Focussed on Modelling in Mechanics

Inverse identification of the material parameters of a nonlinear concrete constitutive model based on the triaxial compression strength testing

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ABSTRACT. The aim of this paper is to perform the inverse identification of the material parameters of a nonlinear constitutive model intended for the modeling of concrete which is known as the Karagozian & Case Concrete model. At present, inverse analysis is frequently used because it allows us to find the optimum parameter values of nonlinear material models. When applying such parameters, the resulting response of the structure obtained from a computer simulation is very similar to the real response of the structure based on the related experimental measurement. This condition then undoubtedly constitutes one of the progressive steps to refine the current numerical approaches. For the purposes of the inverse analysis performed in this paper the experimental data was obtained from the triaxial compression strength tests carried out on the concrete cylinders.

KEYWORDS. Inverse analysis; Optimization; Objective function; Numerical simulation; Nonlinear concrete material model; Experimental data.

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INTRODUCTION

Using nonlinear material models of concrete to examine the behavior of concrete structures exposed to static or dynamic loading currently constitutes a major move within the effort to approach the real character of and processes in the material during computer simulations. Current modern computing systems based on the finite element method, such as ANSYS [1], LS-Dyna [2], and Atena [3], offer a comparatively wide variety of nonlinear material models to simulate different conditions across the entire spectrum of materials. According to their respective properties, these models are applicable in both static [4-7] and dynamic [8-12] numerical simulations.

The actual utilization of nonlinear material models of concrete within tasks of continuum mechanics nevertheless poses certain difficulties. One of such complications, forming the main subject of this paper, lies in the fact that the resulting computer simulation-based response of the structure is heavily dependent on the entered parameter values of the relevant nonlinear model of concrete. Thus, in any case where the parameter values of the employed material model were entered inappropriately, the eventual, computer-simulated response of the structure completely differs from the real response, and this condition constitutes an undesired effect in the given respect. To function properly, a large number of nonlinear models of concrete also require multiple material parameters, some of which carry only a mathematical meaning and therefore remain difficult to acquire due to the lack of a physical substance or can be acquired only via special testing. Such aspects then render the use of certain material models rather problematic; however, modern computing technology enables us to solve the above-mentioned drawbacks by means of inverse analysis, or, in other words, the inverse identification of material parameters.

The inverse analysis combines the numerical and experimental approaches with optimization procedures, and it advantageously facilitates finding the optimum parameter values of the employed material model in such a manner that the computer simulation-based response of the structure is as close as possible to the structure's real response obtained from the related experimental measurement. At present, the set of the most widely preferred techniques for the inverse identification of material parameters includes, among other tools, optimization methods based on the training of artificial neural networks [13]. Considering commercial computing systems, a very powerful instrument to perform inverse analysis currently appears to consist in the optiSLang program [14], which offers a robust algorithm comprising a broad spectrum of optimization procedures suitable for the inverse identification of material parameters [15, 16].

The aim of the research characterized in this paper is to execute the inverse identification of the material parameters of a nonlinear constitutive model, namely, the Karagozian & Case (K & C) Concrete model [17]. The relevant inverse analysis is carried out exploiting a load-displacement curve, one experimentally measured during the triaxial compression strength testing of concrete cylinders. Within the inverse analysis process, we utilize the interaction of nonlinear numerical simulations performed using LS-Dyna software (an explicit solver of the finite element method), with the optimization procedures implemented in the optiSLang program.

EXPERIMENTAL ANALYSIS

Generally, the entire inverse analysis process requires us to define the input data. In the inverse identification of the material parameters of nonlinear models, the basic input information consists in experimental data, which are most often formed by a loading curve defining the response of the real structure to static or dynamic loading.

Triaxial compression strength testing of concrete cylinders

Within this paper, the experimental data consisted in a load-displacement curve obtained from the multiple triaxial compression strength tests presented by the authors of reference [18]. Such testing of cylindrical concrete specimens is one of the methods to examine and verify the physical-mechanical properties of concrete. The dimensions of the concrete cylinders applied in each test were, invariably, 304.8 mm (height) and 152.4 mm (base diameter). The established ultimate uniaxial compressive strength of the concrete used to produce the test cylinders corresponded to 45.4 MPa. During the testing, each cylinder was compressed at a constant velocity, and the loading had a quasi-static character. For this paper, we chose the results for the temporally constant transverse pressure (confinement) of 7 MPa, from which the triaxial stress was induced.

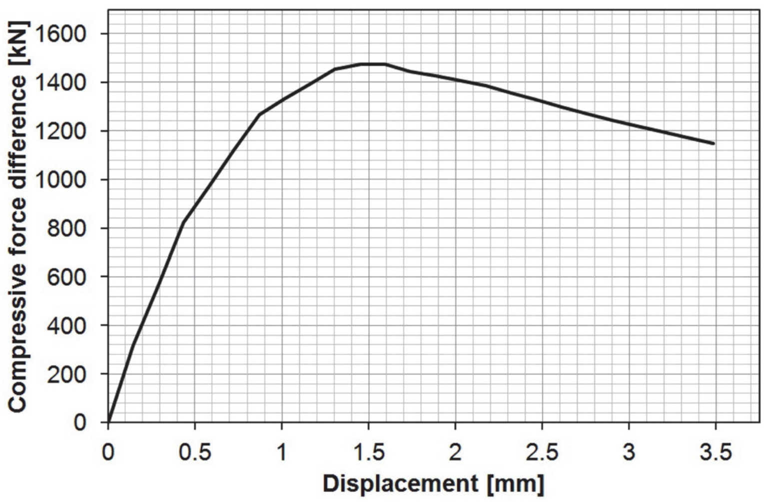

Figure 1: The experimentally-measured load-displacement curve.

Evaluation of the experimental results

The load-displacement curve based on the above-discussed triaxial compression strength tests of concrete is shown in Fig. 1. The curve indicates that, during the compressive loading, the concrete cylinder first exhibited linearly elastic behavior and then elasto-plastic behavior, respectively; further, it is also evident that, after the cylinder had reached its ultimate strength in triaxial compression, compressive strain softening began to occur. This process resulted from cracks disrupting the structure of the relevant concrete cylinder; however, due to the action of the temporally constant transverse pressure of 7 MPa, the total response of the concrete cylinder was very ductile.

NONLINEAR NUMERICAL ANALYSIS

Within the process of the inverse identification of material parameters, the nonlinear numerical analysis was carried out using the LS-Dyna computing system based on the explicit finite element method. To describe the nonlinear behavior of concrete, we selected the K & C Concrete material model, whose parameters were identified. The setting of the computational model, the parameters of the selected nonlinear model of concrete, the solver, and other settings required for the execution of calculations were all written with relevant keywords [17] into the LS-Dyna input file carrying the suffix *.k. Following the formerly defined experimental data, this file embodied the second part of the input information necessary for performing the inverse analysis in the optiSLang program.

Computational model

During the real triaxial compression strength test of a concrete cylinder, the cylindrical specimen was invariably positioned in a triaxial test chamber, between the pressure plates of the test press. For the purposes of the numerical simulations presented within this paper, the boundary conditions were simplified as follows:

- The actual modeling involved only the test cylinder, excluding the pressure plates, and was performed via eight-node 3-D structural finite elements (bricks) designated for an explicit analysis (see Fig. 2);
- the nodes of the lower base of the finite element model of the cylinder were pre-assigned zero displacements in all the directions (see Fig. 2);
- the nodes of the upper base of the finite element model of the cylinder were pre-assigned zero displacements in the horizontal directions (x and y) and linearly increasing displacements in the vertical direction (z), as is shown in Fig. 2. The vertical displacements of the base nodes then simulated the compression of the cylinder at a constant velocity;
- a temporally constant transverse surface pressure was applied directly to the model of the cylinder (see Fig. 2).

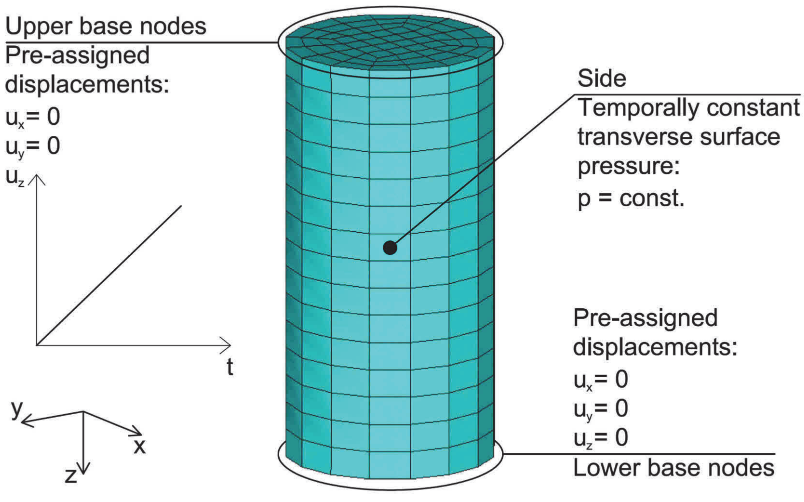

Figure 2: The computational model.

The initial stiffness of the cylinder established from the numerical simulations at simplified boundary conditions corresponded to the initial stiffness of the real cylinder during compression loading, and the assumed simplification thus could be considered correct. The material characteristics in the computational model were defined by means of the K & C Concrete material model.

Karagozian & Case Concrete model

The K & C Concrete model [19] is a three-invariant constitutive model based on three shear failure surfaces: an initial yield surface, a maximum shear failure surface and a residual failure surface. The shear failure surfaces are mutually independent, and their generalized mathematical notation can be expressed as follows [20]:

$$F_i(p) = a_{0i} + \frac{p}{a_{1i} + a_{2i}p} \tag{1}$$

where the index $i = y$ (initial yield surface), m (maximum shear failure surface), or r (residual failure surface). The variables a_{ji} ($j = 0, 1, 2$) are the parameters calibratable from the experimental data, and p denotes the pressure which depends on the first invariant of the stress tensor ($p = -I_1/3$).

The resulting failure surface is, within the model, interpolated between the maximum shear failure surface and either the initial yield surface or the residual failure surface according to the formulas:

$$F(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(F_m(p) - F_y(p)) + F_y(p)] \ \text{ for } \ \lambda \le \lambda_m \tag{2}$$

$$F(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(F_m(p) - F_r(p)) + F_r(p)] \ \text{ for } \ \lambda > \lambda_m \tag{3}$$

where I_1 is the first invariant of the stress tensor, J_2 and J_3 are the second and third invariants of the deviatoric stress tensor, λ denotes the modified effective plastic strain, $\eta(\lambda)$ represents the function depending on the modified effective plastic strain λ, and $r(J_3)$ is the scale factor in the form of the Willam-Warnke equation [21].

The model is implemented in the LS-Dyna software and enables us to consider the failure and various physical-mechanical properties of the material; thus, it is well-suited for the modeling of concrete. Within its parameters, the model facilitates considering also the effect of strain rate on the state of stress; however, this capability can be neglected in the model, making the model's response temporally independent. It then follows from this fact that the K & C Concrete model is suitable for simulating the response of a structure to not only fast dynamic but also quasi-static loading, and this property was utilized in the numerical analysis presented within this paper.

To ensure the correct functioning of the material model, the numerical values of 48 model parameters have to be defined, together with the values of 34 parameters of the equation of state [17]. The appropriate use of the model thus requires us to define the numerical values of 82 parameters, which, with respect to the character of some of the parameters, constitutes a rather difficult task.

No.	Parameter	Unit	Description
1	RO	[Mg/mm^3]	Mass density.
2	PR	[-]	Poisson's ratio.
3	$SIGF$	[MPa]	Maximum principal stress for failure.
4	$A0$	[MPa]	Cohesion.
5	$A2$	[MPa^{-1}]	Pressure hardening coefficient.
6	$A0Y$	[MPa]	Cohesion for yield.
7	$A2Y$	[MPa^{-1}]	Pressure hardening coefficient for yield limit.
8	$A2F$	[MPa^{-1}]	Pressure hardening coefficient for failed material.
9	$B1$	[-]	Damage scaling factor.
10 - 18	$P2$ - $P10$	[MPa]	Pressure 2 - Pressure 10.
19 - 28	$BU1$ - $BU10$	[MPa]	Bulk unloading modulus 1 - Bulk unloading modulus 10.

Table 1: The identified parameters of the K & C Concrete model.

A significant number of the model parameters remain constant despite alterations in the physical-mechanical properties of the material; thus, these parameters need not to be identified, and we utilized this condition within the research discussed herein. However, even after omitting the constant parameters, the numerical values of 28 parameters still wait to be defined, and as they change according to the physical-mechanical properties of the material, this step too is somewhat problematic to perform, especially in view of the character of certain parameters. A survey of these 28 parameters, including the applied units, is given in Tab. 1. Within this paper, these parameters were identified to find, in the available range of options, the most accurate approximation of the experimental data via numerical simulation.

INVERSE ANALYSIS

The inverse identification of material parameters was performed using the optiSLang program and comprised two stages; within the former one, we analyzed the sensitivity of the identified material parameters to the required reference response, and the latter stage was then focused on the optimization.

Sensitivity analysis

As already indicated by its name, the procedure [22, 23] was aimed at analyzing the sensitivity of the variable input parameters to the required reference response, and, subsequently, at reducing the number of the parameters in the design vector to the necessary minimum. The variable input parameters consisted of the identified material parameters of the K & C Concrete model, and the reference response comprised points lying on the experimentally-measured load-displacement curve. Fig. 3 shows the load-displacement curves obtained via numerical simulations for the boundary values of the identified material parameters from their initial range of the variability. These curves constituted the upper and lower bounds, meaning that they enclosed the experimentally-measured load-displacement curve (see Fig. 3), and both of them were based on the test calculations. The above-mentioned initial range of the variability of the individual input variables was also modified within the sensitivity analysis.

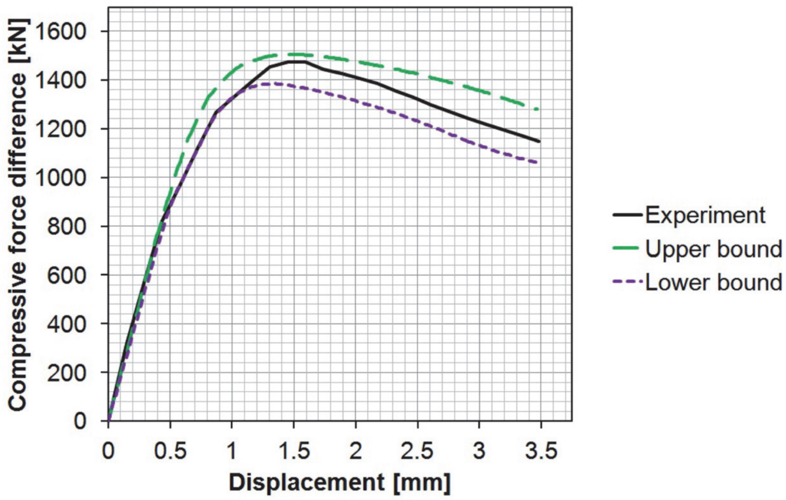

Figure 3: The boundary load-displacement curves.

The actual sensitivity analysis was carried out via the ALHS (Advanced Latin Hypercube Sampling) statistical method [24]; based on this procedure, we generated 500 random realizations of the design vector, which sufficiently covered the given design space. The results produced by the sensitivity analysis indicated that only 9 identified material parameters out of the total of 28 exerted major influence on the resultant form of the load-displacement curve. The initial design vector, which comprised all the original 28 identified parameters, expressed as:

$$\mathbf{X}_{\mathbf{orig}} = \{RO, PR, SIGF, A0, A2, A0Y, A2Y, A2F, B1, P2-P10, BU1-BU10\}^T \tag{4}$$

could eventually be reduced for the subsequent optimization process, assuming the form:

$$\mathbf{X}_{\mathbf{red}} = \left\{PR, SIGF, A0, A2, A2F, B1, P2, P3, BU1\right\}^T \quad (5)$$

The design vector reduction was then utilized during the optimization process as it shortens the computational time and saves the hard disk space without significantly affecting the achievement of the desired results. The reduced design vector $\mathbf{X}_{\mathbf{red}}$ already included only the 9 parameters having a marked impact on the numerical simulation results.
The values of the pre-optimized material parameters and objective function (*LSM*) obtained from the best random realization generated within the sensitivity analysis are presented in Tab. 2.

No.	Parameter	Unit	Sensitivity analysis	Evolutionary Algorithm
1	*RO*	[Mg/mm^3]	$2.468 \cdot 10^{-9}$	$2.400 \cdot 10^{-9}$
2	*PR*	[-]	0.1632	0.1677
3	*SIGF*	[MPa]	3.4787	3.2092
4	*A0*	[MPa]	13.2501	13.9527
5	*A2*	[MPa^{-1}]	$1.755 \cdot 10^{-3}$	$1.846 \cdot 10^{-3}$
6	*A0Y*	[MPa]	10.613	10.130
7	*A2Y*	[MPa^{-1}]	$5.568 \cdot 10^{-3}$	$5.670 \cdot 10^{-3}$
8	*A2F*	[MPa^{-1}]	$2.715 \cdot 10^{-3}$	$2.820 \cdot 10^{-3}$
9	*B1*	[-]	1.5425	1.4752
10	*P2*	[MPa]	23.1475	22.7979
11	*P3*	[MPa]	49.659	49.170
12	*P4*	[MPa]	80.662	93.030
13	*P5*	[MPa]	170.923	176.750
14	*P6*	[MPa]	232.417	266.590
15	*P7*	[MPa]	372.124	378.220
16	*P8*	[MPa]	527.117	578.620
17	*P9*	[MPa]	3362.560	3378.180
18	*P10*	[MPa]	4812.750	5166.940
19	*BU1*	[MPa]	16147.867	18220.000
20	*BU2*	[MPa]	15205.699	17719.000
21	*BU3*	[MPa]	16277.413	17967.000
22	*BU4*	[MPa]	19149.825	18871.000
23	*BU5*	[MPa]	20935.411	22450.000
24	*BU6*	[MPa]	24486.102	26047.000
25	*BU7*	[MPa]	28990.083	29627.000
26	*BU8*	[MPa]	32188.319	32338.000
27	*BU9*	[MPa]	65782.085	72755.000
28	*BU10*	[MPa]	81950.750	88596.000
-	*LSM*	[kN^2]	10312.759	6601.210

Table 2: The resulting identified material parameter values of the K & C Concrete model, obtained from the inverse analysis.

Optimization

The aim of the optimization process consisted in finding such identified parameter values at which the value of the objective function was minimal. The objective function utilized within the entire inverse analysis process was defined as a sum of squares by the formula:

$$LSM = \sum_{i=1}^{n} \left(y_i - y_i^* \right)^2 \tag{6}$$

where, for y_i, we substituted the loading force values obtained from the numerically-simulated load-displacement curve at the given deformations, and y_i^* was substituted with the loading force values obtained from the experimentally-measured load-displacement curve at the same deformations. As already mentioned, the objective function was minimized during the optimization process ($LSM \rightarrow$ min), meaning that we sought such material parameter values at which the load-displacement curve obtained from the numerical simulation exhibited the smallest possible deflection from the reference load-displacement curve produced by the experiment; in other words, we sought such values at which the sum of squares was minimal. It then follows from this description that the inverse analysis was based on minimizing the sum of squares (the Least Squares Minimization) [15].

As pointed out above, the optimization involved only those material parameters that were part of the reduced design vector. The remaining parameters were defined by the constant values from their initial range of the variability. The optimization of the parameters was performed using the optimization procedure known as Evolutionary Algorithm (EA) [14], an optimization approach that exploits processes inspired by biological evolution, including, for example, reproduction, mutation, and recombination. More concretely, we utilized in this context the Evolutionary Algorithm used for global optimization, with the 10 best realizations acquired within the sensitivity analysis serving as the starting point for the discussed algorithm.

The values of the identified material parameters provided by the best generation of the Evolutionary Algorithm are, together with the relevant minimum value of the objective function, presented in Tab. 2.

Fig. 4 below compares the load-displacement curve obtained via the numerical simulation, in which we applied the optimized parameter values of the K & C Concrete model from the Evolutionary Algorithm (EA), with the experimentally-measured load-displacement curve. It is then obvious from the representation that the parameters of the selected material model were identified very accurately because the results of the numerical simulation ensure a very good approximation of the experimental data.

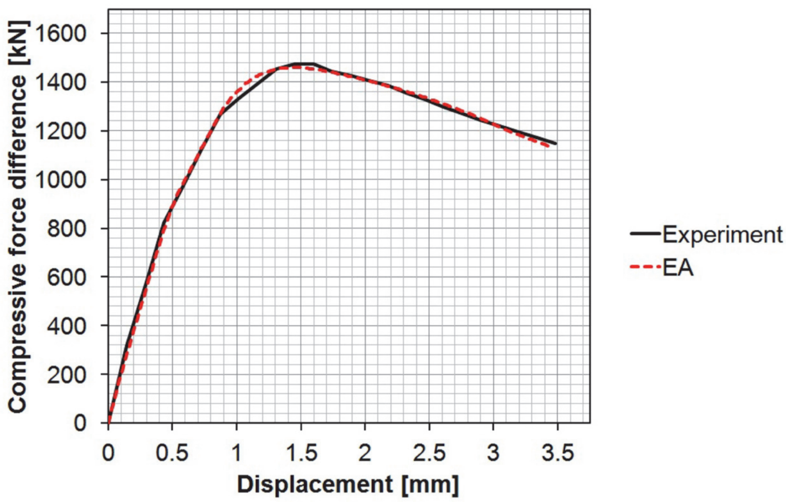

Figure 4: The load-displacement curve for the optimized parameter values compared with the experimental load-displacement curve.

CONCLUSION

This paper was based on performing an inverse analysis to identify the material parameters of the Karagozian & Case Concrete constitutive model. The inverse analysis was carried out utilizing the load-displacement curve experimentally measured during the triaxial compression strength testing of concrete cylinders. The actual results of the inverse analysis conducted by the authors of this paper showed that the K & C Concrete model is a suitable tool to describe the nonlinear behavior of concrete in triaxial compression, and they also indicated that, in any case where the parameters of the model are conveniently selected and entered, we can obtain a very good approximation of the experimental data. Importantly, this claim is proved by the outcome of the numerical simulation performed using the resulting identified parameter values of the material model. The discussed results then approximated the applied experimental data with a high degree of accuracy. Advantageously, the product of the procedure can be exploited for further research concerning the nonlinear numerical response of concrete structures.

Although the experimental data approximation yielded very good accuracy, it has to be pointed out that there still is the possibility of reaching an even better performance rate via operations such as local optimization, the application of an optimization procedure different from that characterized in this paper, increasing the number of the design vector generations, or using a non-reduced design vector within the optimization process. These aspects, which could further improve the accuracy of experimental data approximation and thus lead us towards obtaining more refined values of identified material parameters, will be examined within future research planned by the authors of this paper.

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REFERENCES

[1] ANSYS, ANSYS Mechanical Theory Reference Release 15.0, (2014).
[2] LS-DYNA, Theory Manual, Livemore Software Technology Corporation, Livemore, California, (2016).
[3] ATENA Program Documentation, Cervenka consulting Ltd., (2013).
[4] Kala, J., Hradil, P., Bajer, M., Reinforced concrete wall under shear load – Experimental and nonlinear simulation, International Journal of Mechanics, 9 (2015) 206-212.
[5] Hradil, P., Kala, J., Analysis of the shear failure of a reinforced concrete wall, Applied Mechanics and Materials, 621 (2014) 124-129.
[6] Hokeš, F., Selected Aspects of Modelling of Non-Linear Behaviour of Concrete During Tensile Test Using Multiplas Library, Transactions of the VŠB - Technical University of Ostrava, Civil Engineering Series, 2(15) (2015).
[7] Sucharda, O., Brozovsky, J., Numerical modelling of reinforced concrete beams with fracture-plastic material, Frattura ed Integrita Strutturale, 30 (2014) 375-382. DOI: 10.3221/IGF-ESIS.30.4
[8] Kala, J., Hušek, M., Improved Element Erosion Function for Concrete-Like Materials with the SPH Method, Shock and Vibration, 2016 (2016) 1-13.
[9] Wu, M., Chen, Z., Zhang, C., Determining the impact behavior of concrete beams through experimental testing and meso-scale simulation: I. Drop-weight tests, Engineering Fracture Mechanics, 135 (2015) 94-112.
[10] Král, P., Kala, J., Hradil, P., Verification of the Elasto-Plastic Behavior of Nonlinear Concrete Material Models, International Journal of Mechanics, 10 (2016) 175-181.
[11] Cichocki, K., Domski, J., Katzer, J., Ruchwa, M., Static and Dynamic Characteristics of Waste Ceramic Aggregate Fibre Reinforced Concrete, Transactions of the VŠB - Technical University of Ostrava, Civil Engineering Series, 2(15) (2015).
[12] Králik, J., Králik Jr., J., Seismic analysis of reinforced concrete frame-wall systems considering ductility effects in accordance to Eurocode, Engineering Structures, 31(12) (2009) 2865-2872. DOI: 10.1016/j.engstruct.2009.07.029.
[13] Lehký, D., Novák, D., Inverse reliability problem solved by artificial neural networks, In: Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures, New York, USA, (2013) 5303-5310.
[14] optiSLang, Methods for multi-disciplinary optimization and robustness analysis, Dynardo GmbH, Weimar, Germany, (2014).
[15] Most, T., Identification of the parameters of complex constitutive models: Least squares minimization vs. Bayesian updating, Reliability Conference in München, (2010).

[16] Hokeš, F., Kala, J., Krňávek, O., Nonlinear Numerical Simulation of a Fracture Test with Use of Optimization for Identification of Material Parameters, International Journal of Mechanics, 10 (2016) 159-166.
[17] LS-DYNA, Keyword User's Manual, Livemore Software Technology Corporation, Livemore, California, (2016).
[18] Joy, S., Moxley, R., Material characterization, WSMR-5 3/4-inch concrete, Report to the Defense Special Weapons Agency, USAE Waterways Experiment Station, Vicksburg, MS, (1993) (limited distribution).
[19] Malvar, L. J., Crawford, J. E., Wesevich, J. W., Simons, D., A plasticity concrete material model for Dyna3d, International Journal of Impact, 19 (1997) 847-873.
[20] Youcai, W., Crawford, J. E., Magallanes, J. M., Performance of LS-DYNA Concrete Constitutive Models, 12th International LS-DYNA Users Conference, Detroit, (2012).
[21] Chen, W. F., Han, D. J., Plasticity for structural engineers, Springer-Verlag, New York, (1988).
[22] Kala, Z., Kala, J., Škaloud, M., Teplý, B., Sensitivity analysis of the effect of initial imperfections on the (i) ultimate load and (ii) fatigue behaviour of steel plate girders, Journal of Civil Engineering and Management, 11 (2005) 99-107.
[23] Tvrdá, K., Optimal Design, Reliability and Sensitivity Analysis of Foundation Plate, Transactions of the VŠB - Technical University of Ostrava, Civil Engineering Series, 2(15) (2015).
[24] Huntington, D. E., Lyrintzis, C. S., Improvement to limitations of Latin hypercube sampling, Probabilistic Engineering Mechanics, 13 (1998) 245-253.