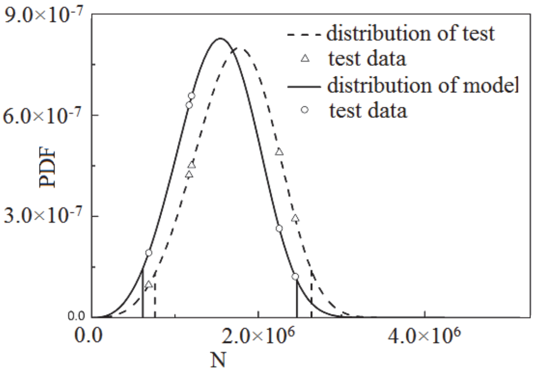
The load retardation effect index has no effect on the fatigue damage produced by the first stress level or the stress level higher than the previous maximum stresses , since $\exp(-f_j)=1$ in Eq. (10). However, the fatigue damage is affected by the load retardation effect index when the current stress level is lower than the previous maximum stresses , since $\exp(-f_j)<1$.

Simulation work (detailed steps presented in section 4.2) has been done by using this modified model to determine the value of v. It is found that the fatigue life distribution of the model matches that of the test well when v=1. The statistical parameters estimated by the modified model are given in Tab. 10.

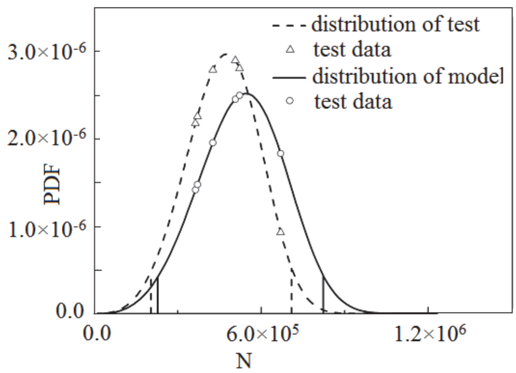
Load level	$\hat{\alpha}$	$\hat{\beta}$ ($\times10^6$)	E ($\times10^6$)	R_E	σ ($\times10^5$)	R_σ	$N_{95/95}$ ($\times10^5$)	R_{N95}	N_t	*Fall into* (E_1, E_2)	*Fall into* (σ_1, σ_2)
5	3.643	1.689	1.523	0.882	4.647	0.960	7.397	0.924	0	yes	yes
3	3.888	0.589	0.533	1.148	1.533	1.177	2.716	1.262	0	yes	yes

Table 10: Comparisons of parameters based on the modified statistical fatigue damage model with those of the fatigue tests.

The predicted PDFs of fatigue life by the modified model considering load sequence effect are compared with the test results under spectrum loading, as shown in Fig. 5. The original test data is also marked in the curve and the fatigue life with 95% intervals are denoted by the vertical lines. It can be found that the test data all fall into the 95% interval of life distributions based on the modified fatigue damage accumulation model under the spectrum loading. The average life and the standard deviation based on the modified model both fall into the 95% interval estimation of test data, and the ratio of model to test is close to 1. The shape parameter obtained by the modified model is closer to 4 indicating that the modified model is more coincident with the fatigue test than the original model. In addition, as shown in Fig.5, the schematic PDFs of predicted fatigue life N_f and the test results are quite close. Therefore, it can be concluded that the modified model can precisely predict the fatigue damage and fatigue life under spectrum loading, and at the same time, properly reflect the stochastic nature and load sequence effects on fatigue behavior.



a) fatigue life distributions under 5-level spectrum loading.



b) fatigue life distributions under 3-level spectrum loading.

Figure 5: Fatigue life distributions under spectrum loading.

Moreover, the mean fatigue lives E and $N_{95/95}$ are also calculated by Palmgren-Miner's linear damage summation rule and are compared with those based on test data, as listed in Tab. 11.

Load level	E ($\times 10^6$)	R_E	$N_{95/95}$ ($\times 10^5$)	$R_{N95,95}$	*E fall into the* (E_1, E_2) interval
5	1.0209	0.5914	4.6588	0.5820	no
3	0.39686	0.8551	1.8111	0.8415	yes

Table 11: Comparisons of parameters based on Palmgren-Miner's linear damage summation rule with those of fatigue tests.

The results illustrate that the fatigue lives calculated by Palmgren-Miner's linear damage summation rule are too conservative, especially for the 5-level spectrum loading. The modified model is more accurate and reasonable than the Palmgren-Miner's rule in engineering applications, and can be easily expanded to the application of other metal materials. The fatigue damage calculation steps are summarized as follows.

1) Conduct the fatigue test under constant amplitude loading with 3 stress levels. At each stress level, the number of test specimens is not less than 6. According to the test data, the p-S-N curve is fitted.
2) The fatigue life distribution is assumed to obey the Weibull distribution, and the parameters are obtained based on the p-S-N curve.
3) Get the consistent index b corresponding to different stress levels through the large number of Monte Carlo sampling using Eq. (2).
4) Calculate the fatigue damage under spectrum loading using Eq. (10).

As for the application of the statistical fatigue damage accumulation model, two aspects should be noted. Firstly, the shape parameter of Al-alloy is different in the data processing. For the experiments, it is assumed to be 4 for both constant and variable amplitude loading conditions; however, for the model simulation, it is estimated by the maximum likelihood method based on the large number of Monte Carlo sampling (not equal to 4, as shown in Tab. 9 and Tab. 10. Secondly, the stress ratio R may be different for each load level in variable spectrum loading. The method based on p-S-N curve and constant life diagram could be applied [20]. The stress level at arbitrary R can be converted to the stress level at the R_0 under which p-S-N curve is available using the constant life diagram [21].

Conclusions

Fatigue damage and fatigue life prediction for structures under variable amplitude loading are discussed in this paper. Several conclusions are made and listed as follows.

The statistical damage accumulation model is based on Palmgren-Miner's linear rule. However, the critical damage is considered as a variable depending on fatigue life distribution rather than a constant. Its mean value equals to unit. Therefore, the model is made self-consistent by introducing a consistent index b determined by numerical simulation and a random disturbance Δ.

The load sequence effects are properly accounted in this model to accurately predict the fatigue life under variable amplitude loading. An exponential function exp$(-f_j)$ is used to slow down the fatigue damage accumulation rate of materials under the low stress levels. This function is dependent on the stress ratio, i.e. $S_{\max}^{j-1} / S_j$ and the multiplier v.

This fatigue damage accumulation model provides a quantitative approach to statistically calculate the fatigue damage and fatigue life by considering the load sequence effects. The predictions by the present model coincide quite well with experimental results, indicating that it can well demonstrate the probabilistic nature of fatigue behavior.

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NOMENCLATURE

b = self-consistent exponent dependent on the fatigue life distribution
b_j = the self-consistent exponent dependent on the fatigue life distribution under jth load level
Δ = a random variable with mean value equal to zero
d = Morrow's plastic work interaction exponent
D = damage value
D_B = damage caused by one loading block
D_{Bj} = damage caused by the jth level of stress
Dc = critical damage value
E = mathematical expectation
m = number of stress levels in a spectrum loading
n = cycle number in one loading block
n_i = cycle number under the ith stress level
n_j = cycle number under the jth stress level
n_f = cycle number to failure
N = fatigue life
$N_{95/95}$ = fatigue life with 95% reliability and 95% confidence level
N_i = fatigue life under the ith stress level
N_j = a random number obeying the distribution of fatigue life under jth stress level
N_k = the number of stress peak to cause failure under the constant stress amplitude of S_k
N_f = the predicted fatigue life under block loading
N_p = the fatigue life of the test sample with reliability of p
R_E = the ratios of mathematical expectation between model and test data
R_σ = the ratios of standard deviation between model and test data
R_{N95} = the ratios of $N_{95/95}$ between model and test data
S = the stress level of the constant amplitude loading
S_j = stress peak of the jth stress level
S_k = stress peak of the kth stress level
S_{max} = the maximum stress in the stress history
$S_{\max}^{j-1}$ = the maximum stress among the stress levels from S_1 to S_{j-1}
v = a constant depending on the material itself and the assumption of fatigue life distribution
a = shape parameter a of Weibull distribution
$\hat{\alpha}$ = the estimation of shape parameter
β = scale parameter of Weibull distribution
β_c = the estimation of characteristic life with confidence level of c
$\hat{\beta}$ = the estimation of characteristic life
μ = mean value of Weibull distribution
σ = standard deviation of Weibull distribution