On adaptive refinements in discrete probabilistic fracture models

J. Eliáš
Brno University of Technology, Faculty of Civil Engineering, Veveří 331/95, Brno, 60200, Czech Republic
elias.j@fce.vutbr.cz

ABSTRACT. The possibility to adaptively change discretization density is a well acknowledged and used feature of many continuum models. It is employed to save computational time and increase solution accuracy. Recently, adaptivity has been introduced also for discrete particle models. This contribution applies adaptive technique in probabilistic discrete modelling where material properties are varying in space according to a random field. The random field discretization is adaptively refined hand in hand with the model geometry.

KEYWORDS. Adaptivity; Discrete model; Probability; Random field.

OPEN ACCESS

Citation: Eliáš, J., On adaptive refinements in discrete probabilistic fracture models, Frattura ed Integrità Strutturale, 39 (2017) 1-6.

Received: 11.07.2016
Accepted: 12.09.2016
Published: 01.01.2017

INTRODUCTION

The adaptivity of model geometry has been originally developed for elastic problems [1,2] and later applied also in inelastic problems with localization [3,4]. The classical rigorous approach involves an error estimation, remeshing criterion, mesh re-generation and transfer of variables onto the new mesh. Recently, the adaptive concept was applied also in discrete modelling [5]. The goal of this work is to extend it for probabilistic discrete models.

Discrete models represent the material via collection of interconnected rigid bodies organized into a net structure. There are several versions of discrete models developed and used for many purposes. In case of simulating fracture in concrete, the lattice models are often employed [6-8]. These models represent the concrete meso-structure by projecting it onto the independently generated lattice. They are excellent in describing fracture phenomena, but applicable only for small laboratory specimens due to their extreme computational demands. Another group of discrete meso-level modelling approaches, sometimes called particle models, generates the network geometry directly according to the meso-structure of concrete [9,10]; typically one node for each mineral aggregate. We focus here on the latter group with geometry generated via Voronoi tessellation [11-14].

Though some reduction of computational cost in particle models is achieved when compared to the lattice models, further reduction would be desirable. It can be done by adaptive construction of the discrete geometry as described in [5]. Availability of adaptive refinement allows starting simulation with coarse discretization and refining it adaptively during the simulation run only in areas where needed.

In some applications of fracture simulations, it might be important to consider additional material randomness (besides the one covered by the random location of nodes in the discrete model) usually represented by a random field [15-18]. An extension of the discrete model by fluctuation of material parameters according to a random field was developed in [18,19]. In this contribution, the adaptive concept is extended for such probabilistic discrete fracture models.

PROBABILISTIC DISCRETE MODEL

The model uses random geometry to avoid directional bias that occurs in any regular structure. Domain of the modeled body is filled with nuclei with randomly generated positions. These nuclei are added sequentially with restricted minimal distance $l_{\min}$. The parameter $l_{\min}$ controls size of the discrete bodies and therefore it should correspond to the size of heterogeneities in the material. In concrete, this is typically a size of the mineral aggregates. Each of the nuclei will serve as one model node with associated six degrees of freedom, three translational and three rotational. The connectivity of the nodes is given by Delaunay triangulation. Dual diagram of Delaunay triangulation called Voronoi tessellation then creates geometry of the rigid bodies. Rigid bodies have common contact facets, which are perpendicular to their connections because of the Voronoi tessellation properties. There is a complex damage-mechanics based constitutive law used at the facets. Its deterministic version has been adapted from [9], where it is also described in detail. The main material parameters for fracture behavior are tensile strength, f_t, and tensile fracture energy, G_F.

The probabilistic extension of the model is elucidated in [19,20]. Here, only brief description of the probabilistic part is given. Both the tensile strength and fracture energy in tension are assumed to be governed by single random field H with mean value 1 and probabilistic distribution with Gaussian core and Weibull left tail. The correlation structure of the random field is given by square exponential function with single parameter, l_e, called the correlation length.

The strength and fracture energy of every model contact with centroid $\boldsymbol{c}$ are given by

$$f_t(\boldsymbol{c}) = \overline{f_t} H(\boldsymbol{c}) \tag{1}$$

$$G_F(\boldsymbol{c}) = \overline{G_F}\left[H(\boldsymbol{c})\right]^2$$

with $\overline{\mathrm{X}}$ being the mean value of the material parameter X. The square in the equation for fracture energy is added to preserve constant material characteristic length [20]. In the adaptive model, new contacts are created after every refinement. Therefore, the random field values at the new contact centers must be generated after every refinement. This is effectively done using kriging. Initially, standard Gaussian random field realizations ($\hat{H}$) are generated on points arranged in a regular grid with spacing $l_e/4$. Random field value at point $\boldsymbol{c}$ is then estimated using the optimal linear estimation method [21]

$$\hat{H}(\boldsymbol{c}) = \sum_{k=1}^{K} \frac{\xi_k}{\sqrt{\lambda_k}} \boldsymbol{\psi}_k^T \boldsymbol{C}_{cg} \tag{2}$$

and finally standard Gaussian field is transformed onto the Weibull-Gauss field ($\hat{H} \to H$) using isoprobabilistic transformation. Vector $\boldsymbol{\xi}$ collects realizations of K independent standard Gaussian variables, λ and $\boldsymbol{\psi}$ are K eigenvalues and eigenvectors of the grid covariance matrix and $\boldsymbol{C_{cg}}$ is the covariance vector between the grid points and point $\boldsymbol{c}$.

ADAPTIVITY

Only brief description of the adaptive concept in deterministic model is given. Deep elucidation is provided in [5]. The refinement criterion is intuitive. It is based on an average stress in the rigid bodies calculated using the fabric stress tensor. For rigid body associated with node i, the average stress components σ_{si} are

$$\sigma_{st} = \frac{1}{V}\sum_j F_s^{(j)} c_t^{(j)} \tag{3}$$

where j runs over all nodes in contact with node i, $\boldsymbol{F}$ is a vector of contact force, $\boldsymbol{c}$ is the centroid of the contact facet and V is a volume of the i-th rigid body. The Mazar's equivalent stress serves as measure of the stress level, σ_{eq}.

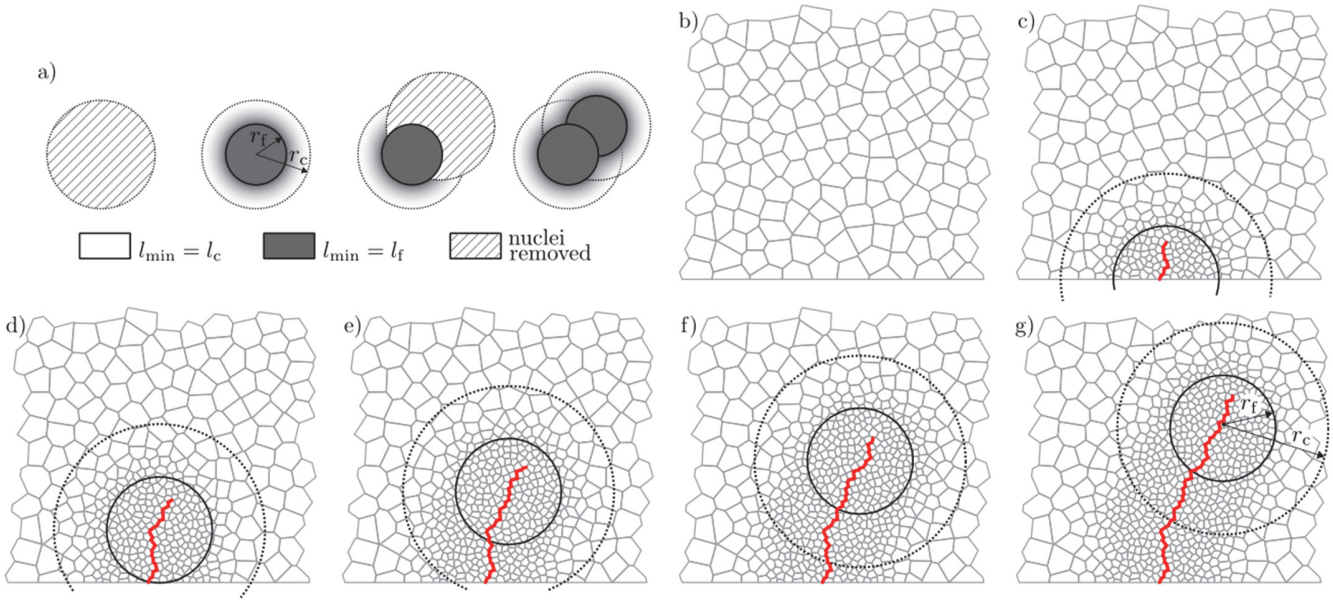

Figure 1: Adaptive refinement of discretization in steps; a) schematic explanation; b)-g) application to a 2D model.

In probabilistic model, the contacts have random strength. Assuming that the random field does not change too much within one discrete body of the model ($l_e < l_{min}$), it is reasonable to estimate strength of hypothetical newly created contacts within the i-th coarse discrete body by strength at node i at coordinates $\boldsymbol{x}_i$.

After every solution step, average stress tensors in all rigid bodies belonging to coarse discretization are evaluated and the Mazar's equivalent stresses are calculated. The refinement takes place whenever the equivalent stress exceeds chosen strength level γ

$$\frac{\sigma_{eq}}{f_t(\boldsymbol{x}_i)} > \gamma \tag{4}$$

The node associated with the rigid body satisfying Eq. (4) serves as a center of the refinement sphere. The safe value of parameter γ was determined as 0.7, i.e. whenever equivalent stress reaches 70% of the tensile strength, the refinement takes place.

The refinement is sketched in Fig. 1. All the nuclei inside the refinement sphere that does not belong to the fine discretization are removed. New nuclei are added into the refinement sphere according to the sequential algorithm described in the previous section. The parameter l_{min} controlling the discretization density changes based on two additional length parameters, r_f and r_c. The linear transition from coarse ($l_{min}=l_c$) to fine ($l_{min}=l_f$) discretization is included within the circular ring of outer (inner) radius r_c (r_f) in order to minimize the shape distortion of the bodies. If the transitional regime is omitted, the sharp change in discretization density would produce significantly elongated body shapes inducing directional bias and anisotropy.

NUMERICAL EXAMPLE

Performance of the proposed adaptive algorithm is demonstrated on simulation of four-point bending test with incorporated material randomness. The computer code used for calculation is an in-house software. The beam geometry is shown in Fig 2. The deterministic model parameters were taken from simulation of experimental series in three-point bending [22]. The average tensile strength is f_t=2.2 MPa, fracture energy in tension is G_F=35 J/m^2 and elastic modulus is 60 GPa. All these parameters are applied on the meso-level, they are not equal to the corresponding macroscopic properties of the model. The adaptive algorithm uses the following parameters: r_f=60 mm, r_c=120 mm, l_f=10 mm and l_c=30 mm. The parameters of the probabilistic extension are arbitrarily chosen according to [19]. The correlation length and the coefficient of variation of the random field is 80 mm and 0.25, respectively.

Three model types are used: (i) the fine model, that uses fine discretization everywhere from the beginning; (ii) the coarse model, that uses coarse discretization all the time; and (iii) the adaptive model, that starts with coarse discretization and refines it adaptively.

Fig. 3 shows on the left-hand side identical responses of one simulation using the fine model and one simulation using the adaptive model with the same refined meso-structure and also the same random field realization. The resulting crack patterns as well as the random field applied are shown in Fig. 4.

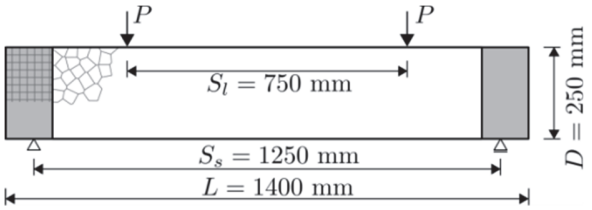

Figure 2: Dimensions of the simulated beam loaded in four-point bending.

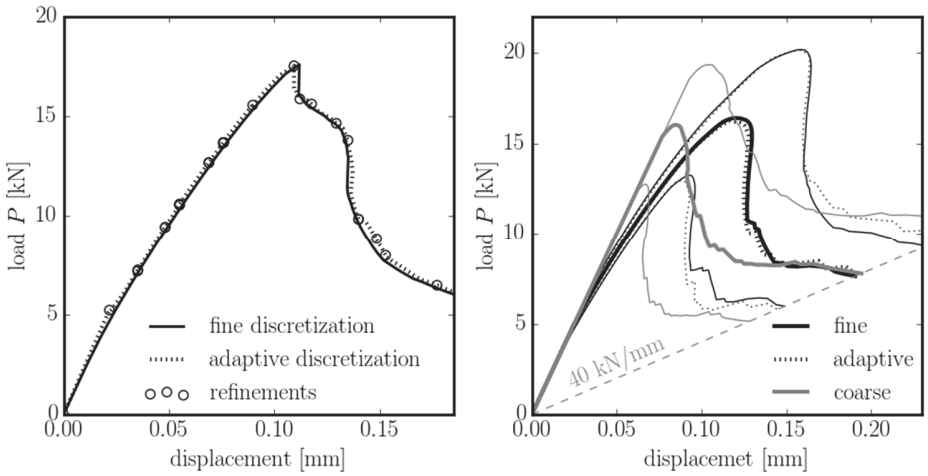

Figure 3: Left: Load-displacement response of one four-point-bending test simulation using the fine model and the adaptive model with the same refined meso-structure. Right: Average response of 30 simulation of four-point-bending test.

All three model types were then compared statistically. The same 30 realizations of the random field were used for every model type. The average responses together with standard deviations are shown in Fig. 3 on the right hand side. The fine and the adaptive model exhibit the same behavior while the coarse model deviates from them. In average, the computational time consumed by the adaptive model was only 47% of the time consumed by the fine model.

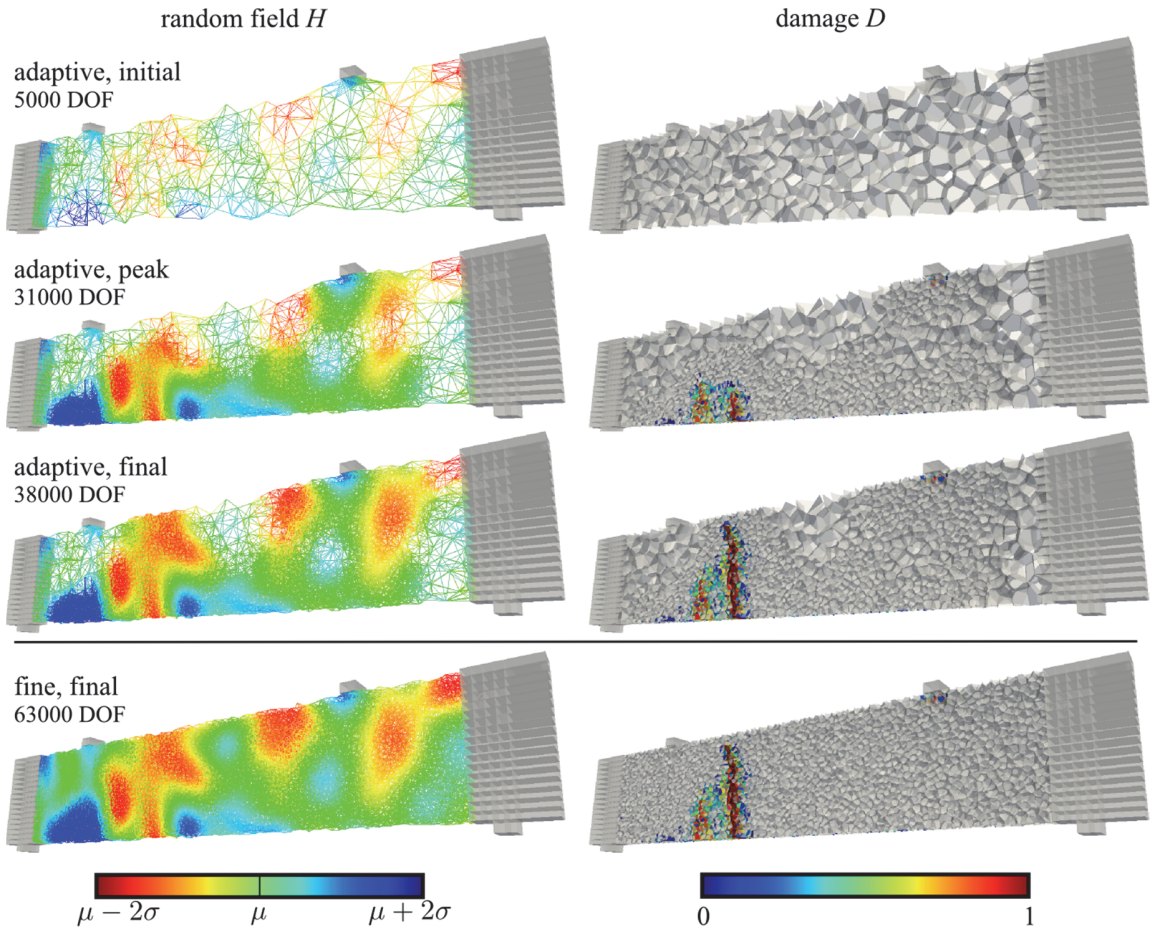

Figure 4: Damage patterns and random field discretizations developed during the simulation of four-point bending test for the adaptive and fine model.

Conclusions

The probabilistic discrete model has been extended by an adaptive technique that allows significant reduction of the computational time with no effect on the obtained results. The probabilistic model had both fracture energy and tensile strength assigned according to the random field. The random field discretization was adaptively refined on the run hand to hand with the discretization of the model. The adaptive algorithm was verified by simulating four-point bending test.

Usage of the adaptive concept is limited to the specific types of the discrete models that have elastic behavior independent on discretization density. Moreover, the presented concept is available only for static models. In dynamics, the translations and rotations and their first and second order derivatives cannot be computed from scratch and needs to be somehow estimated from replaced coarse mesh via some transfer algorithm.

Acknowledgement

The financial support provided by the Ministry of Education, Youth and Sports of the Czech Republic under the project LO1408 “AdMaS UP - advanced Materials, Structures and Technologies” under “National Sustainability Programme I” is gratefully acknowledged.

REFERENCES

[1] Babuška, I., Rheinboldt, W.C., A-posteriori error estimates for the finite element method. Int. J. Numer. Meth. Eng., 12 (1978) 1597-1615. DOI: 10.1002/nme.1620121010.

[2] Zienkiewicz, O.C., Zhu, J.Z., A simple error estimator and adaptive procedure for practical engineering analysis. Int. J. Numer. Meth. Eng., 24 (1978) 337-357. DOI: 10.1002/nme.1620240206.

[3] Selman, A., Hinton, E., Bičanič, N., Adaptive mesh refinement for localised phenomena. Comput. Struct., 63 (1997) 475-495. DOI: 10.1016/S0045-7949(96)00372-0.

[4] Patzák, B., Jirásek, M., Adaptive resolution of localized damage in quasi-brittle materials. J. Eng. Mech.-ASCE, 130 (2004) 720-732. DOI: 10.1016/S0045-7949(96)00372-0.

[5] Eliáš, J., Adaptive technique for discrete models of fracture. Int. J. Solids Struct., accepted for publication. DOI: 10.1016/j.ijsolstr.2016.09.008.

[6] Man, H.-K., van Mier, J.G.M., Damage distribution and size effect in numerical concrete from lattice analyses. Cement Concrete Comp., 33 (2011), 867-880. DOI: 10.1016/j.cemconcomp.2011.01.008.

[7] Sands, C.M., An irregular lattice model to simulate crack paths in bonded granular assemblies. Comput. Struct., 162 (2016) 91-101. DOI: 10.1016/j.compstruc.2015.09.006.

[8] Eliáš, J., Stang, H., Lattice Modeling of Aggregate Interlocking in Concrete. Int. J. Fracture, 175 (2012) 1-11. DOI: 10.1007/s10704-012-9677-3.

[9] Cusatis, G., Cedolin, L., Two-scale study of concrete fracturing behavior. Eng. Fract. Mech., 74 (2007) 3-17. DOI: 10.1016/j.compstruc.2015.09.006.

[10] Cusatis, G., Pelessone, D., Mencarelli, A., Lattice discrete particle model (LDPM) for failure behavior of concrete. I: Theory. Cement Concrete Comp., 33 (2011), 881-890. DOI: 10.1016/j.cemconcomp.2011.02.011.

[11] Eliáš, J., Le, J.-L., Modeling of mode-I fatigue crack growth in quasibrittle structures under cyclic compression. Eng. Fract. Mech., 96 (2012) 26-36. DOI: 10.1016/j.engfracmech.2012.06.019.

[12] Gedik, Y.H., Nakamura, H, Yamamoto, Y., Kuneida, M., Evaluation of three-dimensional effects in short deep beams using a rigid-body-spring-model. Cement Concrete Comp., 33 (2011) 978-991. DOI: 10.1016/j.cemconcomp.2011.06.004.

[13] Veselý, V., Frantík, P., Vodák, O., Keršner, Z., Localization of Propagation of Failure in Concrete Specimens Assessed by Means of Acoustic and Electromagnetic Emission and Numerical Simulations, Transactions of the VŠB – Technical University of Ostrava, Civil Engineering Series, 11 (2011) 1213-1962. DOI: 10.2478/v10160-011-0036-5.

[14] Frantík, P., Veselý, V., Keršner, Z., Parallelization of lattice modelling for estimation of fracture process zone extent in cementitious composites. Adv. Eng. Softw., 60-61 (2013) 48-57. DOI: 10.1016/j.advengsoft.2012.11.020.

[15] Georgioudakis, M., Stefanou, G., Papadrakakis, M., Stochastic failure analysis of structures with softening materials. Eng. Struct., 61 (2014) 13-21. DOI: 10.1016/j.engstruct.2014.01.002.

[16] Grassl, P., Bažant, Z.P., Random lattice-particle simulation of statistical size effect in quasibrittle structures failing at crack initiation. J. Eng. Mech.-ASCE, 135 (2009) 85-92. DOI: 10.1061/(ASCE)0733-9399(2009)135:2(85).

[17] Vořechovský, M., Sadílek, V., Computational modeling of size effects in concrete specimens under uniaxial tension. Int. J. Fracture, 154 (2008) 27-49. DOI: 10.1007/s10704-009-9316-9.

[18] Vořechovská, D., Vořechovský, M., Analytical and Numerical Approaches to Modelling of Reinforcement Corrosion in Concrete, Transactions of the VŠB – Technical University of Ostrava, Civil Engineering Series, 14 (2014) 20-30. DOI: 10.2478/tvsb-2014-0003.

[19] Eliáš, J., Vořechovský, M., Skoček, J., Bažant, Z.P., Stochastic discrete meso-scale simulations of concrete fracture: comparison to experimental data. Eng. Fract. Mech., 135 (2015) 1-16. DOI: 10.1016/j.engfracmech.2015.01.004.

[20] Eliáš, J., Kaděrová, J., Vořechovský, M., Interplay of probabilistic and deterministic internal lengths in simulations of concrete fracture. in: Saouma, V., Bolander, J., Landis, E. (Eds.) 9th International Conference on Fracture Mechanics of Concrete Structures, Berkley, USA, (2016). DOI: 10.21012/FC9.155.

[21] Li, C.-C., Der Kiureghian, A., Optimal discretization of random fields. J. Eng. Mech.-ASCE, 119 (1993) 1136-1154. DOI: 10.1061/(ASCE)0733-9399(1993)119:6(1136).

[22] Gregoire, D., Rojas-Solano, L.B., Pijaudier-Cabot, G., Failure and size effect for notched and unnotched concrete beams. Int. J. Numer. Anal. Met., 37 (2013) 1434-1452. DOI: 10.1002/nag.2180.